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APPLIED MECHANICS REVIEWS

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RECENT DEVELOPMENTS IN ELASTIC WAVE PROPAGATION

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INTRODUCTION AND SCOPE

Interest in elastic waves has grown considerably in recent years. Aside from the fact that this subject offers intrigue and challenge, there are several practical reasons for the expansion. One of the strongest, at least from the mechanics and engineering point of view, has been the continually growing need for information on the performance of structures subjected to high rates of loading. In geophysics the expanding research activity in elastic waves has also had strong underlying practical reasons, such as the need for more accurate information on earthquake phenomena and improved prospecting techniques. Seismologists are also currently concerned with the important nuclear detection problem. Developments in the related fields of acoustics and electromagnetic waves, and in applied mathematics in general, have also influenced the interest and progress made in the study of elastic waves. Last, but not least, the electronic computer has been of considerable influence. As in many other fields, it has given numerical information for otherwise intractable problems.

Wave propagation problems in elastic media are, in general, exceedingly more complex when they involve boundaries than otherwise. The complexity stems from partial mode conversion of a wave upon reflection from a boundary (like compression into compression and shear) and multiple reflections of the so-created waves between neighboring boundaries. The latter is responsible for dispersion which is characterized, in the case of harmonic waves, by a frequency or phase velocity dependence on wavelength. Dispersion is responsible, for instance, for the change in shape of a pulse as it travels in a bounded elastic solid in one-dimensional propagation. The bulk of modern day study of elastic waves is concerned with problems involving boundaries and dispersion.

The broad interest in this subject makes it clear that no one general survey could do it justice. In restricting this survey the author has focused on transient wave propagation in the homogeneous, isotropic, linear elastic solid in a vacuum. Further, only the solid of infinite or semi-infinite extent, in the propagation direction, has been considered. Time and space imposed further restrictions on literature coverage within these rather broad bounds. It was therefore natural for this author to draw most heavily on papers representing contributions from the Western countries. The body of the article is broken down into sections on the rod, plate, cylindrical shell, half-space, and infinite medium. Some semblance of continuity with the earlier work in the field is given through the existing surveys mentioned in these sections. In drawing on some earlier works this article serves to complement rather than duplicate the coverage given in certain of these surveys. General literature coverage through 1959 may be assumed, and into early 1960 for the more readily accessible journals.

RODS

The cylindrical rod of circular cross section is a classical example of a medium involving the complexities of dispersive propagation. Its dispersive nature was first pointed out by Pochhammer (1) in 1876. Pochhammer's solutions of the equations of motion from the linear theory of elasticity (frequently called the "exact" theory), for the cases of compressional, flexural, and torsional waves in an infinite rod, had the form of traveling harmonic wave trains. The requirement of a traction free rod surface generated the frequency equations for these cases. The real roots or branches of each of these equations are infinite in number. Each of these branches (sometimes called a mode of wave transmission) is a relation between frequency, or phase velocity, and real wave number over the whole spectrum of the latter. Pochhammer's frequency equations are basic to both the steady vibration and transient wave propagation problems of the rod.

Further contributions to our understanding of wave propagation in an elastic rod (plate, and cylindrical shell) have come not only from exact theory work, but approximate theory and experimental studies, and combinations of these as well. The strong interest in approximate theories exhibited in the literature stems from the complexity of the exact equations. Attempts are made in these theories to retain the essential physics of the exact equations along with the simplifying features of the elementary theories such as one-dimensionality, plane sections remain plane, etc. Comparison is usually made between the exact and approximate theory modes of transmission to ascertain the range (frequency and wavelength) of validity of the latter. The goal is usually a good approximation up to moderately high frequencies and out to moderately short waves.

Pochhammer's work is discussed in detail by Love (2) and Pfeiffer (3). Later work on the rod problem is reviewed by Schoch (4), Kolsky (5), Davies (6, 7), and Ewing, Jardetsky, and Press (8). More recent work is reviewed by Abramson, Plass, and Ripperger (9), and Kolsky (10).

Compressional Waves. Initial work with Pochhammer's frequency equation for compressional (axially symmetric) waves

was concerned with the numerical evaluation of the lower modes of transmission and the physics of the waves they governed. Chiefly through the efforts of Davies (11) it was established, for inputs of moderate high frequency density, that the faster traveling low frequency-long waves from the lowest mode could account for the important features of the transient disturbance at rod stations remote from the source. Davies also showed the extreme radial variation of the stresses and displacements across the rod section for the shorter waves from the lowest mode. Later Holden (12) investigated the higher modes, noting the qualitative agreement between the nature of compressional waves in a rod and those in an infinite plate (in plane strain), i.e., the similarity in form of the Pochhammer and Rayleigh-Lamb frequency equations. Holden gave curves for the first five modes. He also pointed out important general features of the high mode spectrum such as the asymptotic approach of the phase velocity, at high frequency and short wavelength, to the velocity of shear waves in an infinite medium.

Adem (13) evidently was the first to point out that the Pochhammer frequency equation also governed waves associated with complex wave numbers. He noted such waves would be important near the rod origin (or end) since they obviously involved spatial attenuation. Complex wave numbers are a topic of much current activity in both rods and plates. Adem, assuming large wave number, showed there are an infinite number of such roots for compressional waves. Recently Onoe (14a) [see also (14b)] has shown that complex wave numbers are a general feature of the Rayleigh-Lamb frequency equation for compressional and flexural (symmetric and antisymmetric) waves in an infinite plate. He has shown that parts of more general branches, corresponding to complex wave numbers, emanate from points of zero slope in the real and imaginary parts of these branches. The close analogy between compressional waves in the rod and plate then, implies such features would probably also be true of Pochhammer's frequency equation. Later work by Mindlin and McNiven (15) gives curves for the first three general branches of the Pochhammer frequency equation for compressional waves for small wave number, showing the imaginary and complex portions involved. Mindlin and McNiven point out that Oliver (16) has shown experimentally the existence of edge waves corresponding to complex wave numbers. These waves gradually lose their energy to the rod interior, through coupling with other propagating waves; hence the name, leaking mode.

Quite recently formal mathematical solutions have been written, in accord with the exact theory, for certain basic semi-infinite rod problems involving time-dependent mixed end conditions. Multi-integral transforms, and residue theory for partial inversion, have yielded the solutions. Far field numerical information was obtained with the aid of the stationary phase or saddle-point methods of integration. Skalak (17) solved the longitudinal impact problem (end step uniform axial velocity, zero shear stress) with this technique. He found the long waves from the lowest (real) mode formed the main contributions to the strains, in agreement with Davies' (11) findings for the far field. Skalak's approximate solution has the form of simple integrals of the Airy function, representing a non-decaying (with time), dispersive disturbance. Skalak also noted the simple Poisson's ratio coupling between the strains (elementary theory behavior in the far field). The transform method given by Folk, Fox, Shook, and Curtis (18a) applies generally to end conditions of the mixed type. They applied it to the case of end pressure shock under radial constraint, getting results equivalent to those in Skalak's problem for the main contribution. The contributions corresponding to shorter waves from the first two modes were also evaluated in (18a). The authors in (18a) point out their technique will not yield solutions for problems with nonmixed end conditions. This still leaves the important pressure shock problem (end step axial stress, zero shear stress) unsolved in the exact theory. In an experimental counterpart of (18a) Fox and Curtis (18b) extended earlier shock tube experiments by Curtis. This experiment is the analog of the theoretical pressure shock problem. The good agreement found in (18b) between measured surface strains and the approximations in (18a) implies the far field solution is independent of the lateral boundary condition at the source; an important point, in view of the fact that the exact theory pressure shock problem is unsolved.

Redwood (19) has recently contributed a solution that can explain the propagation of a high-frequency compressional pulse applied to the end of a semi-infinite rod (and for the related case in a semi-infinite plate). Earlier Hughes, Pondrom, and Mims [see (9)] showed such a pulse splits up into a main signal, traveling with almost the infinite medium dilatational velocity, and a train of equally spaced secondary signals behind it. Redwood points out the continuous group velocity spectrum (from zero to dilatational wave velocity) of the Pochhammer frequency equation would not appear to predict the separated signals. His special wave train solution of the exact equations leads to a modification of Pochhammer's frequency equation. A high-frequency approximation then, based on complex wave numbers, does predict the separated signals and their observed spatial attenuation. Redwood's paper and (9) serve as good sources for other earlier work on this problem.

Davies (11) showed the usefulness of approximate theories in studying transient wave propagation in a rod with his solution of Love's equation [see (2), pg. 428] for the pressure shock problem. Love's equation improves the elementary theory (one-dimensional wave equation) by also considering the radial inertia of the rod element. The resultant one mode of wave transmission is in good agreement with the lowest mode of the exact theory for the low frequency-long waves. It is therefore not surprising, from what was written earlier, that Davies found good agreement of his solution and the observed far field response. Mindlin and Herrmann (20) gave a theory for compressional waves in a rod which incorporates, in addition to radial inertia, radial shear of the rod element. This theory approximates the first two modes (longitudinal and radial) of the exact theory (and the coupling between them), comparing well with the latter for low frequency-long waves. The author (21a) evaluated, with the aid of the Laplace transform, contour, and numerical integration, the problem of a semi-infinite rod subjected to a radially-constrained end pressure shock in accord with the Mindlin-Herrmann theory. This evaluation and a related stationary phase solution, showed the predominance of the theory's fundamental period of radial vibration (corresponding to the cut-off frequency at infinitely long waves of the radial mode). Related experimental shock tube work by the author and Nisewanger (21b), in which axial strain and radial displacement response were measured, showed the tail of the disturbance at a station had such character. It was also clear, however, that greater accuracy in predicting this long period-long wave response would require other higher modes of transmission. The experiments also gave evidence of higher mode activity in the early response, particularly for the near field. More recently the author (22) has given (with the aid of asymptotic expansions of contour integrals) approximate far field solutions for the longitudinal and pressure shock problems, showing their equivalence according to the Mindlin-Herrmann and Love theories, and basic equivalence with the exact theory results (17, 18a). The latter implies the exact theory solutions for the pressure shock and longitudinal impact problems would be the same. (22) shows the excellent agreement between these far field solutions (exact and approximate theory) and the corresponding radial and axial strain records taken from (21b).

Mindlin and McNiven (15) have recently contributed a higher-order theory taking into account coupling between longitudinal, radial, and axial shear modes of compressional motion, which these authors point out can occur at moderately high frequencies. The frequency spectrum of the theory has

three branches with real, imaginary, and complex parts, in good agreement with the corresponding branches of the exact theory for small wave number.

Flexural Waves. The development of information from Pochhammer's frequency equation for flexural waves followed a pattern similar to that for compressional waves. Davies (11) analyzed, with the aid of a stationary phase solution, the propagation of a flexural pulse on the basis of the lowest mode, pointing out that moderately high frequency short waves form the leading disturbance in contrast to the low frequency long waves for the compressional case. Abramson (23) recently gave the numerical evaluation of the second and third modes. In a later paper Ripperger and he [see (9)] made use of the second mode to extend Davies' description of the propagating flexural pulse. Important is the fact that the second mode governs earlier arriving moderately high frequency short waves than the first. More recently Pao and Mindlin (24) have given an approximate method, not requiring detailed numerical computation, for constructing in detail any of the branches. The method employs a simpler grid of curves (called bounds), which the actual branches will not cross except at certain intersection points, and information on the asymptotic behavior of the branches at long and short-wave length. Pao and Mindlin discuss the characteristics of the first four and higher modes.

Curtis and Devault (25) have extended the exact theory solution technique in (18a) to the problem of a semi-infinite rod subjected to a sudden flexural disturbance, end conditions again being of the mixed type. In this more complicated problem finite sine and cosine transforms are also needed in the method to eliminate the circumferential variable in the problem. Curtis and Devault noted the main contributions to the flexural disturbance came from the lowest mode, and their shock tube experimental results agreed.

Approximate theory work was primarily devoted to studies employing the well-known Timoshenko beam in which, in addition to elementary bending, the effects of shear force and rotatory inertia of the beam element on the displacement are taken into account. The resulting two modes of wave transmission are in good agreement with the corresponding exact theory modes up to moderately high frequencies and out to moderately short waves. As a result the Timoshenko theory predicts with good accuracy the beam response to a pulse with moderate high frequency density. It is applicable to rod (or beam) sections other than circular.

As in the case of the approximate compressional wave theories, the Timoshenko theory afforded the opportunity to derive and analyze solutions for infinite and semi-infinite beam problems involving transient inputs. Techniques employing integral transforms and contour integration proved fruitful in getting and evaluating exact solutions of this type, and several authors contributed. Uflyand (26) first showed the Laplace transform and contour integration would yield exact traveling wave solutions for the theory, and gave the displacement solution for the problem of an infinite beam subjected to a step shear load. Dengler and Goland (27) pointed out that Uflyand's solution was based on incorrect boundary conditions under the load. They solved the same problem, avoiding the question of boundary conditions by working with the nonhomogeneous Timoshenko equation and a limiting distributed impulsive load. The author (28a) modified Uflyand's method, and gave meaning to his solution by employing the known coupled equations of the Timoshenko theory and the correct boundary conditions. He (28b) also used this method to solve the problem of a semi-infinite beam subjected to a half sine moment input on its end. Later Goland, Wickersham and Dengler (29) gave a correction to the solution derived in (27), evaluated it numerically, and showed it compared favorably with experimental results (square beam response to falling steel ball impact) provided very high frequency activity in the latter was ignored (in agreement with the high frequency-short wave restrictions on the theory). Boley and Chao (30) used a technique similar to that in (28a, 26) to derive and evaluate a number of solutions for semi-infinite Timoshenko beams corresponding to a variety of end loadings. They compared the solutions with related results from the elementary beam theory (Bernoulli-Euler) and noted the shear force response was in good agreement only for the later times at a station. This agrees with the fact that the two theories are only in agreement for the very low frequency-long waves of the lowest mode, these being the relatively slower traveling groups represented.

Other solution techniques were also productive. Leonard and Budiansky (31) used a numerical procedure based on the method of characteristics to derive traveling wave solutions for various infinite (and finite) Timoshenko beam problems. For mathematical simplicity, however, the solutions were all based on the special (nonphysical) case of equality of the two characteristic wave velocities of the theory (compressional and modified shear). Plass and Steyer (32) extended the technique in (31) to the more general case of different characteristic velocities. Here again numerical results, for semi-infinite beam problems involving various end loadings of half-sine time character, showed good agreement with experimental results except where the pulses were very short. The experiments, due to Ripperger [see (9)], involved an eccentrically impacting ball on the beam end.

Anderson (33) used the stationary phase method to study the moment and shear force amplitude spectra for the cases of concentrated moment and shear force impulse, centrally applied to an infinite Timoshenko beam. Interesting was the reciprocity Anderson found between the contributions of the two modes in the two cases. For instance, in the case of moment input the largest moment amplitudes were associated with the high frequency short waves of the upper mode, whereas in the case of shear force input the largest shear force amplitudes were associated with the high frequency short waves of the lower mode. Jones (34) derived a long time solution for Uflyand's problem with the aid of the Fourier transform and stationary phase method. His spatial plot of moment amplitude (for fixed long time) from both modes of the theory shows the maximum disturbance occurs near the source, being associated with the slow traveling low frequency-long waves of the lowest mode (in agreement with Anderson's findings). Jones showed results from the Bernoulli-Euler theory compared well in this region, which might have been expected from the good agreement of the two theories for low frequency long waves. Jones also found sizeable amplitudes associated with the leading moderately high frequency short waves from the neighborhood of the group velocity maximum of the lower mode. Curtis and Devault (25) showed their solution from the exact theory (and experiments) exhibited large amplitudes associated with the same maximum.

Torsional Waves. It is well known that the lowest mode of Pochhammer's frequency equation for torsional waves is nondispersive [see (2)], i.e., all waves of this mode have a phase and group velocity equal to the infinite medium shear wave velocity. Thus the need for higher-order physical considerations for the torsional case has been considerably less than in the compressional or flexural cases. The limited amount of literature on torsional waves reflects this. Some attention, however, has been given the higher modes recently. Davies (7) discusses curves for these given by Owen. On the basis of the higher modes the leading disturbance is composed of very high frequency short waves, since the group velocity of each of these modes increases monotonically from zero for the longest to the infinite medium shear wave velocity for the shortest waves. Davies and Owen [see (7)] measured rod disturbances at large distance from the source and found no evidence of higher mode activity. Very recently, however, Jones (35) pointed out the importance of the higher modes near the wave front of a disturbance, and in the vicinity of a source. He treated, on the basis of the exact theory, the infinite rod

initial value problem, and the problem of the semi-infinite rod subjected to a step shear stress over a small outer annulus of the end. Exact series solutions, and an approximate solution valid for small times behind the wave front, were given for displacements near the rod surface.

PLATES

Compressional and flexural wave dispersion in an infinite elastic plate is governed by the Rayleigh-Lamb frequency equation (36a, b) which was given in 1889. The Rayleigh-Lamb solution had the form of traveling straight-created harmonic wave trains, representing the plane strain solution from the exact theory. The similarity in form of the Rayleigh-Lamb and Pochhammer frequency equations for compressional waves has already been mentioned, so that one could expect many common features in the properties and study of waves in a plate and rod. A detailed account of the Rayleigh-Lamb frequency equation and physics of the associated waves, as well as an up-to-date review of contributions made toward their understanding, was very recently given by Mindlin (14b). Earlier reviews and discussions may be found in (3, 4, 6, and 8).

General Studies (Compressional and Flexural Waves). Lamb (37) first analyzed the lowest symmetric and antisymmetric modes of wave transmission. Goodman (38) treated the infinite plate problem in cylindrical co-ordinates and showed that the Rayleigh-Lamb frequency equation was again generated. He pointed out that in the far field circular-crested (38) and straight-crested waves (Rayleigh-Lamb) are the same. Goodman gave phase velocity curves for the first four modes of the Rayleigh-Lamb equation for both compressional and flexural (symmetric and antisymmetric) waves. Later Tolstoy and Usdin (39) presented an interesting interpretation of this equation in terms of multiple reflecting plane P (dilatational) and SV (vertically polarized shear) waves of plane strain. They gave group velocity curves for the first two modes. As in the rod case, the lowest mode demonstrated leading low frequency long waves for a disturbance composed of symmetric waves, and leading moderately high frequency short waves for one composed of antisymmetric waves.

Mindlin (40) [see also (14b)] recently made a comprehensive study of the Rayleigh-Lamb problem (for both symmetric and antisymmetric waves). In this work first a study was made of the simpler problem of mixed conditions on the plate faces (zero normal displacement and shear stress). Since no mode conversion occurs upon reflection from such a boundary, a frequency spectrum for uncoupled dilatational and equivoluminal waves is generated in this problem. Mindlin produces the Rayleigh-Lamb spectrum by relaxing the plate face conditions from mixed to free. The spectrum for the simpler mixed condition problem forms a grid, certain intersection points of which are common to the Rayleigh-Lamb spectrum. As in (24) [(40) was the forerunner] this grid (of bounds), and asymptotic information for long and short waves, bring out approximately the essential features of the Rayleigh-Lamb spectrum [as pointed out in (14b), Holden (12), Mindlin, and Onoe also contributed earlier on this technique]. Mindlin evaluates the spectrum numerically up to moderately high frequency and out to moderately short wavelength. Among other new features he noted the terrace-like character of the spectrum that develops, at large wave number, and phase velocity near the infinite medium dilatational wave velocity, through coupling of equivoluminal and dilatational modes.

Tolstoy and Usdin (41) extended their earlier work (39), and discovered that phase and group velocities of opposite sign were associated with the low frequency-long wave end of the second symmetric and third antisymmetric modes. They also studied the very high mode spectrum giving the group velocity curves for the fiftieth symmetric and antisymmetric modes. These curves exhibit an interesting oscillatory character in the region in which Mindlin found his terraces, which Tolstoy and Usdin point out is consistent with his findings.

Imaginary roots (wave numbers) of the Rayleigh-Lamb equation, corresponding to stationary spatially attenuated waves, were apparently first found by Agarwal and Shaw (42). They found such roots associated with the second general branch of the frequency equation for symmetric waves. Later Lyon (43) evaluated numerically the imaginary roots of several higher branches for both symmetric and antisymmetric waves. He gives Fay and Fortier's earlier evaluation of the real roots associated with these branches also. Mindlin and Onoe (14a), extending their earlier work and methods [(40), and see (14b)], have studied and given curves for the real and imaginary roots of the Rayleigh-Lamb equation for the first twelve symmetric and antisymmetric branches (out to moderately large wave number). They show that the branches, of the simpler problem of mixed face conditions, form upper and lower bounds on frequency and wave number for the Rayleigh-Lamb modes.

The findings in (14a) on complex wave numbers have already been mentioned. Sherwood (44) also found such roots in the frequency equation. He gave curves for the complex roots of several higher symmetric and antisymmetric wave branches. His work is in basic agreement with (14a). Shaw (45) observed such edge waves in experiments with circular disks.

Progress has also been made recently in deriving and evaluating solutions, in accord with the exact theory, for axially symmetric infinite plate problems involving transient inputs. The contributing authors have used basically the same attack. It employs ray theory and the Laplace transform. The transformed solution is obtained in the form of an infinite integral. Expansion of the integrand, containing the Rayleigh-Lamb frequency equation, into exponential functions leads to a solution in the form of an infinite series of multiple reflecting P- and SV-waves. Inversion is accomplished through Cagniard's ingenious method (46), which employs one or more transformations that make possible an extraction of the solution by inspection. This general ray theory attack has distinct advantages (over a modal solution) for the near field for restricted times. Cagniard's inversion technique is, of course, an important added mathematical convenience. In all the problems treated inversion was carried out for points on the axis of symmetry which gave much greater simplicity through elimination of the Bessel functions. Mencher (47) used this technique to determine and evaluate the normal displacement at the epicenter of an infinite plate subjected to a step point source of pressure at its center. Knopoff (48) treated the case of a step surface point load excitation, and got the solution for the motion on the opposite face directly under the load (epicenter). His solution predicted a strong P- and weaker S-pulse. On the surface away from the load Knopoff used seismic model experiments and found that the S-pulse becomes much larger than the P-pulse. Knopoff and Gilbert (89) studied the motion on the plate face opposite the load, as a function of distance from the load, through first motion approximations (see discussion in section on Half Space).

Broberg (49), Davids (50a), and Pytel and Davids (50b) contributed further to this problem. Broberg derived the solution for the vertical displacement at the epicentral point, due to an impulsive normal load source. He took into account six waves: the incident P and S, and the four reflected waves generated by these. The displacement response is a delta function at the P-wave front. This is followed by a monotonic rise (almost linear) until a step release occurs at the S-wave arrival time. Asymptotic return to zero follows. Broberg's experiments were in agreement with this behavior. He also treated the related line load plate problem. In this case only a step response occurs at the P-wave front. Davids' work differs from Broberg's on two counts. Information is obtained for the interior field along the axis of symmetry (and not just the epicenter), and for the normal stress. His results for the normal stress response at various depths is quite similar to Broberg's results for the vertical displacement, except for the difference in inputs. Pytel and Davids gave numerical informa-

tion for the vertical displacement for the interior field along the axis of symmetry. They also gave further normal stress information.

Thiruvenkatachar (51), extending his earlier work on the half-space problem, solved the related problem of an impulsive normal distributed surface loading (over a small circular area) using the same technique as in (47–50). Mention should also be made of the work of Fulton and Sneddon (52), who have used multi-integral transforms (Fourier) to write the formal solution, in multi-integral form (inversion integral statement), for the problem of the infinite plate subjected to general transient surface loadings on both faces. The special cases of axial symmetry and plane strain, and symmetric loading in the latter, are also formulated. Inversion was not attempted for any transient cases, however.

Mindlin (40) has given a general method for constructing approximate plate theories for compressional and flexural waves with a systematic means of defining the level of approximation. The method converts the exact (three-dimensional) equations to an infinite series of two-dimensional equations, through expansion of the displacement in an infinite series of powers of the thickness co-ordinate and integration over the plate thickness. The series form of the resultant theory (governing equations, stress, strain, etc.) guides the choice on what to include in certain orders of approximation. For a particular approximation a comparison of the frequency spectrum with that of Rayleigh and Lamb is used to define the restrictions on frequency (highest) and wavelength (shortest). Mindlin discussed in detail in (40) the lower-order approximate theories.

Compressional Waves. Further studies of interest concentrated on symmetric waves. In the exact theory Holden's (12) work on the frequency equation, and Redwood's work (19) on a modification of this equation, have already been cited. Exact theory contributions in symmetric wave problems, involving transient inputs, were also made recently. Curtis and Folk (53) have shown the method of (18a), discussed earlier, may be used for solving semi-infinite plate problems involving time-dependent mixed end conditions. Recently the author (54) derived and evaluated numerically the far field displacement solution for the basic symmetric loading case (step, normally opposed surface point loads) of the axially symmetric infinite plate problem treated formally in (52). The solution was obtained through integral transforms (Laplace and Hankel), inversion being accomplished by residue theory and a stationary phase approximation. The work points out the displacement (horizontal and vertical) response, to a moderate input (lower frequencies), is dominated by the early arriving low frequency long waves from the lowest mode in the Rayleigh-Lamb frequency equation.

Of note in approximate theory studies was the higher-order theory (than generalized plane stress) for symmetric waves, written by Kane and Mindlin (55). It is a two-mode theory. Mindlin and Medick (14c) have improved this theory through a higher-order approximation. Their theory is the plate analog of the rod theory given in (15), and the similarity in form of the spectra (for real, imaginary, and complex wave numbers) may be noted. As in the case of the rod, the frequency spectrum closely approximates the exact theory spectrum, having similar restrictions. Gazis and Mindlin (64) have shown that, according to this theory, an incident straight-crested compressional wave, upon meeting the edge of a semi-infinite plate, generates three reflected waves: a pure compressional and two waves corresponding to localized edge motion (associated with the complex roots as noted earlier). Gazis and Mindlin have pointed out this edge mode is always coupled with unattenuated (spatially) lowest mode compressional deformation, and its energy gradually leaks into the plate interior through propagation of the latter. They show their results are in fair agreement with Shaw's observations (45) of such waves.

Transient compressional-wave solutions in approximate theory work were written for the plane stress theory governing nondispersive waves (the plate analog of the one-dimensional wave equation for the rod). Kromm (57) solved the problems of an infinite plate subjected to sudden uniform radial pressure or radial velocity in a small circular hole at the center. He found, for instance, a sizeable increase in circumferential stresses over their static values near the hole. A problem closely related to Kromm's radial pressure case, the sudden punching of a circular hole in a stretched elastic plate, was recently solved and evaluated by the author (58). The greater circumferential stresses near the hole in these problems could possibly create radial cracks. (58) gives a direct Laplace transform technique for solving the plane stress displacement equation of motion, rather than the special device used by Kromm.

Flexural Waves. Approximate theory work yielded more on flexural waves. Uflyand (26) wrote the analog of the Timoshenko beam displacement equation of motion for plates. Noting the beam and axially symmetric plate cases were the same, except for constants, Uflyand applied the transform analysis for his beam problem to the related infinite plate problem (step shear load). Incorrect boundary conditions again invalidated his solution, however. Mindlin (59) gave the correct boundary conditions in a later more complete derivation and analysis of the theory. He also showed, on the basis of the lowest mode, the close agreement between this theory and the exact as in the analogous beam case. A correct solution for Uflyand's plate problem was supplied later by Lubkin (60) who drew partly on the analyses in (26, 28). He used the nonhomogeneous displacement equations of motion, getting the solution as a limiting case of distributed loading as in (27). The author (61) recently solved Uflyand's problem using his earlier analysis (28) with the Timoshenko beam and the boundary conditions given by Mindlin (59). Numerical evaluation of the exact near field solution brought out the rapid spatial decay of the moments and shear force, and the predominance of the lower frequency long waves of the theory. A criterion was also given for judging the accuracy of this response based on a comparison of approximate and exact theory stationary phase solutions. Kane (62) used the theory, as given by Mindlin, to study the reflection of a straight-crested flexural wave at the edge of a semi-infinite plate.

Torsional Waves. In-plane torsional waves in a plate fall into the general category known as *SH* (horizontally polarized shear) waves. *SH*-waves in a plate are discussed by Ewing, Jardetsky, and Press (8), who evaluate numerically the first two higher modes. A more extensive evaluation is given in (14b), and was also given recently by Flinn (63), who noted the frequency spectrum was equivalent to that for torsional waves in a rod (the latter was discussed earlier). Flinn treated the problem of *SH*-waves in an infinite plate emanating from a sudden rotary disturbance in a buried spherical cavity. He derived both a solution in the form of mode superposition and one in the form of an infinite series of multiple wave reflections, and demonstrated their equivalence. Flinn evaluated the first three modes of the former, for a near field station, showing their amplitudes decayed sharply with time after the wave front arrival. Buchwald (64) also treated *SH*-waves in a plate (and a two-layered plate). He derived a stationary phase solution for the (horizontally polarized) displacement for general initial conditions.

Approximate theory work was contributed by Goodier and Jahsman (65) who treated the problems of a suddenly applied shear stress and rotary velocity on the wall of a small circular hole in an infinite plate in plane stress [analog of Kromm's compressional wave problems (57), the equation of motion having the same form, governing nondispersive waves as noted earlier]. They used Kromm's method of solution, and studied numerically the shear stress and rotary particle velocity fields for both problems. It should be pointed out that the plane stress theory here approximates the lowest mode of *SH*-wave

transmission for long waves, just as its counterpart in compressional waves approximates the lowest mode there. An interesting distinction lies in the fact that the exact theory lowest mode for *SH*-waves in a plate is nondispersive, whereas that for compressional waves is dispersive.

CIRCULAR CYLINDRICAL SHELLS

Progress in the study of transient wave propagation along an infinitely long circular cylindrical shell was limited to solutions of the wave train type and understandably so, when one considers the high-order partial differential equations involved in both the exact and approximate theories on this problem. In the exact theory work Ghosh [66, see also ref. (3)] first wrote the frequency equation for axially symmetric waves (including the torsional case). Wave train analyses for any of the wave types, however, were contributed only recently. Fay made such a study for axially symmetric (from here on, taken to mean torsionless) waves, but an error in his work was pointed out in later work on the problem by Herrmann and Mirsky (67a). They evaluated the lowest mode of wave transmission showing, as might be expected, the phase velocity of the limiting low frequency long waves was the same as that for the lowest compressional mode of the rod of circular section, i.e., the "bar" velocity $\sqrt{E/\rho}$. The very high frequency short waves behave as they do in a plate (or rod), traveling with the velocity of the Rayleigh surface wave (see section on Half Space). More recently Greenspon (68a, b) and Gazis (69) have made extensive analyses of the frequency equations for both axially symmetric and nonaxially symmetric waves. In the latter case Greenspon (68a) evaluated numerically for several shell wall thicknesses, the first branch of wave transmission (along the cylinder) for the first three circumferential modes (of which there are an infinite number, each of which has an infinite number of branches of wave transmission). The spectrum associated with the first of these circumferential modes ($n = 1$; $n = 0$ is the axially symmetric wave case) corresponds to Pochhammer's flexural wave spectrum for the circular rod. The first branch of this mode for the very thick shell is, as might be expected, like the rod (beam) case; phase velocity rises monotonically with wave number from zero to the Rayleigh surface wave velocity. It follows that the Timoshenko beam theory would be a good approximation in this case. On the other hand, this curve for the thin shell exhibits a characteristic minimum at moderately short waves. In (68b) Greenspon extended his work to the axially symmetric case ($n = 0$), higher circumferential modes ($n = 4,5$), and higher branches of wave transmission. He also presented for $n = 0, 1, 2$ (and lowest branch) a study of displacement and stress distribution across the shell wall thickness (for fixed wall thickness and wave number). Gazis derived the general frequency equation for the circular cylindrical shell, through the use of displacement potentials associated with dilatational and equivoluminal waves, and described particular types of motion generated by particular cases of coupling of these potentials. In a second part of his work he evaluated numerically several higher branches for the nonaxially symmetric wave cases $n = 1,2$, and made comparisons of these for several shell thicknesses.

The free vibrations of an infinitely long thin circular cylindrical shell were studied by Rayleigh (70). Love and others also contributed to the early work on this problem [see ref. (2), section 334]. Rayleigh's work and later related studies [see ref. (67a)] accounted for just membrane effects, or also the additional effects due to bending. More recently a good deal of effort has been spent on the study of frequency equations of various approximate theories that incorporate the higher-order effects of shear force and rotatory inertia on the displacement. Again here, as in rods and plates, the theme is the creation of a theory (again through approximate displacements) that can be used to model more accurately the higher frequencies and shorter waves. For axially symmetric waves Herrmann and Mirsky (67a) and Lin and Morgan (71) wrote Timoshenko thin shell theories, and studied the influence of the higher-order effects. Herrmann and Mirsky showed the lowest mode was in close agreement with their evaluation of the corresponding exact theory mode (mentioned earlier) for the thin shell. Lin and Morgan studied all three modes of the theory and found an insensitivity to shell thickness for low frequency-long waves. Naghdi and Cooper (72a) studied the same problem employing modifications (with shear force and rotatory inertia) of the classical thin shell theory due to Love [see ref. (2), chapter 24], and the theory due to Donnell. The modified Love theory is similar to the Timoshenko theory used in (67a, 71). Naghdi and Cooper found good agreement between the theories for all three modes of wave transmission [closely resembling those of the Timoshenko theory (67a, 71)], and for the displacement amplitude spectrum of the first two of these modes. Mirsky and Herrmann (67b) extended their work in (67a) to a thick shell by accounting for the additional effect of transverse normal stress. They found the lowest mode of the theory compared very favorably with the corresponding exact theory mode for practically all thicknesses.

Progress in approximate theory work on nonaxially symmetric waves in the cylindrical shell was made through extensions of the techniques used in the simpler axially symmetric case. Mirsky and Herrmann (67c) wrote a general Timoshenko shell theory for nonaxially symmetric waves which contained their former theory for axially symmetric waves as a special case. They made a numerical study of this theory, containing five modes of wave transmission (representing coupled flexural and torsional motions), for the circumferential modes $n =$ 1,2, and 6 and different shell thicknesses. Gazis (69) compared his exact theory results for $n = 1,2$ with those of Mirsky and Herrmann, and found their theory to be a very good approximation for the lowest mode of wave transmission even for relatively thick shells. The second and third modes were also fairly good approximations. Cooper and Naghdi (72b) also extended their work (72a) to the case of nonaxially symmetric waves. Again they found good agreement between the modified Love [similar again to the Timoshenko theory (67c)] and Donnell theories in similar comparisons of transmission modes (for $n = 1, 2, 3, 4, 5, 6, 10$) and associated displacement amplitudes, but here over a restricted intermediate range of wavelengths. Yu (73) studied the nonaxially symmetric wave case with a set of equations he derived similar to the Donnell theory equations (but not involving the simplifications in the strains of the latter). Yu made an extensive study of the modes of transmission of this theory with and without shear force and rotatory inertia considerations for different circumferential modes and shell thicknesses.

Approximate theory work on pure torsional waves in the infinitely long thin circular cylindrical shell was carried out in the works of Mirsky and Herrmann (67c) and Cooper and Naghdi (72b). In the former it was shown, in the case of $n = 0$ (axial symmetry), the three flexural uncouple from the two torsional modes. These torsional modes are similar in character to their close counterparts in the rod, the lower being nondispersive (phase velocities, independent of wavelength, equal to the shear wave velocity) and the upper dispersive. Cooper and Naghdi's work exhibited the same characteristics.

HALF SPACE

Wave propagation in an infinite, and semi-infinite homogeneous, isotropic, elastic medium, subjects of long-standing interest in seismology, have become, more recently, also of considerable importance in mechanics and engineering. The underlying practical reason, of course, is the need for nuclear-blast-resistant structures. The complexities of the half-space problem stem from mode conversion of the body compressional (P) and shear (S) waves at the free surface, and the generation there of the Rayleigh surface wave (74). Rayleigh's important

discovery of the surface wave demonstrated a fundamental difference in the character of the half-space and infinite-medium problems. Lamb (75) was the first to study the propagation of a pulse in the half-space. Such studies are usually referred to as Lamb's problem. He treated several cases: surface normal line (plane problem) and point load (axially symmetric problem) sources and internal dilatational line and point sources. Lamb obtained his solutions for various pulse inputs through Fourier synthesis of the steady-state solutions. For the surface source problems Lamb evaluated the surface displacements (horizontal and vertical), bringing forth the important fact that the largest disturbance in the far field was due to the Rayleigh wave. He noted the nondispersive nature of this disturbance and, in the case of the point load excitation, that its amplitude decayed as $1/\sqrt{r}$ typical of annular divergence.

Pfeiffer (3) discusses the early work on wave propagation in the half-space. Later contributions are reviewed by Schoch (4) and Bullen (76). More recent discussions on the problem were given by Broberg (77), Ewing and Press (78), Petrashen (79a), and Petrashen and Uspenski (79b), the latter two giving an account of Russian contributions. Ewing, Jardetsky, and Press (8) give the most complete, recent discussion of the problem and related literature (including fairly late Russian work). Brekhovskikh's book (80), the Russian counter part of (8), also treats the problem. Goodier's survey (81) on elasticity is a good source for further detail on earlier Russian work. The recent *Handbuch Der Physik* article by Sneddon and Berry (82) should also be mentioned for its review of methods (complex variable, integral transforms) used in solving certain half-space problems.

Surface Source Problems. Of note in the recent contributions to Lamb's problem is the closed form solution derived by Sauter (83) for the entire displacement field due to an impulsive two-component (normal and tangential) line load on the surface. Sauter's solution, which he obtained through integral superposition of plane harmonic waves and contour integration, was given in component form representing contributions to wave-front-determined regions of the field. Three fronts determine the field: cylindrical fronts (emanating from the source) for the compressional and the slower traveling shear wave, and plane fronts for two symmetric shear waves (known as von Schmidt head waves) generated by the compressional front at the free surface. The latter extend from their surface generation points to tangency points on the cylindrical shear wave front. Later Broberg (77) numerically evaluated Sauter's solution for the case of the normal line load impulse. He computed the vertical displacement at a surface station which exhibited the Rayleigh wave infinite discontinuity, pointed out by Sauter, in agreement with the findings of Lamb (the nature of this singularity is determined by the nature of the input function). Broberg also evaluated the vertical displacement along an interior line, parallel to the surface directly below the load (in the plane of symmetry). It showed an infinite discontinuity at the compressional wave front with sizeable disturbances occurring later. Similar character was exhibited in Broberg's approximate evaluation of the vertical displacement and tangential strain on the axis of symmetry for the case of the normal point load impulse. To get these results he made use of Sauter's solution, superposing line loads through the center of a small circle (and within its bounds) to approximate the point load.

Pekeris (84a) recently solved Lamb's problem for the surface displacements due to a step normal point load which he treated as a limiting case of the buried source problem. Through the Laplace transform, contour integration, and an inversion technique similar to Cagniard's, he gave the vertical displacement in simple closed form and the horizontal displacement in terms of elliptic integrals. Pekeris' numerical evaluation, for a surface station, showed infinite discontinuities in both displacements at the Rayleigh wave arrival time. He noted a $1/r$ spatial decay for the displacements, except near the Rayleigh wave infinity, where they had the typical surface wave $1/\sqrt{r}$ behavior noted by Lamb. Petrashen (79a) and Petrashen and Uspenski (79b), extending the earlier work of Petrashen on Lamb's problem and the related plate problem [see (8) and (81)], have given formal solutions for dynamic surface loadings of all types time-dependent normal and tangential line, point and related distributed loads, and twisting moment in the surface plane. Petrashen's method of solution employs integral transforms and contour integration. In (79a) he discusses in detail the mathematical foundations of the method (treated briefly in his earlier work), and the use of the saddle point method of integration for getting approximations to his integral solutions.

More recent work of interest on the plane problem has been done by Miles (85a), Craggs (85b), and Sherwood (86). Miles has formulated Busemann's method of conical flows for two-dimensional problems in elastic wave propagation. The method employs homogeneous solutions which are applicable in problems not involving a characteristic length, or if one is involved it must be derived from a parameter to which the solution must be proportional. Miles treats certain diffraction problems (to be discussed later) and Lamb's line load problem as special cases fitting into this category. The method reduces the equations of motion to either Laplace's equation (in two dimensions) or the one-dimensional wave equation. Solutions are then derived through analytic function or characteristics theory, respectively. Craggs employed essentially the same method to derive solutions for the two-component line and distributed load problems. He studied the Rayleigh wave singularity in detail in these problems. Sherwood used multi-integral transforms (Laplace) to solve both the normal and tangential impulsive line load problems. Of particular interest are his extensive numerical results in both problems for both radial and tangential displacements for the entire field. Sherwood also studied the normal load case with two-dimensional model experiments. These experiments vividly demonstrated such features as the P, S, and head wave arrivals, and a Rayleigh wave amplitude increase with the approach to the surface (in agreement with the theory).

Further work of interest on Lamb's problem for the normal point load has also been done more recently. Lang (87) extended Sauter's method to this problem. Analogously Lang gets the solution for the displacement in component form (here in terms of elliptic integrals) representing the contributions to the regions of the field determined, in this case, by compressional and shear spherical fronts and a head wave conical front. He shows his solution for the surface displacements reduces to that given by Pekeris (84a). Baron, Bleich, Dimaggio and others (88a) pointed out the existence of an elastodynamic reciprocity theorem (due to Rayleigh and Graffi) and used it to derive numerical information for the interior vertical displacement field for the step normal point load problem. Through the theorem they were able to use the numerical results of Pekeris and Lifson (see below) for the vertical surface displacement due to a buried step point vertical load. They noted the relative insignificance of the Rayleigh wave in the interior, and its increasing significance as the surface was approached. These authors also point out that in a similar way information for the interior horizontal displacement field could be obtained from Chao's work (see below) on the horizontal point load problem. Knopoff and Gangi (88b) have also written on the elastodynamic reciprocity theorem. They point out its limitations, i.e. reciprocity is not obtained when the scalar product of the vectors, representing the source excitation and the displacement field at the receiver, is zero. Knopoff and Gilbert (89) have given first motion approximate information on the amplitude coefficients of the interior vertical displacement field in the step normal point load problem.

Their solution (a power series in delay times) is derived through a Laplace transform, the saddle point method of integration, and a (Tauberian) limit theorem. Knopoff and Gilbert point out their method is applicable to many of the problems in theoretical seismology, for which a high frequency, short time approximation suffices. Interesting also was the work of Goodier, Jahsman, and Ripperger (90) pointing out that Lamb's asymptotic solution for the far field horizontal surface displacement can be used to measure force-time impacts on an elastic half-space. Their method, exploiting the nondispersive character of the Rayleigh wave (predominant in the solution), involves numerical solution of the integral equation for the sum of the surface strains through stepwise measurement of this sum. They compared their method with direct force-time measurements and found good agreement.

Chao (91), with the aid of integral transforms and Cagniard's inversion technique, has recently given an exact evaluation of the surface displacements for the step tangential surface point load excitation of the half-space. He also evaluated the displacement (horizontal) along the normal axis below the load. Analogously, but in a reciprocal way to Pekeris' work, Chao's solutions for the horizontal displacements are in simple closed form, and that for the vertical surface displacement in terms of elliptic integrals. His numerical evaluation exhibits infinite Rayleigh wave discontinuities in the vertical displacement and radial component of the horizontal displacement on the surface. The tangential component of the horizontal displacement exhibits a finite shear wave jump. Chao points out the reciprocal relation that exists between the horizontal surface displacement due to a vertical surface load, and the vertical surface displacement due to a horizontal surface load, i.e., Pekeris' curve for the former and Chao's for the latter are identical, in agreement with the reciprocity theorem noted earlier.

Of particular interest in nuclear blast-half space wave studies are the problems involving traveling line and point surface loads. Sneddon (92) treated the problem of the line load (normal or shear) moving with constant velocity under the assumption that the load has been moving for a long time, which yields the steady-state solution (reflected in the fact that the observer traveling with the load sees no change in the displacement wave field). Sneddon's work was restricted to the case where the line load moves with velocity less than the infinite medium shear wave velocity (so-called subsonic case). Cole and Huth (93) extended Sneddon's work to the transonic (line load velocity less than dilatational but greater than shear wave velocity) and supersonic (line load velocity is greater than both infinite medium wave velocities) cases. They present a numerical study of the stress and acceleration fields for the subsonic case, as a function of the ratio of load velocity to shear wave velocity in its approach to one (the transonic case). Their results showed the maximum stresses and accelerations increased with load velocity, but the Rayleigh wave singularity influencing these results was not assessed. Interesting are Cole and Huth's comparisons of the stresses with their static counterparts (Boussinesq-Flamant solution). For a case of traveling load velocity near the Rayleigh wave velocity, they show multifold increase of the stresses over the corresponding static values. Miles (94) has recently written a formal integral transform solution for the response of an elastic half-space to a traveling radially symmetric surface pressure signal, for which the peak pressure and velocity decay with distance from the origin. Miles points out the variable velocity of the signal, in this model of the blast wave problem, removes the Rayleigh wave singularity inherent in the model assuming constant velocity (Sneddon, Cole and Huth). Through a far field approximation he showed his solution reduced to that of Cole and Huth, plus a correction term for the Rayleigh wave singularity. Miles shows the latter is important only when the blast wave velocity (measured directly above the point of observation) is close to the Rayleigh wave velocity.

More recently Ang (95) derived the transient solution for the subsonic case of the moving normal line load problem. This closed form solution, which accounts for the effects of the initial motion, was obtained through a Laplace transform of the problem and subsequent solution of dual integral equations for the transformed potentials (through residue theory). Cagniard's method produced the inversions. The field patterns for Ang's problem resemble those of the fixed normal line load case, his solution for the vertical normal stress reflecting the component waves involved and the Rayleigh wave singularities. Ang points out the transonic and supersonic regime cases of his problem, and the transient moving shear line load problem, could be carried out similarly.

Mention should be made of the contributions made by Mitra (96) and Flitman (97) on other half-space problems of interest. Mitra derived the displacement solution in the form of elliptic integrals, for the problem of the half-space subjected to an impulsive twisting moment in the plane of its surface. He showed the tangential component of the horizontal surface displacement (only component in this problem) was of limited extent in time (related to finite spatial distribution of shear input), with finite jumps at each end, and decayed rapidly with distance. Flitman solved the dynamic mixed boundary-value problem of the die on the elastic half space (plane problem). Flitman's method was to construct (on the boundary) the normal stress under the die and the vertical displacement outside of it, hence reducing the problem to other already solved problems. Toward this end he employed a double Laplace transform, solved an integral equation for the auxiliary problem of the semi-infinite die, and used this solution in connection with a continuation process in a characteristics plane to construct the desired solution. He found a logarithmic Rayleigh wave singularity in the vertical surface displacement. Interesting also was Flitman's observation that compressional or shear waves, incident at the boundary under the die, underwent no mode conversion (typical of mixed conditions). Flitman's paper has a good bibliography of other Russian work on dynamic mixed boundary-value problems.

Buried Source Problems. Several investigations are of note in the more recent literature on the buried point source problem. Pinney (98) used Fourier and Fourier-Bessel integrals, and contour integrations, to derive solutions for the surface displacements for step pressure and shear sources. He presented an extensive numerical evaluation of his integral solutions. Interesting, for instance, were the maximums in the vertical displacement response to the compressional source, that occurred soon after the initial jump, and their rapid decay with distance. Dix (99) used Cagniard's method in the step pressure source problem to derive and evaluate the vertical displacement response at stations above the source on the axis of symmetry. He showed both shear and compressional wave energy arrived at these stations, after reflection of the incident pulse at the surface. He points out that through this mechanism energy is held near the source and surface long enough to account for the generation of long period surface waves. In (84a) Pekeris gave the transformed (Laplace) solutions for the surface displacements in the problem of a buried point vertical force (step). He also inverted these in another paper (84b) with an operational technique similar to that given in (84a). In a later paper by Pekeris and Lifson (84c) numerical evaluation and analysis of the solutions brought out several interesting points. Their results, for instance, demonstrated the total reflection phenomenon associated with the *SV*-wave generated at the source, i.e., beyond a certain distance from the epicenter (determined by the medium properties, i.e., compressional and shear wave veloc-

ities) an incident SV-wave at the surface generates a reflected SV-wave (carrying all the incident energy) and a diffracted SP-wave traveling along the surface. Pekeris and Lifson noted there was no Rayleigh wave within this critical epicentral distance. They also noted that their solution approached the form of the surface pulse solution, with its infinite Rayleigh wave amplitude, for large values of distance to source depth. Mention should be made of the fact that Petrashen, and Petrashen and Uspenski, also treated in (79a, b) buried source problems for all types of loadings (like those of their surface loading cases), in a manner similar to their treatment of the surface loading cases.

Garvin (100) extended early work on the buried line pressure source problem, which was concerned primarily with the far field evaluation, by getting simple closed form solutions for the surface displacements (valid everywhere). Again here the Laplace transform and Cagniard's inversion technique were used. Garvin gave an extensive numerical evaluation of his solution, showing the development of the Rayleigh wave and S-wave in the far field. Later Mitra (101) used the same technique to derive the solution for the related problem in which the pressure source is asymmetric about its line of application.

Surface Waves. A topic of current interest in the half-space problem is the dispersion induced in the Rayleigh wave through deviation of the free surface from a plane. The dispersion is created through reflection of the Rayleigh wave, and generation of compressional and shear waves at the irregular boundary, which are needed to satisfy the boundary conditions there. Ewing, Jardetsky, and Press (8) discuss curvature effects on wave propagation, including Rayleigh waves, and review the related literature. Brekhovskikh (102) has recently investigated the dispersive induced attenuation of the Rayleigh wave. He used a perturbation technique (parameter: small depth of surface irregularity to length of Rayleigh wave), expressing the boundary stresses as first-order approximations (where the zeroth order is the plane boundary case of vanishing stresses). For a two-dimensional surface irregularity Brekhovskikh found the Rayleigh wave amplitude decayed exponentially with the propagation co-ordinate. He found the attenuation coefficient dependent on the ratio of the wavelength of shear wave to that of the surface irregularity (when the latter was harmonic), and that certain peak attenuation values were associated with energy conversion into dispersive shear waves propagating away from the boundary. Viktorov (103) and de Bremaecker (104) investigated this problem with two-dimensional models [introduced by Oliver Press, and Ewing, see (8), pg. 72, ref. 35]. Viktorov studied the reflection of Rayleigh waves from single surface defects: a slit, a semicircular groove, and a corner. de Bremaecker concentrated on the latter case, studying also wave path bending as a function of corner angle. Both noted a good portion of the incident Rayleigh wave energy was lost upon reflection, in agreement with Brekhovskikh's findings.

Eringen and Samuels (105) studied the general two-dimensional problem of the half-space with a slightly irregular boundary (various irregularities were considered). They also used a perturbation technique, in connection with a double Fourier transform, to write the solutions for the stresses and displacements. Eringen and Samuels found, for the case of a uniformly pulsating pressure on a sinusoidal boundary, perturbation created stress and displacement surface waves that were dispersive. They showed (numerically) the amplitude dependence of these waves on phase velocity.

INFINITE MEDIUM

Poisson (106) and Stokes (107) were the first to recognize the basic underlying feature of wave propagation in an infinite elastic medium, namely, that two fundamentally different types of waves (dilatational and equivoluminal) could propagate. Lamé (108) first gave the general solution to the vector displacement equation of motion composed of both the gradient of a scalar potential and the curl of a vector potential, where these potentials satisfied wave equations and reflected the two basic waves involved. In Stokes' classical work (107) he also derived the displacement field due to an arbitrary time-dependent point force. Love (2) and Pfeiffer (3) discuss these and other early works on the infinite medium problem. A modern account is given in (8). Recent studies of interest have been made on the general problem, and on specific problems involving boundaries such as those of a cavity source, diffraction, and scattering. Sternberg (109) presented a study on the theory of integration of the governing displacement equations of motion. He investigated in detail the work done on proving the completeness of Lamé's solution and contributed to its interpretation. Sternberg reviews general integration methods (as well as related special cases) employing dynamic stress functions (Iacovache, and others), and shows their relation to Lamé's solution. Eason, Fulton, and Sneddon (110) have used multi-Fourier integral transforms to study the waves generated by time-dependent body forces in an infinite elastic solid. The general solution for the isotropic solid involves a four-dimensional transform. Eason, Fulton, and Sneddon treated several special two- and three-dimensional problems involving fixed, and moving, impulsive, line, point, and distributed body forces, and studied some of these numerically. The reader should also note that the work quoted on the half-space buried source problems (98, 99, 84, 79, 100, 101) applies equally to the related infinite medium problems, since the latter are basic to the former.

Cavity Source Problems. Wave propagation in an infinite elastic solid emanating from a uniform pressure source in a spherical cavity was studied quite early by Sezawa, Kawasumi, Yosiyama, Nishimura, and other Japanese seismologists, and later by Sharpe, Blake, Selberg, Vanek, and others. Surveys of this work are given in (8) and (77) along with discussions of the problem. In the later work Blake wrote the solution for the case of a step-exponential decay pressure input (explosive source). Selberg derived the solution for the step pressure input. His response curves, when compared to the static stresses at a station, exhibited a dynamic increase in the tensile tangential stress and a corresponding decrease in the compressive radial stress. In Vanek's work he also considered a case of nonuniform pressure distribution in the cavity. More recent contributions on the spherical cavity source problem have been made by Das Gupta (111), Goldsmith and Allen (112), Eringen (113), and Broberg (77). Das Gupta derived the solution for the stresses and displacements for the case of an impulsive couple applied to the cavity wall. Goldsmith and Allen gave an extensive numerical evaluation of Blake's solution for the displacement, velocity, and the stresses for various combinations of cavity radius and input attenuation coefficient. With the aid of Fourier transforms Eringen has given the solution for the general problem, in which the cavity wall is subjected to arbitrary dynamic tractions, and for various special cases. He evaluated the stresses and displacement for the case of uniform impulsive cavity pressure. Broberg also treated and evaluated the solution for this latter problem.

Selberg also gave the solution (through the Laplace transform) for the more difficult plane strain problem involving a uniform pressure (step and step-exponential decay inputs) in a cylindrical cavity. The stresses here exhibit features similar to those in the spherical cavity case as might be expected. It also should be pointed out that Kromm's work (57) directly applies to this cylindrical cavity case, because of the equivalence (except for the elastic constants) of the dynamic plane stress and plane strain problems [Bishop (114)].

The work of Viktorov (115) on the cylindrical cavity problem should also be mentioned. He studies Rayleigh waves propagating along the cavity wall in the circumferential direction, and found them to be dispersive as in the related case of Rayleigh waves on the surface of a cylindrical rod [also treated by Viktorov, but earlier by Sezawa and others; see ref. (8)].

Diffraction and Related Crack Problems. Transient diffraction of a plane pulse by a plane boundary or a wedge is a topic of long standing interest in acoustics and electromagnetic wave theory. Fruitful studies on its more complicated analog in the infinite elastic medium, however, have only recently been made. Goodier (81) points out that progress has been made on two-dimensional transient diffraction problems that have homogeneous solutions [re: discussion on Miles work (85a)]. He reviews the work done by Maue and Fridman on such problems, and compares the methods used in detail. Maue solved the problem of the sudden opening of a transverse semi-infinite crack in an initially static uniform tension field, and evaluated the stresses. The problem is equivalent to the diffraction by a half plane of two normally incident, and oppositely directed, plane step compressional stress waves. Maue's method led to an integral equation which he solved with the Wiener-Hopf technique. Fridman carried out his work with a technique introduced earlier in the Russian literature by Smirnov and Sobolev (method of functional invariants) which, as Goodier points out, also draws on the homogeneity of solutions. Fridman solved the basic problems of the diffraction of a plane compressional (P) pulse, and of a plane shear (SV) pulse, by a stress free (weak) and by a fixed (rigid) half plane. In this work Fridman treats the general case of arbitrary angle of incidence. Miles' method (homogeneous solutions) has already been discussed in connection with his solution of Lamb's line load problem. As mentioned earlier he applied it also to the problems of the transient diffraction of a plane SH-pulse by a wedge-shaped cavity, and of a plane P- and SV-pulse by a weak half-plane. In the latter two problems Miles gave the distributions of the velocities (radial and tangential) along their respective diffracted wave fronts for the case of normal incidence.

Contributions were also made on three-dimensional diffraction theory and problems. Knopoff (116) solved the vector displacement equation of motion by using a technique similar to that used by Kirchhoff in solving the inhomogeneous wave equation of optical theory. The method yielded a general solution (a representation theorem) for the acceleration vector in terms of retarded body forces and retarded accelerations and strains on the surface. Knopoff also applied this technique tc study the diffraction of elastic waves (both P and S) through large apertures. De Hoop (117) also worked along these lines, drawing on recent developments in acoustics and electromagnetic theory. De Hoop wrote a three-dimensional (and two-dimensional) representation theorem for the transformed (Laplace) displacement in terms of volume and surface integrals involving the Green's function associated with the (transformed) displacement equation of motion. Through the representation theorems the diffraction problems are reduced to the solution of integral equations. De Hoop treats several two-dimensional problems involving diffraction of a plane pulse from a weak or rigid half-plane. In these problems the integral equations arising were solved with the Wiener-Hopf technique. The solution is given its transient character through an inversion that employs (basically) Cagniard's method. Filippov (118) used the method of functional invariants to solve the three-dimensional problem of transient diffraction of a plane P-pulse, under arbitrary incidence, by a rigid half-plane.

Also of interest were further solutions of problems involving the two-dimensional crack carried out by Ang (119a) and Ang and Williams (119b). Ang studied the elastic stress wave field generated by a force moving along a half-plane crack. Ang's method followed that due to Clemmow. It involved a Laplace transformation of the problem, assumed Fourier integral solutions (plane wave super position) for the transformed potentials, reduction to dual integral equations (through consideration of boundary conditions) and solution by the Wiener-Hopf technique, and inversion by Cagniard's method. With a similar technique Ang and Williams studied the stress wave field due to a line discontinuity in one displacement component, moving in a shear free half-plane which is perpendicular to this displacement. In both (119a) and (119b) the wave front patterns and stress singularities were analyzed.

Scattering Problems. Recent work on the problems of scattering of plane waves by spherical obstacles embedded in an infinite elastic solid has been done by Knopoff (120 a,b). He studied formally the scattering of an incident plane P-sinusoidal wave by an elastic sphere, embedded in a different elastic solid, and evaluated the special cases of plane P- and S-wave scattering by a rigid sphere [range: radius $a \ll$ wavelength λ to $0(\lambda)$]. Knopoff computed the scattered wave fields for the far field (from obstacle) giving the angular amplitude distributions of the displacement components. He noted in the incident P-wave problem, for instance, that amplitudes of the scattered S-wave were about three times those of the scattered P-wave. Knopoff's work contains a discussion of related earlier work by Sezawa, Ying, Truell, and others. Nagasse (121) has treated the problem of a point source in the infinite elastic medium surrounding a spherical cavity. The problem is quite complicated mathematically, being governed by three separated wave equations, reflecting the three basic waves (dilatational and two equivoluminal) involved. Through series solutions and residue theory Nagasse presented a study of the scattered wave fields.

On scattering by a cylindrical obstacle White's work (122) is one of note. He treated formally the general three-dimensional problems of plane compressional and shear waves, obliquely incident on an elastic cylinder (with respect to its axis) embedded in a different elastic medium. He evaluated the special two-dimensional case of normal incidence on a cylindrical cavity (and also studied it experimentally), and found a significant amount of scattered energy underwent mode conversion. White discusses earlier work by Kato, and others on the problem. Gilbert (123) contributed further on the cylindrical cavity problem, treating the scattering of a plane compression pulse for both normal and oblique incidence. Gilbert considered only the portion of the cylinder on which the wave impinges (shadow zone forthcoming). His method employed a high frequency approximation (like that used in geometrical optics) and ray theory considerations to determine the reflected waves. He found the presence of the cavity produced a maximum amplification of the field stresses of approximately two, and that the maximum stress was the hoop stress. The authors in (88a) also contributed on the cylindrical cavity scattering problem, treating it through superposition of the free field stresses (no cavity) and stresses to negate these on a cylindrical boundary (the cavity wall). The problem was formulated to include general time and space variation and the higher circumferential modes, the method involving a Laplace transform, separation, and contour integration. The axially symmetric pressure loading case was treated numerically, evaluation of the hoop stress giving the same curve for a step input as in Selberg's work [see (77)] except for sign.

Gilbert and Knopoff (124) obtained the exact formal solution to the two-dimensional problem of scattering of compressional waves, from a line source in an infinite elastic medium, by an embedded rigid cylinder. They evaluated their integral solution asymptotically for the early (wave front) motion, for the illu-

minated zone, through the saddle point method of integration. For the early motion in the shadow region, integration was accomplished through a method developed for high-frequency approximations in analogous scattering problems in acoustic and electromagnetic wave theory. Gilbert and Knopoff point out that, in the illuminated region, the geometric optics approximation results, observed events being the direct *P*-, reflected *P*-, and reflected *S*-waves. In the shadow zone they are the diffracted *P*- and diffracted *S*-wave. The fronts of the latter two waves travel around the cylinder with the *P*-wave velocity.

CLOSING REMARKS

This survey has pointed out that extended information exists now on the exact theory frequency equations governing compressional, flexural, and torsional waves in an infinite elastic rod, plate, and cylindrical shell. Recent efforts have established the character of the higher real branches of the frequency spectrum (real frequency versus real wave number) and associated harmonic waves in these cases (7, 12, 14ab, 15, 23, 24, 38-41, 43, 63, 68, 69). The existence and character of the imaginary and complex branches of the spectrum (real frequency versus imaginary and complex wave numbers) and their waves, in the cases of compressional and flexural waves in the rod and plate, have also been established (13-15, 43, 44). As pointed out earlier this information is basic in solving propagation problems in the semi-infinite and infinite rod, plate, and cylindrical shell, involving time-dependent boundary conditions. Use of it in this connection, however, has only really begun. Integral and multi-integral transforms have yielded formal solutions for problems of this type in the case of the rod and plate (17, 18a, 25, 46-54, 63, 89), although problems involving non-mixed end conditions (for example, rod pressure shock problem) remain a challenge. Numerical evaluation (involving the frequency equations) in the problems solved, has been limited to the far field and the lower real branches of the frequency spectrum, or to the axis of symmetry (in the plate), use being made of ray theory and Cagniard's method of inverting the Laplace transform; an exception is the evaluation of plate face motion away from the axis of symmetry obtained through first motion approximations (89). These evaluations are typical of general techniques used in the study of elastic wave propagation, the former involving the modal form of the solution usually used for far field and long time information (in which many wave reflections are involved), and the latter involving the multiple reflections form of the solution suited to near field and short time data (in which a much smaller number of reflections are involved). Similar information for the near and far field, involving the higher real branches, and lower imaginary and complex branches of the frequency spectrum, is of interest and perhaps will be forthcoming.

Approximate theories for the rod and plate (20, 26, 40, 59) have been of value in studying these problems. With the aid of integral transforms and other techniques exact and approximate solutions, and comparisons of these with exact theory and experimental results, have established that the lower-order approximate theories, such as the Timoshenko beam and the Mindlin-Herrmann rod (and their counterparts in plates), are good approximations provided the proper high frequency and short wave restrictions are imposed (11, 21, 22, 26-34, 57, 58, 60, 61, 65). Work along these lines, with the higher-order theories written recently (14c, 15), is possible and could be productive in the future. They offer the chance to study transient waves governed by the higher real branches and lower imaginary and complex branches of the frequency spectrum. The coupled partial differential equations are of sixth order, however, and time might be more profitably spent on the exact theory attack. It should also be pointed out that similar solutions can be obtained from approximate theories for the shell (67, 71-73), but again the higher-order (sixth and greater) equations must be dealt with.

Extended information on wave propagation in the elastic half-space also exists now. As noted earlier, through integral transforms, Cagniard's inversion technique, homogeneous solutions, and other means, surface displacements for most of the cases in Lamb's problem (surface and buried sources of most types) have been evaluated, detail being given on the *P*-, *S*-, and Rayleigh wave contributions (77, 83-87, 91, 96-98, 100, 101). The interior displacement field in the cases of surface line load sources has also been evaluated (86). Noticeably lacking however, with the exception of computations for points on the axis of symmetry (77, 91, 99) and some first motion approximations (89), is the interior displacement field for the point load cases. Some information of this type has been obtained through evaluated surface displacements in a buried source problem (84c) and an elastodynamic reciprocity theorem (88a). More is needed and can be derived this way, and through other solutions and techniques mentioned earlier (46, 79, 87). Progress has been made in problems dealing with moving loads (92-95). Here again more work on solutions and their evaluation is indicated, particularly in the transient cases (95). Important also is the problem dealing with propagation in the half-space with an irregular surface, where advances have been made through perturbation techniques (102, 105) and model studies (103, 104). One would expect futher information on this problem from these methods in the near future. Concern over underground protective construction makes the problem of wave scattering by a cavity or obstacle in the half-space of current importance. Considerable work has been done on the related infinite medium problems, cylindrical and spherical cavities and obstacles being treated (88a, 120-124).

Important advances have been made in the theory and solution of problems on the diffraction of a transient plane pulse by a plane boundary or wedge in an infinite elastic solid. In two-dimensional diffraction and related crack problems, methods employing homogeneous solutions with and without solutions of integral equations of the Wiener-Hopf type have been productive (81, 85). Other techniques in these problems involving the Laplace transform, Wiener-Hopf-type integral equations, and Cagniard's method of inverting the Laplace transform have also been productive (117, 119). In addition three-dimensional representation theorems for the displacement and the acceleration vectors have been written, and contributions made on particular problems (116-118). One would expect further solutions in diffraction problems to be forthcoming from these techniques. Work on evaluation of the displacement and stress fields is needed in the solutions already obtained. Recent studies on the problem of the cavity source in the infinite medium are of note. Detailed information on the stress and displacement fields has been given for both the cylindrical and spherical cavities involving the uniform pressure source (57, 77, 112, 113). Some work has been done on the cases involving nonuniform pressure and other cavity inputs (77, 111, 113, 115).

Some brief remarks are in order on the important related subject of wave propagation in layered elastic media. This subject is the main theme in the books by Ewing, Jardetsky, and Press (8) and Brehkovskikh (80), and in the papers by Petrashen and Uspenski (79), where detailed coverage of the literature (Western and Russian) through 1956 may be found. Important in Petrashen and Uspenski's work (79b) is the method given for deriving and evaluating transient solutions for the *n*-layered space (plane boundaries) for all types of loadings. The method uses the basic half-space and infinite medium solutions [discussed earlier for (79)], along with consideration of the reflection and refraction of waves at the boundaries, ray theory arguments being used to define the

times of interest. Of further interest to this article is the recent activity in thermoelasticity dealing with the transient propagation problem. Discussions of the problem and reviews of the literature may be found in the papers by Sternberg and Chakravorty (125), Lessen (126), Nowacki (127), and Boley (128).

The reader will also be interested in certain articles and books that have appeared lately relating to some of the methods that have been mentioned here, which are certain to be of importance in future work. Brillouin's book (129) on propagation in dispersive media, containing discussions on wave groups and the saddle point method of integration, is of note. The papers by Karal and Keller (130) and Babich and Alekseev (131) on a ray theory method for solving general propagation problems in homogeneous and inhomogeneous elastic media, and Friedlander's book (132) on the related attack in acoustics problems should also be noted. Angona (133) gives an up-to-date account of one-, two-, and three-dimensional modeling techniques and the related literature. In this connection Ivakin's study (134) of similitude criteria for the piecewise nonhomogeneous elastic solid should be mentioned. Important also are the techniques being developed in photoelasticity for studying elastic waves. These were reviewed recently by Hetenyi (135).

REFERENCES

1 Pochhammer, L., Ueber die fortpflanzungsgeschwindigkeiten schwingungen in einem unbegrenzten isotropen Kreiszylinder, *J. fur Math.* **81**, 324-336, 1876.

2 Love, A. E. H., A treatise on the mathematical theory of elasticity, Cambridge Univ. Press, Fourth Edition, 1927.

3 Pfeiffer, F., Elastokinetik, Handbuch der Physik **6**, Verlag J. Springer, Berlin, 1928.

4 Schoch, A., Schallreflexion, schallbrechung und schallbeugung, *Ergeb. Exact. Naturw.* **23**, 127-234, 1950; AMR **5**(1952), Rev. 3572.

5 Kolsky, H., Stress waves in solids, Oxford, Clarendon Press, 1953; AMR **7**(1954), Rev. 418.

6 Davies, R. M., Stress waves in solids, *Applied Mechanics Reviews* **6**, 1-3, 1953.

7 Davies, R. M., Stress waves in solids, Surveys in Mechanics, G. I. Taylor 70th Anniversary Vol., Cambridge Univ. Press, 64-138, 1956; AMR **11**(1958), Rev. 28.

8 Ewing, W. M., Jardetzky, W. S., and Press, F., Elastic waves in layered media, McGraw-Hill Book Co., Inc., New York, 1957; AMR **12**(1959), Rev. 562.

9 Abramson, H. N., Plass, H. J., and Ripperger, E. A., Stress wave propagation in rods and beams, Advances in Appl. Mech. **5**, Academic Press, Inc., New York, 111-194, 1958; AMR **11**(1958), Rev. 3914.

10 Kolsky, H., Experimental wave propagation in solids, Structural Mechanics, Proc. 1st Symp. on Naval Struct. Mech., Pergamon Press, New York, 233-262, 1960.

11 Davies, R. M., A critical study of the Hopkinson pressure bar, *Phil. Trans. Roy. Soc. Lond. (A)* **240**, 375-457, 1948; AMR **2**(1949), Rev. 988.

12 Holden, A. N., Longitudinal modes of elastic waves in isotropic cylinders and slabs, *Bell Syst. Tech. J.* **30**, 1, 956-969, 1951; AMR **5**(1952), Rev. 1309.

13 Adem, J., On the axially-symmetric steady wave propagation in elastic circular rods, *Quart. Appl. Math.* **12**, 261-275, 1954; AMR **8**(1955), Rev. 1300.

14a Mindlin, R. D., and Onoe, M., Mathematical theory of vibrations of elastic plates, Proc. 11th Annual Symp. Freq. Control, U. S. Army Sign. Corps Engr. Labs., Ft. Monmouth, N. J., 17-40, 1957.

b Mindlin, R. D., Waves and vibrations in isotropic, elastic plates, Structural Mechanics, Proc. 1st Symp. on Naval Struct. Mech., Pergamon Press, New York, 199-232, 1960.

c Mindlin, R. D., and Medick, M. A., Extensional vibrations of elastic plates, *J. Appl. Mech.* **26**, 561-569, 1959; AMR **13**(1960), Rev. 2798.

15 Mindlin, R. D., and McNiven, H. D., Axially symmetric waves in elastic rods, *J. Appl. Mech.* **27**, 145-151, 1960.

16 Oliver, J., Elastic wave dispersion in a cylindrical rod by a wide-band short-duration pulse technique, *J. Acoust. Soc. Amer.* **29**, 189-194, 1957; AMR **10**(1957), Rev. 2440.

17 Skalak, R., Longitudinal impact of a semi-infinite circular elastic bar, *J. Appl. Mech.* **24**, 59-64, 1957; AMR **10**(1957), Rev. 3555.

18a Folk, R., Fox, G., Shook, C. A., and Curtis, C. W., Elastic strain produced by sudden application of pressure to one end of a cylindrical bar, I. Theory, *J. Acoust. Soc. Amer.* **30**, 552-558, 1958; AMR **12**(1959), Rev. 2337.

b Fox, G., and Curtis, C. W., II (of 18a). Experimental observations, *J. Acoust. Soc. Amer.* **30**, 559-563, 1958; AMR **12**(1959), Rev. 2338.

19 Redwood, M., Velocity and attenuation of a narrow band, high-frequency compressional pulse in a solid wave guide, *J. Acoust. Soc. Amer.* **31**, 442-448, 1959.

20 Mindlin, R. D., and Herrmann, G., A one-dimensional theory of compressional waves in an elastic rod, Proc. 1st U. S. Nat. Congr. Appl. Mech., ASME, New York, 187-191, 1952; AMR **5**(1952), Rev. 1308; See also Proc. 2nd U. S. Nat. Congr. Appl. Mech., ASME, New York, 233, 1955.

21a Miklowitz, J., The propagation of compressional waves in a dispersive elastic rod, Part I—Results from the theory, *J. Appl. Mech.* **24**, 231-239, 1957; AMR **11**(1958), Rev. 1129.

b Miklowitz, J., and Nisewanger, C. R., II (of 21a), experimental results and comparison with theory, *J. Appl. Mech.* **24**, 240-44, 1957; AMR **11**(1958), Rev. 1130.

22 Miklowitz, J., On the use of approximate theories of an elastic rod in problems of longitudinal impact, Proc. 3rd U. S. Nat. Congr. Appl. Mech., ASME, New York, 215-224, 1958; AMR **13**(1960), Rev. 165.

23 Abramson, H. N., Flexural waves in elastic beams of circular cross section, *J. Acoust. Soc. Amer.* **29**, 42-46, 1957; AMR **10**(1957), Rev. 2799.

24 Pao, Y. H., and Mindlin, R. D., Dispersion of flexural waves in an elastic, circular cylinder, Paper 59-A-84, ASME Ann. Meet., 1959.

25 Curtis, C. W., Propagation of an elastic strain pulse in a semi-infinite bar, Inter. Symp. Stress Wave Propagation in Materials, Interscience Publishers, Inc., New York, 15-43, 1960. See also Devault, G. P., and Curtis, C. W., Problem of elastic bar with mixed time-dependent end conditions of general form, *J. Acoust. Soc. Amer.* **31**, p. 635, 1959.

26 Uflyand, Ya. S., The propagation of waves in the transverse vibration of bars and plates, *Prikl. Mat. Mekh.* **12**, 287-300, 1948 (in Russian); AMR **3**(1950), Rev. 33

27 Dengler, M. A., and Goland, M., Transverse impact of long beams, including rotatory inertia and shear effects, Proc. 1st U. S. Nat. Congr. Appl. Mech., ASME, New York, 179-186, 1952; AMR **6**(1953), Rev. 2703.

28a Miklowitz, J., Flexural wave solutions of coupled equations representing the more exact theory of bending, *J. Appl. Mech.* **20**, 511-514, 1953; AMR **7**(1954), Rev. 1026.

b Miklowitz, J., Flexural waves in beams according to the more exact theory of bending, NAVORD Report no. 2049, 1953.

29 Goland, M., Wickersham, P. D., and Dengler, M. A., Propagation of elastic impact in beams in bending, *J. Appl. Mech.* **22**, 1-7, 1955; AMR **6**(1953), Rev. 3666.

30 Boley, B. A., and Chao, C. C., Some solutions of the Timoshenko beam equations, *J. Appl. Mech.* **22**, 579-586, 1955; AMR **9**(1956), Rev. 2462.

31 Leonard, R. W., and Budiansky, B., On travelling waves in beams, NACA Tech. Note 2874, Jan. 1953; AMR **6**(1953), Rev. 2704.

32 Plass, H. J., and Steyer, C. C., Studies in longitudinal and bending waves in long elastic rods, Univ. Texas, Defense Research Lab. Report DRL-376, CM-860, 1956. See also *J. Appl. Mech.* **25**, 379-385, 1958; AMR **12**(1959), Rev. 1214.

33 Anderson, R. A., Wave groups in the flexural motion of beams predicted by the Timoshenko theory, *J. Appl. Mech.* **21**, 388-394, 1954; AMR **8**(1955), Rev. 2992.

34 Jones, R. P. N., Transient flexural stresses in an infinite beam, *Quart. J. Mech. Appl. Math.* **8**, 373-384, 1955; AMR **9**(1956) Rev. 1022.

35 Jones, R. P. N., The generation of torsional stress waves in a circular cylinder, *Quart. J. Mech. Appl. Math.* **12**, 325-336, 1959; AMR **13**(1960), Rev. 4536.

36a Rayleigh, Lord, On the free vibrations of an infinite plate of homogeneous isotropic elastic matter, *Proc. Lond. Math. Soc.* **20**, 225-234, 1888-1889.

b Lamb, H., On the flexure of an elastic plate (Appendix), *Proc. Lond. Math. Soc.* **21**, 85, 1889-1890.

37 Lamb, H., On waves in an elastic plate, *Proc. Roy. Soc. Lond. (A)* **93**, 114-128, 1917.

38 Goodman, L. E., Circular-crested vibrations of an elastic solid bounded by two parallel planes, Proc. 1st Nat. Congr. Appl. Mech., ASME, New York, 65-73, 1952; AMR **6**(1953), Rev. 2997.

39 Tolstoy, I., and Usdin, E., Dispersive properties of stratified elastic and liquid media: A ray theory, *Geophysics* **18**, 844-870, 1953.
40 Mindlin, R. D., An introduction to the mathematical theory of vibrations of elastic plates, Monograph, U. S. Army Signal Corps Engr. Labs., Ft. Monmouth, N. Y. Signal Corps Contract DA-36-039, Sc-56772, 1955.
41 Tolstoy, I., and Usdin, E., Wave propagation in elastic plates: low and high mode dispersion, *J. Acoust. Soc. Amer.* **29**, 37-42, 1957.
42 Agarwal, R. R., and Shaw, E. A. G., Axially symmetric vibrations of a finite isotropic disk, IV, *J. Acoust. Soc. Amer.* **26**, 341-42, 1954; AMR **7**(1954), Rev. 3816.
43 Lyon, R. H., Response of an elastic plate to localized driving forces, *J. Acoust. Soc. Amer.* **27**, 259-265, 1955; AMR **9**(1956), Rev. 380.
44 Sherwood, J. W. C., Propagation in an infinite elastic plate, *J. Acoust. Soc. Amer.* **30**, 979-984, 1958; AMR **12**(1959), Rev. 2872.
45 Shaw, E. A. G., On the resonant vibrations of thick barium titanate disks, *J. Acoust. Soc. Amer.* **28**, 38-50, 1956.
46 Cagniard, L., Reflexion et refraction des ondes seismiques progressives, Gauthier-Villars & Cie., Paris, 1939; see also paper by C. H. Dix, The method of Cagniard in seismic pulse problems, *Geophysics* **19**, 722-738, 1954. (A translated (in English, by E. A. Flinn) 2nd Edition of Cagniard's book, with additions by C. H. Dix, is forthcoming from McGraw-Hill Book Co., New York.)
47 Mencher, A. G., Epicentral displacement caused by elastic waves in an infinite slab, *J. Appl. Phys.* **24**, 1240-46, 1529, 1953.
48 Knopoff, L., Surface motions of a thick plate, *J. Appl. Phys.* **29**, 661-670, 1958; AMR **12**(1959), Rev. 5318.
49 Broberg, K. B., A problem on stress waves in an infinite elastic plate, *Trans. Roy. Inst. Tech., Stockholm*, Rep. no. 139, 1959; AMR **13**(1960), Rev. 3355.
50a Davids, N., Transient analysis of stress-wave penetration in plates, *J. Appl. Mech.* **26**, 651-660, 1959; AMR **13**(1960), Rev. 2821.
b Pytel, A., and Davids, N., Further transient analysis of stress wave propagation in plates, Proc. 4th Midwest Conf. Solid Mech., Univ. Texas Press, 358-381, 1959; AMR **13**(1960), Rev. 685.
51 Thiruvenkatachar, V. R., Recent research in stress waves in India, Inter. Symp. Stress Wave Prop. Materials, Interscience P b., Inc., New York, 1-14, 1960.
52 Fulton, J., and Sneddon, I. N., The dynamical stresses produced in a thick plate by the action of surface forces, *Proc. Glasgow Math. Assoc.* **3**, 153-163, 1956.
53 Curtis, C. W., Propagation of elastic and plastic deformations in solids, OOR Report, Contr. DA-36-034-ord-1456, Sup. 2, Proj. TB 2-0001 (187), Lehigh Univ., Sept. 1956.
54 Miklowitz, J., Transient compressional waves in an infinite elastic layer overlying a rigid half space, Space Technology Labs. Rep., STL-TR-60-0000-19285, Sept. 30, 1960, Los Angeles, Calif.
55 Kane, T. R., and Mindlin, R. D., High-frequency extensional vibrations of plates, *J. Appl. Mech.* **23**, 277-283, 1956.
56 Gazis, D. C., and Mindlin, R. D., Extensional vibrations and waves in a circular disk and a semi-infinite plate, Paper 60-APM-3, presented at 23rd Nat. Conf. Appl. Mech. ASME, Penn State Univ., June 1960.
57 Kromm, A., On the propagation of shock waves in discs with a circular hole, *ZAMM* **28**, 104-114, 297-303, 1948 (in German); AMR **3**(1950), Rev. 1857.
58 Miklowitz, J., Plane-stress unloading waves emanating from a suddenly punched hole in a stretched elastic plate, *J. Appl. Mech.* **27**, 165-171, 1960.
59 Mindlin, R. D., Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates, *J. Appl. Mech.* **18**, 31-38, 1951; AMR **4**(1951), Rev. 2826.
60 Lubkin, J. L., Propagation of elastic impact stresses, ONR Progress Rep. no. 5, Contr. Nonr-704(00), Midwest Res. Inst., Kansas City, Missouri, Apr. 1954.
61 Miklowitz, J., Flexural stress waves in an infinite elastic plate due to a suddenly applied concentrated transverse load, Presented at 10th Inter. Congr. Appl. Mech., Stresa, Italy, Sept. 1960 (in press, *J. Appl. Mech.*).
62 Kane, T. R., Reflection of flexural waves at the edge of a plate, *J. Appl. Mech.* **21**, 213-220, 1954; AMR **7**(1954), Rev. 2760.
63 Flinn, E. A., III, Exact transient solution of some problems of elastic wave propagation, Ph. D. Thesis, Calif. Inst. of Tech., Pasadena, 1960.
64 Buchwald, V. T., Transverse waves in elastic plates, *Quart. J. Mech. Appl. Math.* **11**, 498-508, 1958; AMR **12**(1954), Rev. 2881.
65 Goodier, J. N., and Jahsman, W. E., Propagation of a sudden rotary disturbance in an elastic plate in plane stress, *J. Appl. Mech.* **23**, 284-286, 1956; AMR **9**(1956), Rev. 1384.
66 Ghosh, J., Longitudinal vibrations of a hollow cylinder, *Bull. Calc. Math. Soc.* **14**, 31-40, 1923-24, and Torsional vibrations of a circular tube, same Journ. **13**, 217-20, 1922-23.
67a Herrmann, G., and Mirsky, I., Three dimensional and shell theory analysis of axially-symmetric motions of cylinders, *J. Appl. Mech.* **23**, 563-568, 1956; AMR **9**(1956), Rev. 2113.
b Mirsky, I., and Herrmann, G., Axially symmetric motions of thick cylindrical shells, *J. Appl. Mech.* **25**, 97-102, 1958; AMR **11**(1958), Rev. 1989.
67c Mirsky, I., and Herrmann, G., Nonaxially symmetric motions of cylindrical shells, *J. Acoust. Soc. Amer.* **29**, 1116-1123, 1957; AMR **11**(1958), Rev. 1131; errata: same Journal **31**, p. 250, 1959.
68a Greenspon, J. E., Flexural vibrations of a thick-walled circular cylinder according to exact theory of elasticity, *J. Aero/Space Sci.* **27**, 37-40, 1960; AMR **13**(1960), Rev. 3910.
b Greenspon, J. E., Vibrations of thick shells in a vacuum, ONR Rep. Proj. 385-412, Tech. Rep. No. 1, Feb. 1959; see also *J. Acoust. Soc. Amer.* **31**, 1682-1683, 1959.
69 Gazis, D. C., Three-dimensional investigation of the propagation of waves in hollow circular cylinders I: Analytical Foundation, and II: Numerical Results, *J. Acoust. Soc. Amer.* **31**, 568-578, 1959; AMR **13**(1960), Rev. 2823.
70 Rayleigh, Lord, The theory of sound, 2nd Ed., **1**, Chap. 10A, Dover Publ., New York, 1945.
71 Lin, T. C., and Morgan, G. W., A study of axisymmetric vibrations of cylindrical shells as affected by rotatory inertia and transverse shear, *J. Appl. Mech.* **23**, 255-261, 1956; AMR **9**(1956), Rev. 1722.
72a Naghdi, P. M., and Cooper, R. M., Propagation of elastic waves in cylindrical shells, including effects of transverse shear and rotatory inertia, *J. Acoust. Soc. Amer.* **28**, 58-63, 1956; AMR **9**(1956), Rev. 2848.
b Cooper, R. M., and Naghdi, P. M., Propagation of nonaxially symmetric waves in elastic cylindrical shells, *J. Acoust. Soc. Amer.* **29**, 1365-1373, 1957.
73 Yu, Y. Y., Vibrations of thin cylindrical shells analyzed by means of Donnell-type equations, *J. Aero/Space Sci.* **25**, 699-715, 1958; AMR **12**(1959), Rev. 3839.
74 Rayleigh, Lord, On waves propagated along the plane surface of an elastic solid, *Proc. Lond. Math. Soc.* **17**, 4-11, 1887.
75 Lamb, H., On the propagation of tremors over the surface on an elastic solid, *Phil. Trans. Roy. Soc. Lond.* (*A*) **203**, 1-42, 1904.
76 Bullen, K. E., An introduction to the theory of seismology, Cambridge Univ. Press, 2nd Ed., 1953; AMR **7**(1954), Rev. 2044.
77 Broberg, K. B., Shock waves in elastic and elastic-plastic media, Kungl. Fortifikationsforvaltningen Befastningsbyran, Rep. 109:12, Stockholm, 1956: AMR **10**(1957), Rev. 1023.
78 Ewing, W. M., and Press, F., Surface waves and guided waves, Handbuch der Physik **47**, Springer-Verlag, Berlin, 75-118, 1956.
79a Petrashen, G. I., On the rational method of solving problems in dynamic elasticity for layered isotropic media with plane parallel boundaries (in Russian), Uch. Zap. LGU no. 208 (Rep. State Univ. Leningrad), 5-57, 1956; *Ref. Zh. Mekh.* no. 3, 1957, Rev. 3433; AMR **12**(1959), Rev. 1808.
b Petrashen, G. I., and Uspenskii, I. N., The propagation of waves in layered isotropic elastic media (in Russian), Uch. Zap. LGU no. 208 (Rep. State Univ. Leningrad), 58-141, 1956; *Ref. Zh. Mekh.*, no. 5, 1957, Rev. 5913; AMR **12**(1959), Rev. 1807.
80 Brekhovskikh, L. M., Waves in layered media (in Russian), Izdat. Akad. Nauk SSSR, Moscow, 1957.
81 Goodier, J. N., The mathematical theory of elasticity, Surveys in Appl. Math. I, John Wiley and Sons, Inc., New York, 1-47, 1958; AMR **12**(1959), Rev. 633.
82 Sneddon, I. N., and Berry, D. S., The classical theory of elasticity, dynamic problems, Handbuch der Physik **6**, Springer-Verlag, Berlin, 107-122, 1958.
83 Sauter, F., Der elastische Halbraum bei einer mechanischen beeinflussung seiner oberflache, *ZAMM* **30**, 203-215, 1950; AMR **4**(1951), Rev. 1949.
84a Pekeris, C. L., The seismic surface pulse, *Proc. Nat. Acad. Sci.* **41**, 469-480, 1955.
b Pekeris, C. L., The seismic buried pulse, *Proc. Nat. Acad. Sci.* **41**, 629-639, 1955.
c Pekeris, C. L., and Lifson, H., Motion of the surface of a uniform elastic half-space produced by a buried pulse, *J. Acoust. Soc. Amer.* **29**, 1233-1238, 1957; errata: same Journal **30**, 365, 1958.
85a Miles, J. W., Homogeneous solutions in elastic wave propagation, Presented at Inst. Geophys. 10th Annual Conf., Univ. Calif., Berkeley, Dec. 1957, Space Technology Labs. Rep. GM-TR-0165-00350, EM 8-6, Apr. 2, 1958.
b Craggs, J. W., On the propagation of stress in a half-space, ONR Tech. Rep. no. 44, Contr. Nonr-562 (10), NR-064-406, Brown Univ., Oct., 1958.
86 Sherwood, J. W. C., Elastic wave propagation in a semi-infinite solid medium, *Proc. Phys. Soc. Lond.* **71**, 207-219, 1958.
87 Lang, H. A., The complete solution for an elastic half-space under a point step load, Rep. no. P-1141, Rand Corp., Oct. 1957.
88a Baron, M., Bleich, H. H., Dimaggio, F. L., and others, An informal progress report on some theoretical investigations on the

vulnerability of deep underground openings in rock, Proc. 2nd Protective Constr. Symp. 2, Rand Corp., 925-961, Mar. 1959, see also *J. Appl. Mech.* 26, 678-679, 1959.

b Knopoff, L., and Gangi, A. F., Seismic reciprocity, *Geophysics* 24, 681-691, 1959.

89 Knopoff, L., and Gilbert, F., First motion methods in theoretical seismology, *J. Acoust. Soc. Amer.* 31, 1161-1168, 1959; AMR 13(1960), Rev. 2826.

90 Goodier, J. N., Jahsman, W. E., and Ripperger, E. A., An experimental surface-wave method for recording force-time curves in elastic impact, *J. Appl. Mech.* 26, 3-7, 1959; AMR 13(1960), Rev. 177.

91 Chao, C. C., Dynamic response of an elastic half space to tangential surface loadings, Paper 60-APM-29, Presented at 23rd Nat. Conf. Appl. Mech., ASME, Penn State Univ., June 1960.

92 Sneddon, I. N., Stress produced by a pulse of pressure moving along a surface of a semi-infinite solid, *Rend. Circ. Mat. di Palermo* 2, 57-62, 1952; AMR 6(1953), Rev. 383.

93 Cole, J., and Huth, J., Stresses produced in a half plane by moving loads, *J. Appl. Mech.* 25, 433-436, 1958; AMR 12(1959), Rev. 6027.

94 Miles, J. W., On the response of an elastic half-space to a moving blast wave, Paper 60-APMW-9, Presented at West Coast Conf. Appl. Mech., ASME, Calif. Inst. of Tech., June 1960.

95 Ang, D. D., Transient motion of a line load on the surface of an elastic half-space, GALCIT Rep. no. SM 59-14, Calif. Inst. of Tech., June 1959.

96 Mitra, M., Disturbance produced in an elastic half-space by an impulsive twisting moment applied to an attached rigid circular disc, *ZAMM* 38, 40-43, 1958; AMR 11(1958), Rev. 4870.

97 Flitman, L. M., Dynamic problem of the die on an elastic half-space, *Appl. Math. Mech.* 23, 997-1008, 1959 (ASME transl. *Prikl. Mat. Mekh.* 23, 697-705, 1959).

98 Pinney, E., Surface motion due to a point source in a semi-infinite elastic medium, *Bull. Seism. Soc. Amer.* 44, 571-590, 1954; AMR 8,(1955), Rev. 1880.

99 Dix, C. H., The mechanism of generation of long waves from explosions, *Geophysics* 20, 87-103, 1955.

100 Garvin, W. W., Exact transient solution of the buried line source problem, *Proc. Roy. Soc. Lond.*, (A) 234, 528-541, 1956.

101 Mitra, M., Exact transient solution of the buried line source problem for an asymmetric source, *ZAMP* 9a, 322-331, 1958; AMR 12(1959), Rev. 4967.

102 Brekhovskikh, L. M., On the attenuation of Rayleigh waves during propagation along an uneven surface, *Soviet Physics—Doklady* 4, 150-153, 1959 (Amer. Inst. Phys. Transl.).

103 Viktorov, I. A., The effects of surface defects on the propagation of Rayleigh waves, *Soviet Physics—Doklady* 3, 304-306, 1958 (Amer. Inst. of Phys. Transl.); AMR 12(1959), Rev. 5522.

104 de Bremaecher, J. Cl., Transmission and reflection of Rayleigh waves at corners, *Geophysics* 23, 253-266, 1958.

105 Eringen, A. C., and Samuels, J. C., Impact and moving loads on a slightly curved elastic half-space, *J. Appl. Mech.* 26, 491-498, 1959; AMR 13(1960), Rev. 2820.

106 Poisson, M., Memoire sur l'equilibre et le mouvement des corps elastiques, *Mem. Acad. Sci. Paris* 8, 356, 623, 1829.

107 Stokes, G. G., On the dynamical theory of diffraction, *Trans. Camb. Phil. Soc.* 9, 1-62, 1851-56.

108 Lamé, M. G., Lecons sur la theorie mathematique de l'elasticite des corps solides, Bachelier, Paris, 1852.

109 Sternberg, E., On the integration of the equations of motion in the classical theory of elasticity, ONR Rep., Contr. Nonr-562(25), NR-064-431, Tech. Rep. no. 7, Brown Univ., July 1959.

110 Eason, G., Fulton, J., and Sneddon, I. N., The generation of waves in an infinite elastic solid by variable body forces, *Phil. Trans. Roy. Soc. Lond.*, (A) 248, 575-607, 1955-56; AMR 9(1956), Rev. 3523.

111 das Gupta, S. C., Waves and stresses produced in an elastic medium due to impulsive radial forces and twist on the surface of a spherical cavity, *Geofis. Pura. Appl., Milano* 27, 3-8, 1954; AMR 8(1955), Rev. 356.

112 Goldsmith, W., and Allen, W. A., Graphical representation of the spherical progagation of explosive pulses in elastic media, *J. Acoust. Soc. Amer.* 27, 47-55, 1955; AMR 9(1956), Rev. 2470.

113 Eringen, A. C., Elasto-dynamic problem concerning the spherical cavity, *Quart. J. Mech. Appl. Math.* 10, 257-270, 1957; AMR 11(1958), Rev. 395.

114 Bishop, R. E. D., On dynamical problems of plane stress and plane strain, *Quart. J. Mech. Appl. Math.* 6, 2, 250-254, 1953; AMR 7(1954), Rev. 34.

115 Viktorov, I. A., Rayleigh-type waves on a cylindrical surface, *Soviet Physics—Acoust.* 4, 131-136, 1958 (Amer. Inst. Phys. Transl.); AMR 12(1959), Rev. 3856.

116 Knopoff, L., Diffraction of elastic waves, *J. Acoust. Soc. Amer.* 28, 217-229, 1956; AMR 9(1956), Rev. 3524.

117 DeHoop, A. T., Representation theorems for the displacement in an elastic solid and their application to elastodynamic diffraction theory, D. Sc. Thesis, Technische Hogeschool, Delft, 1958.

118 Filippov, A. F., A three-dimensional problem of diffraction of an elastic wave at a sharp edge, *Appl. Math. Mech.* 23, 989-996, 1959 (ASME Transl. *Prikl. Mat. Mekh.*).

119a Ang, D. D., Elastic waves generated by a force moving along a crack, *J. Math. Phys.* 38, 246-256, 1960.

b Ang, D. D., and Williams, M. L., The dynamic stress field due to an extensional dislocation, Proc. 4th Midwest Conf. Solid Mech., Univ. Texas, 36-52, 1959; AMR 13(1960), Rev. 176.

120a Knopoff, L., Scattering of compressional waves by spherical obstacles, *Geophysics* 24, 30-39, 1959; AMR 13(1960), Rev. 4537.

b Knopoff, L., Scattering of shear waves by spherical obstacles, *Geophysics* 24, 209-219, 1959.

121 Nagasse, M., Diffraction of elastic waves by a spherical surface, *J. Phys. Soc. Japan* 11, 279-301, 1956; AMR 9(1956), Rev. 3522.

122 White, R. M., Elastic wave scattering at a cylindrical discontinuity in a solid, *J. Acoust. Soc. Amer.* 30, 771-785, 1958.

123 Gilbert, J. F., Elastic wave interaction with a cylindrical cavity, Rep. to WDIE, AFBMD, Contr. AF 04(647)-342, Dec. 1959, E. H. Plesset Assoc. Inc., Los Angeles, Calif.

124 Gilbert, F., and Knopoff, L., Scattering of impulsive elastic waves by a rigid cylinder, *J. Acoust. Soc. Amer.* 31, 1169-1175, 1959; AMR 13(1960), Rev. 3923.

125 Sternberg, E., and Chakravorty, J. G., On inertia effects in a transient thermoelastic problem, *J. Appl. Mech.* 26, 503-509, 1959; AMR 13(1960), Rev. 3291.

126 Lessen, M., Thermoelastic waves and thermal shock, *J. Mech. Phys. Solids* 7, 77-84, 1959; AMR 12(1959), Rev. 4971.

127 Nowacki, W., Some dynamic problems of thermoelasticity, *Arch. Mech. Stos.* 11, 259-283, 1959.

128 Boley, B. A., Thermo-elasticity and thermo-plasticity, Structural Mechanics, Proc. 1st Symp. Naval Struct. Mech., Pergamon Press, New York, 378-406, 1960.

129 Brillouin, L., Wave propagation and group velocity, Academic Press, New York, 1960.

130 Karal, F. C., Jr., and Keller, J. B., Elastic wave propagation in homogeneous and inhomogeneous media, *J. Acoust. Soc. Amer.* 31, 694-705, 1959; AMR 13(1960), Rev. 4533.

131 Babich, V. M., and Alekseev, A. S., On the ray method of calculation of the intensity of wave fronts, *Izv. Akad. Nauk SSSR, Ser. Geofiz.* 17-31, 1958 (in Russian).

132 Friedlander, F. G., Sound pulses, Cambridge Univ. Press, 1958.

133 Angona, F. A., Two-dimensional modeling and its application to seismic problems, *Geophysics* 25, 468-482, 1960; AMR 13(1960), Rev. 4934.

134 Ivakin, B. N., Similarity in elastic wave phenomena, Parts I and II, *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, 1269-1281, 1384-1388, 1956 (in Russian); *Ref. Zh. Mekh.* no. 8, 1957, Revs. 9358, 9359; AMR 12(1959), Revs. 3339, 3334 (English Transl. of Pt. I by V. Chobotov, Space Technology Labs., Los Angeles, Calif.).

135 Hetenyi, M., Photoelasticity and photoplasticity, Structural Mechanics, Proc. 1st Symp. Naval Struct. Mech., Pergamon Press, New York, 483-505, 1960.

Analytical Methods in Applied Mechanics

(See also Revs. 6160, 6164, 6165, 6166, 6167, 6171, 6175, 6191, 6193, 6251, 6277, 6279, 6290, 6350, 6380, 6510, 6528, 6585, 6592, 6593, 6629, 6657, 6694, 6712)

6138. Vinti, J. P., and Dressler, R. F., The functional synthesis of linear plots, *J. Res., Nat. Bur. Stands.* **64** C (*Engng. Instrumentation*) 2, 115-119, Apr./June 1960.

Authors present the most general mathematical formulation that would represent linear relationships between experimentally determined parameters that plot linearly on Cartesian graph paper, semi-log paper, or log-log paper. Method was developed to represent experimental data on elastic shells.

G. W. Housner, USA

6139. Conway, H. D., The approximate analysis of certain boundary-value problems, *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 275-277, June 1960.

A simple method is given which is suitable for the approximate analysis of certain boundary-value problems, including, for example, the small deflections of clamped plates and the torsion of prismatic bars. The analysis is particularly simple and lends itself well to the use of the digital computer. The method is applied to four problems, the uniformly loaded, clamped square, and equilateral-triangle plates, and the torsion of bars of square and hexagonal cross section. The results agree well with the exact solutions, where these are known. Method could be extended to fluid-flow problems, buckling and vibration of plates, plane stress- and strain problems, etc.

From author's summary by V. Kopriva, Czechoslovakia

6140. Campanato, S., Boundary value problem of linear differential equation of the elasticity type, Parts 1 and 2 (in Italian), *Ann. Scu. Norm. Sup., Pisa* **13**, 2, 223-258, Apr./June 1959; **13**, 3, 275-302, July/Sept. 1959.

Paper studies boundary-value problems for a linear system of partial derivative equations of the second order, which generalizes the classical equations of elasticity. The problems of Dirichlet, Neumann, Picone, etc., are treated for this general system and existence theorems for each case are proved. The paper requires a high mathematical background and will present difficulties for the designers. The results are only of mathematical interest.

D. Gh. Ionescu, Roumania

6141. Solomon, L., Connection between the Prandtl constant, geometrical torsional rigidity and Green function (in French), *Bull. Math.* (50) **2**, 3, 329-341, 1958.

In the case of multiply connected domain the limiting values of the Prandtl function at the inner boundaries have been defined as Prandtl's constants. The relations have then been established as per title. The significance of these relations in solving torsion problems has not been discussed fully.

S. C. Das, India

6142. Bass, J., Turbulent solution of certain partial differential equations (in French), *C. R. Acad. Sci., Paris* **249**, 16, 1456-1457, Oct. 1959.

6143. Butler, D. S., The numerical solution of hyperbolic systems of partial differential equations in three independent variables, *Proc. Roy. Soc. Lond.* (*A*) **255**, 1281, 232-252, Apr. 1960.

An original method of integration is described for quasi-linear hyperbolic equations in three independent variables. The solution is constructed by means of a step-by-step procedure, employing difference relations along four bicharacteristics and one time-like ordinary curve through each point. From these difference relations the derivatives of the dependent variables at the unknown point are eliminated. The solution at any point can then be computed, with an error proportional to the step size cubed, without referring to conditions outside its domain of dependence.

The application of the method to the systems of equations governing unsteady plane motion and steady supersonic flow of an inviscid, nonconducting fluid is discussed in detail. As an example of the use of the method, the flow over a particular delta-shaped body has been computed.

From author's summary by R. Conti, Italy

6144. Esch, R. E., A necessary and sufficient condition for stability of partial difference equation problems, *J. Assn. Comput. Mach.* **7**, 2, 163-175, Apr. 1960.

In a volume by R. D. Richtmeyer ["Difference methods for initial value problems," New York, Interscience, 1957], several conditions for the stability of numerical methods for solving partial differential equation problems are formulated. These include the so-called von Newmann necessary condition and several alternative sufficient conditions. Present paper studies the one-dimensional heat-conduction equation and the one-dimensional wave equation. The author employs the Jordan canonical matrix decomposition to develop a simple condition which is both necessary and sufficient and which includes Richtmeyer's conditions as special cases.

Y. L. Luke, USA

6145. Takeda, S., Koido, S., Iinuma, K., Onuma, K., Susuki, H., Goto, S., Contributions to the Laplace transformation method, Tech. College, Hosei Univ., Tokyo, Rep. 4, 66 pp., Aug. 1959.

This report is a printed version of seven seminars on what might be called Applied Laplace Transforms. The first chapter contains a theorem for finding the transform of an arbitrary function terminated at an arbitrary point. In the second chapter several direct and inverse transforms of functions that are integrated several times are given. The third chapter is devoted to transforms of exponential functions whose index includes sine or cosine functions. The last four chapters contain detailed solutions for the thermometer problem, straight beam problems and micron-pressure gage response problem.

The authors indicate that it is their intent to present material that is not mathematically sophisticated but is useful to those engineers who apply the Laplace transform method. The principal contribution of this report would seem to be the tables of the transforms that are worked out. Students of beam theory will undoubtedly be interested in the detailed analysis of the beam problem that includes column loads.

E. E. Covert, USA

6146. Wittrick, W. H., Some simple transformation functions for square and triangular holes with rounded corners, *Aero. Quart.* **11**, 2, 195-199, May 1960.

The numerical coefficients of the second and third terms of a three-term approximation to the Schwartz-Christoffel transformation for infinite regions bounded, respectively, by the square and the equilateral triangle are replaced by parameters whose variation causes variation in the curvature of the vertices of the figure. This is certainly not a new idea [e.g., M. Greenspan, *Quart. Appl. Math.* 1944, p. 60; R. M. Evan-Iwanowski, *J. Appl. Mech.* p. 336, 1956; AMR **10**(1957), Rev. 1050] but it is useful to have the two formulas worked out explicitly.

H. Deresiewicz, Italy

6147. Roitenberg, Ia. N., Some problems of the theory of dynamic programming, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 4, 943-955, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Let [1] $\sum_{k=1}^{n} f_{jk}(D)y_k = x_j(t) + q_j(t)$, $(j = 1, \ldots, n)$, be equations of motion of a "continuous" dynamical system, with prescribed

initial conditions at t_0, where $D = d/dt$, and f_{jk} are polynomials whose coefficients are functions of t. Author treats problem of determining the additive modifications $q_j(t)$ of original driving functions $x_j(t)$ such that preselected generalized coordinates $y_{p_1}, \ldots, y_{p_m}$, $m \leq n$, will be prescribed functions $r_{p_1}(t), \ldots, r_{p_m}(t)$, respectively, for $t \geq t_1 > t_0$, t_1 a prescribed value of t.

Author outlines theoretical approximative method, asserted suitable for computer utilization, of selecting $q_j(t)$ as step functions constant over interval (t_0, t_1) and over each of "small" subintervals in subdivision of (t_1, ∞), after first reducing [1] to system of first-order equations by classical transformation (i.e. $z_{k1} = y_k$, $z_{k2} = Dy_k$, etc.) (Reviewer's note: Here author misleadingly seems to imply that *any* of new dependent variables z_{kb} may be ascribed arbitrary behaviors, $r(t)$, rather than, as it appears to the reviewer, at most one for each k, presumably z_{k1}, i.e. y_k.) Resulting system's classical integral form of solution utilizing fundamental matrix of corresponding homogeneous system is used to determine the $q_j(t)$ as solutions of systems of linear algebraic equations resulting from step function assumption.

Analogous "impulsive" problem, where operator D^s in [1] is replaced by difference operator T^s defined by $T^s y_k(t) = y_k(t + s\tau)$, τ a positive constant, yielding difference equation analog of [1], is treated in entirely parallel fashion.

Not discussed are: (a) influence of selection of subdivision of interval (t_1, ∞) on quality or approximation to solution; (b) consistency (i.e. solvability) of linear algebraic equation systems determining $q_j(t)$; (c) computer experience with the method on typical examples.

R. F. Rinehart, USA

Book—6148. Brillouin, L., Science and information theory, New York, Academic Press Inc., 1956, xvii + 320 pp. $6.80.

During the last fifteen years a new scientific theory had been developed. It is called information theory or communication theory. Mainly of a mathematical and probabilistic nature, it provides important results concerning the general problem of communication. The book under review shows that this theory has applications to several other fields of science. About half of the volume deals with problems that have been connected with information theory from the beginning: coding, analysis of signals, language, computers. The other half is devoted to physics: thermodynamics and the general theory of observations. The work of the author and some other scientists is presented here in a rounded-off form. From the author's preface: "It (information theory) enables one to solve the problem of Maxwell's demon and to show a very direct connection between information and entropy.... Whenever an experiment is performed in the laboratory it is paid for by an increase of entropy and a generalized Carnot principle states that the price paid in increase of entropy must always be larger than the amount of information gained."

The knowledge of mathematics and physics required from the reader is about equal to the level of a B.Sc. degree.

For both physicists and information theorists this book is very important. It also provides a good introduction into the meaning and possibilities of information theory. As a minor deficiency reviewer considers the fact that the presentation of the theory is not always as clear and concise as might be wished.

A. J. Stam, Holland

Book—6149. Henderson, F. M. Elliptic functions with complex arguments, Ann Arbor, Mich., University of Michigan Press, 1960, v + 38 pp. + charts. $8.

Let $z = x + iy$, $w = u + iv$, $u(z) = \int_0^z [(1 - t^2)(1 - k^2t^2)]^{-1/2}\, dt$,

$sn(w) = z$, $E(w,k) = \int_0^w [(1 - k^2t^2)/(1 - t^2)]^{1/2}\, dt \cdot K = K(k) = u(\pi/2)$

and $E = E(k) = E(\pi/2,k)$ are the complete elliptic integrals of the first and second kinds, respectively. Also $K' = K(k')$ and $E' = E(k')$ where $k'^2 = 1 - k^2$. All tables are to four significant figures. Table I gives $sn(w)$ for all pairs of $u/K = 0(.1)1.0$, $v/K' = 0(.1)1.0$, and for each $k = \sin\alpha$, $\alpha = 1°, 5°(5°)85°, 89°$. For each k, there is also a graph on the page facing the corresponding table for that k. Tables II and III are similar and deal with the related Jacobian elliptic functions $cn(w)$ and $dn(w)$, respectively. For u, v and k as above, Table IV gives $E_R'\left(\frac{u}{K}, \frac{v}{K'}, \alpha\right) = E_R/E$ and $E_I'\left(\frac{u}{K}, \frac{v}{K'}, \alpha\right) = E_I/(K' - E')$ where $E(w) = E_R + iE_I$. For each k, graphs are also provided as above.

Tables I, II and III are new. However, the author seems unaware of a previous table closely related to Table IV. E. N. Fox and J. M. McNamee ["The two-dimensional potential problem of seepage into a cofferdam," *Phil. Mag.* (7) **39**, pp. 165–203, 1948] have studied $f_1(u/K, v/K', \alpha) + if_2(u/K, v/K', \alpha) = E[E_R' - u/K] + i(K' - E')[E_I' - v/K']$. For u/K, v/K' and α above and also $\alpha = 2°, 4°, 86°, 87°, 88°$, they tabulated f_1 to three significant figures. They show that it is not necessary to tabulate f_2 since it is f_1 with changed parameters. For the convenience of the reader, the reviewer records the connections between the present functions.

$$E_R'(u/K, v/K, k) = (K/E - 1)[E_I'(1 - v/K', 1 - u/K, k') - 1] + (u/K)(K/E)$$

$$E_I'(u/K, v/K, k) = [E'/(K' - E')][E_R'(1 - v/K, 1 - u/K, k') - 1] + (v/K')[K'/(K' - E')]$$

An introduction gives a short summary of formulas on elliptic integrals and functions. Construction of the tables and charts is discussed. A worthy feature is a survey of applications. These include conformal mapping, percolation under a dam, and free boundary problems.

Y. L. Luke, USA

6150. Lowell, H. H., Tables of the Bessel-Kelvin function ber, bei, ker, kei, and their derivatives for the argument range 0(0.01)107.50, NASA TR R-32, 291 pp., 1959.

The functions of title are modified Bessel functions of the first and second kind of order zero and their derivatives along the ray $\pi/4$. The functions enter into numerous applied problems and present tables are the most extensive available. Ber and bei and their derivatives are given in floating-point form generally to 13 or 14 significant figures. Likewise for ker and kei, but the number of significant figures varies from 9 to 13 as explained in the introduction. Appendix A gives a list of symbols and definitions. Appendix B tabulates the function at special arguments to facilitate investigation of various interpolation schemes by the user—a task which should have been performed by the author. Appendix C gives coefficients in floating-point form of powers of x in both the ascending and asymptotic series for each function. The typography is excellent and the tables are attractively arranged.

Y. L. Luke, USA

6151. Saermark, K., A note on addition theorems for Mathieu functions (in English), *ZAMP* **10**, 4, 426–428, July 1959.

6152. Johnson, R. C., Ball, W. E., Burggrave, W. F., Heiny, R. L., Russell, J. L., and Sweeny, R. F., Mathematics, computers, operations research, and statistics, *Indust. Engng. Chem.* **52**, 4, 359–367, Apr. 1960.

Computing Methods and Computers

(See also Revs. 6143, 6152, 6171, 6213, 6358, 6502, 6505, 6651)

Book—6153. Hollingdale, S. H., High speed computing; methods and applications, New York, The Macmillan Co., 1959, xii + 244 pp. $5.

This slim volume includes more information about electronic digital computers than one might expect from its size, despite the fact that it does not presuppose any prior knowledge of the field on the part of the reader. It begins with a general introduction to computing and computers, goes on to explain the various ways of representing numbers, and introduces the reader to the elements of machine programming. This is followed by an historical sketch of a few of the earliest machine developments, and by detailed descriptions of two machines—EDSACI and DEUCE—which, though not the most modern, contain most of the characteristic features of up-to-date machines. Next we find a systematic functional description of storage elements, and a brief introduction to the logical design of arithmetic and control circuits. One chapter is devoted to miscellaneous observations on the operation of a computing service, and four chapters to computer applications.

From the author's preface we quote the statement "the reader is assumed to have little formal mathematics beyond that normally acquired at school." This is, if anything, an understatement, even though the author has reference to British schools with their rather high requirements in mathematics. In fact most of the book should be intelligible to high school students, and this is one of its most valuable features. This reviewer knows of no comparable text that is as easy and attractive to read.

In the historical part of the book, there are interesting references to papers of Babbage's contemporaries, especially of Lady Lovelace, the "first programmer," which shed new light on early ideas in this field. It is perhaps unfortunate that there is no reference to the pioneering work of Stibitz at Bell Telephone Laboratories, nor to the early computer development at Birbeck College in London. While it is in keeping with the character of the book to give isolated examples rather than complete coverage, incompleteness in historical matters can be distorting.

The description of the EDSAC and DEUCE computers in chapters 5 and 6 is more detailed than anything else in the book, and some readers will perhaps wish to skip parts of it. It should not trouble American readers that both these machines are British, and that many of the examples of applications throughout the book are taken from British experience; the differences between British and American conditions are minor. That both machines are rather old-fashioned is perhaps unavoidable, given the rapid development of the computer field and the long time lapse between the first plan for a manuscript and the appearance of the finished book.

The last three chapters contain brief discussions of specialized and isolated problem areas to which computers have been applied: the simulation of sequences of random events (Monte Carlo method), the control of industrial processes, and the translation of languages. Why just these? They are quite atypical, i.e., they do not teach the reader how to find other applications by analogy. They are so narrow that only a selected few will be interested in them for their own sake—the one on random processes is so narrow as to omit applications of this method to military and other operational problems, which account for a good part of its use. What the reader does learn is to respect the versatility of digital computers, to "expect the unexpected" in new computer applications. Nevertheless, many readers will wish to get a more complete survey of machine uses. Chapter 10, which describes a variety of "typical" applications, is rather too short and superficial to satisfy this desire.

There is no mention of numerical analysis, nor of any problems of design engineering. Both omissions are appropriate to a book as elementary as this. It is just this elementary character of the book, together with its brevity, which distinguishes it from others and makes it so appropriate for beginners and amateurs with limited technical background.

F. L. Alt, USA

6154. Brinkerhoff, J. R., Evaluation of preflight risks by means of very high speed digital system simulation, *ARS J.* **30,** 5, 493–495 (Tech. Notes), May 1960.

This paper gives a brief description of the mathematical and programming techniques employed in using a comprehensive digital computer simulation of a complex system to provide, in a short time, accurate data on the chance of successful operation of the system in a specific test. A mathematical model of the system is obtained by writing equations and logical relations specifying the behavior of each part of the system as a function of the states of the other parts and parameters determined by external factors. These equations are then programmed on a computer and after setting parameter values and starting conditions, the equations are solved and the solution is checked against an applicable criterion to determine whether performance is satisfactory. In practice, this procedure may become extremely complicated, and it is often abandoned in favor of other methods which analyze the various parts of the system separately or which make broad simplifying assumptions and generalizations to enable simple analytical treatment of the system.

From author's summary

6155. Bellar, F. J., Jr., An iterative solution of large scale systems of simultaneous linear equations, AFOSR-TN 60-550 (Univ. Maryland, Inst. Fluid Dynam. Appl. Math. TN-BN-211), 9 pp., May 1960.

An algorithm proposed by Lanczos for the generation of an approximate solution of a large-scale system of simultaneous linear equations is improved so that the number of iterations required for a desired degree of accuracy is minimized.

From author's summary

6156. Collar, A. R., Iterative solutions of the equations for plane oblique shock waves, *J. Roy. Aero. Soc.* **63,** 587, 669–672 (Tech. Note), Nov. 1959.

In flow at Mach number M let ε be inclination of shock that deflects flow by angle δ. Given M and δ, then one can find ε from shock polar $x^3 + Cx^2 - Ax + (B - AC) = 0$, where $x = \cot \varepsilon$ and A, B, C are positive functions of M and δ. Author proves that sequence of positive x_n defined by $x_{n+1}^2 = A - B/(x_n + C)$, $x_0 = A^{1/2}$ converges to weak shock value (when it exists).

J. Giese, USA

6157. Di Pasquale, S., Generalization of the method of finite differences for the solution of equations with partial derivatives that can be brought down to a Laplace equation (in Italian), *G. Gen. Civ.* **97,** 6, 486–497, June 1959.

In numerical solution of Laplace's or Poisson's equation in two dimensions on a simply connected region, boundary points generally require special treatment since commonly used rectangular lattices usually do not include boundary points. Author proposes to use different lattices, somewhat similar to polar coordinates constructed as follows. Let O be a point inside the region, and let $A_0, A_1, \ldots, A_{n-1}$ be points on boundary. Divide each triangle $OA_{k-1}A_k$ into N^2 congruent triangles, whose vertices form desired lattice. At interior points P of $OA_{k-1}A_k$ Laplacian is approximated in terms of functional values at P and its six neighbors. Same formula applies to all lattice points of $OA_{k-1}A_k$, and coefficients depend on shape of triangle. Point O and points of OA_k require special treatment. Author interprets his formula for Laplacian on skewed hexagonal lattice in terms of membrane analogy.

J. H. Giese, USA

6158. Weisfeld, M., Orthogonal polynomials in several variables (in English), *Numerische Mathematik* **1,** 1, 38–40, Jan. 1959.

Generalization of the method of Householder ["Principles of numerical analysis," McGraw-Hill, 1953, p. 221] and Stiefel [Nat. Bur. of Standards, AMS no. 49, pp. 1–22].

Two polynomials $f(x_1, \ldots, x_n)$ and $g(x_1, \ldots, x_n)$ are mutually orthogonal if

$$\int\int \ldots \int wfg \, dx_1 \ldots dx_n = 0,$$

where $w(x_1, \ldots, x_n)$ is a fixed weight function. Now let a monomial $x_1^{j_1} x_2^{j_2} \ldots x_n^{j_n}$ be denoted by φ_j, with suffix $j = (j_1, \ldots, j_n)$. The set J of all possible j's can be ordered as follows: $(i_1, \ldots, i_n) < (j_1, \ldots, j_n)$ if and only if $i_1 + \ldots + i_n < j_1 + \ldots + j_n$ or $i_1 + \ldots + i_n = j_1 + \ldots + j_n$ and, for some $k \leqq n$, $i_k + \ldots + i_n < j_k + \ldots + j_n$. Consider the sequence of monomials $\Phi = \{\varphi_j : j \varepsilon J\}$. (For $n = 3$, e.g., we have the sequence $\varphi_{000} = 1$, $\varphi_{100} = x_1$, $\varphi_{010} = x_2$, $\varphi_{001} = x_3$, $\varphi_{200} = x_1^2$, $\varphi_{110} = x_1 x_2$, $\varphi_{101} = x_1 x_3$, $\varphi_{020} = x_2^2$, $\varphi_{011} = x_2 x_3$, $\varphi_{002} = x_3^2$, $\varphi_{300} = x_1^3$, $\varphi_{210} = x_1^2 x_2, \ldots$.) Its orthogonalization is a sequence $\psi = \{\psi_j : j \varepsilon J\}$ such that any two ψ_j's are mutually orthogonal and each φ_i can be written as a linear combination of elements of $\{\psi_k : k \leqq i\}$. Paper gives a method of constructing ψ.

An important special case is noted where the products $b_i(x_1, \ldots, x_p) \cdot k_j(x_{p+1}, \ldots, x_n)$ constitute the set of orthogonal polynomials in n variables, $b_i(x_1, \ldots, x_p)$ and $k_j(x_{p+1}, \ldots, x_n)$ being members of the sets of orthogonal polynomials in p and $n - p$ variables, respectively.

S. Moriguti, Japan

6159. Milnes, H. W., and Potts, R. B., Numerical solution of partial differential equations by boundary contraction, *Quart. Appl. Math.* **18**, 1, 1-13, Apr. 1960.

The numerical solution of partial differential equations, with given boundary conditions, is made difficult by the large number of netpoints required, usually much too large for the fast memory of electronic computers. The new method described here, of "boundary contraction," avoids this difficulty as follows: Starting inward from the given boundary, we draw similar curves in the interior of the domain (e.g., if the boundary is a circle, we draw a family of concentric circles inside the domain); we then deduce function values on the nearest "contracted boundary" from the given function values on the outside boundary. The process can be repeated step by step, and we need not "remember" function values in regions already covered by this contraction procedure. Eventually, we contract right down to the center of the region, and the process is terminated. In general, the memory requirements for partial differential equations in N dimensions are reduced, by this method, to the ones for conventional methods in $n - 1$ dimensions. Solutions involving a total number of netpoints of the order of 250,000 were obtained, without going beyond the core memory of an IBM 704 computer. The stability of the process is discussed in detail, and explicit stability conditions are given.

J. M. Blatt, Australia

Analogies

(See also Revs. 6141, 6511, 6520, 6521, 6693)

6160. Standart, G., The theory of analogy, *Collection Czechoslov. Chem. Commun.* **24**, 323-340, 1959.

This paper deals with the application of dimensional considerations in solving differential equations. The author defines ordinary dimensional analysis as the reduction of the number of variables in a problem by means of the π theorem without specific consideration of the governing equations. Similarity is defined as the reduction of the number of parameters in the set of equations describing a problem by division of the variables by appropriate reference values, i.e., by the transformation $x_i = a_i X_i$. The author discusses the more general transformation $x_i = b_i + \sum_j a_{ij} X_j$, which he calls the method of analogy. Reviewer notes that the usage is not the conventional one; also the author's division between similarity and dimensional analysis is debatable; however, these terms are employed in a consistent and reasonably well-defined manner.

While the use of the general transformation had been suggested previously by other investigators, no example had been previously given in which results were obtained different from those obtained from the similarity transformation. This paper presents a number of nontrivial problems in heat and mass transfer where an improvement is obtained. A typical example is the treatment of unsteady heat conduction in an infinite bar with the conductivity a linear function of temperature. Dimensional considerations suggest a a variable $(Xt^{-1/2})$ which reduces the partial differential equation to a nonlinear ordinary differential equation. This was integrated numerically for a number of cases by Beutler and Knudren [*Chem. Engng. Progr. Symposium*, Ser. 49, no. 5, p. 115, 1953]. The applicability of these numerical solutions is extended by similarity considerations and even further by the analogy transformation.

Reviewer believes this paper to be an interesting contribution to the subject.

W. Squire, USA

6161. Cremer, L., and Klotter, K., New aspects of the electromechanical analogies (in German), *Ing.-Arch.* **28**, 27-38, Mar. 1959.

Development of mechanical models of electrical networks primarily for classroom demonstrations is described. An effort has been made to construct models which conform topologically with analogous electrical networks. This simplifies the translation procedure from one system to another.

E. G. Newman, USA

6162. Keropyan, K. K., Employment of the Wheatstone bridge for the solutions of some problems in building mechanics (in Russian), *Trudi Rostovsk.-n/D. Inzh.-Stroit. In-ta* no. 6, 181-185, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 7114.

The known analogy between the equations for the coefficients of influence in elastic systems and the electromotive force of the Wheatstone bridge opens up the possibility of modelling the calculations. It is proposed to use the compensation method for determining the difference in potentials in order to simplify the static and dynamic computations for the problems regarding constructional mechanics, including the determinations for transpositions, lines of influence, deflection moments, intersecting forces, etc.

I. S. Roginskii

Courtesy Referativnyi Zhurnal, USSR

6163. Gray, R. B., Experimental smoke and electromagnetic analog study of induced flow field about a model rotor in steady flight within ground effect, NASA TN D-458, 32 pp., Aug. 1960.

Hovering and steady low-speed forward-flight tests were run on a 4-foot-diameter rotor at a ground height of 1 rotor radius. Measurements were made of the rotor rpm, collective pitch, and forward-flight velocity. Smoke was introduced into the tip vortex and the resulting vortex pattern was photographed from two positions. Using the data obtained from these photographs, wire models of the tip vortex configurations were constructed and the distribution of the normal component of induced velocity at the blade feathering axis that is associated with these tip vortex configurations was experimentally determined at 45° increments in azimuth position from this electromagnetic analog.

From author's summary

Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 6147, 6161, 6183, 6198, 6199, 6200, 6201, 6280, 6380, 6570, 6614, 6617, 6619, 6627)

Book—6164. Breneman, J. W., Mechanics, 3rd ed., New York, McGraw-Hill Book Co., Inc., 1960, x + 262 pp. $5.50.

Book provides introduction to mechanics for students whose mathmatical preparation is limited to algebra and trigonometry.

Topics include equilibrium of coplanar, noncoplanar force systems, friction, centroids and moments of inertia, and dynamics of a particle by force, mass and acceleration; by work and energy; and by impulse and momentum. Simple mechanisms involving screws, pulleys, gears, and gear trains are introduced. Limitations of methods are not emphasized. For example no mention is made of statically indeterminate problems, assumptions involved in solving truss problems are not completely stated, and the difference between moment about a point and about an axis is not too clear. About 390 problems are included with answers to half of them. Many other examples are solved. Several typographical errors were noted.

Appendix includes properties of selected structural and geometrical areas and trigonometric functions. Reviewer believes this book gives a satisfactory introduction to elementary statics but the coverage of dynamics (without using calculus) is too limited to be of practical value.

W. B. Stiles, USA

Book—6165. Champion, F. C., and Davy, N., Properties of matter, 3rd ed., New York, Philosophical Library, Inc., 1959, xvi + 334 pp. $10.

There is need for a reference book that covers the many topics included in the broad title "Properties of Matter" for use by advanced undergraduate and graduate students and practicing engineers and scientists. Such a book should cover these topics in a concise but complete and, especially important, clear manner. The purpose of this book seems to be to provide such a reference.

In general the book does a creditable job. There are eleven chapters (276 pages) on properties of matter which are entitled (1) The acceleration due to gravity, (2) The Newtonian constant of gravitation, (3) Elasticity, (4) Compressibility of solids and liquids, (5) Seismic waves, (6) Capillarity, (7) Surface films, (8) Kinetic theory of matter, (9) Osmotic pressure, (10) Diffusion, and (11) Viscosity. In addition there is a chapter on Units and dimensions and one on Errors of measurement. In each chapter there is a discussion of the basic topic and pertinent subtopics. Considerable space is then devoted to various methods of measuring the properties discussed. Frequently a number of methods are described, and advantages, disadvantages, and limitations are pointed out. Diagrams are used extensively to assist in understanding the various methods. Examples and exercises, with answers, are given so that the student can insure his understanding of the work.

The book also has its short comings. Each reader will undoubtedly find additional topics which should be included. The reviewer is doubtful if twenty per cent of the book should be devoted to capillarity. The space devoted to gravity and elasticity might also be considered excessive. However, a more serious criticism pertains to the lack of adequate references and bibliography. In the entire book there are only 89 footnote references and 31 general references. Of these, twelve of the footnotes and eleven of the general references are for the two supplementary chapters. Seventeen of the footnotes refer to one book [Roberts, "Heat and thermodynamics," fourth edition]. Frequently names are given in the text without a work being referenced. It is also noted that, although this book is supposedly an up-to-date version of the second edition (1951), only six of the referenced works were identified with dates later than 1950.

There are instances where methods and equipment more precise and convient than those described are available. For example, the use of the Ewing extensometer (a rather cumbersome instrument weighing six pounds with a sensitivity of 2×10^{-5} inches) for determining elastic modulus is described, but the Tuckerman extensometer (a relatively simple and rugged device weighing about two ounces with a sensitivity of 2×10^{-6} inches) is not mentioned, although it has been described in the literature and used for more than thirty years. The reviewer found a few sections of the book to be more confusing than helpful, notably the section on Relativity and the Law of Gravitation (pages 50–52). Also the reviewer would prefer to have "standard deviation" rather than "probable error" used in the Errors of measurement chapter.

This book, in spite of its weaknesses, will be a valuable reference for many people.

R. L. Bloss, USA

Book—6166. Shames, I. H., Engineering mechanics, Vol. 1, Statics, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1959, xxxi + 272 pp. $6.35.

This is an undergraduate text presenting a course in statics in which vector methods are heavily emphasized. Vector algebra, to the extent to which it is required, is developed in Chap. 2. A detailed discussion of force, moment, couple and equivalent force systems is given in Chaps. 3 and 4. The equations of equilibrium and free body diagrams are introduced in Chap. 5. Their application to trusses, beams, cables and frictional forces are treated in Chaps. 6 and 7. Graphical methods in structural problems are omitted. Centers of gravity and moments of inertia are discussed in Chap. 8. The concept of a tensor is touched upon lightly here and is further developed with reference to the stress tensor in the following chapter, which is devoted to continuum mechanics (elasticity, hydrostatics, aerostatics and viscous friction). Reviewer feels that this discussion of tensors will not be meaningful to the average undergraduate, and that in places it will be quite confusing, especially in the absence of a definition of a tensor. The book concludes with a chapter on the methods of virtual work and potential energy.

Each chapter contains illustrative examples and a set of problems to which answers are provided. Reviewer regrets that greater emphasis was not placed on three-dimensional problems, to which the vector approach might be more suitable, than on two-dimensional problems, to which its application is at times, as the author conservatively admits, "somewhat clumsy."

T. P. Mitchell, USA

Book—6167. Shames, I. H., Engineering mechanics; Vol. 2: Dynamics, 3rd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc. 1960, xv + 359 pp. + appendix + index $6.75.

In the second volume of this work the following sections are included: Elements of kinematics, Integration of Newton's laws for rectilinear translation, Central-force motion, General motion of a particle, Energy methods, Methods of momentum, The inertia tensor, Euler's equations of motion, Energy considerations for rigid bodies, Motion of a body about a fixed point and momentum equations for deformable media.

In this tome, vector calculus is covered and in the kinematic portion a smattering of differential geometry of space curves is given, which, however, falls short of a satisfactory exposition. The unit mobile triad is not introduced and consequently some very enlightening kinematic and dynamic problems on space curves, like helices, have been left out. The intrinsic or natural coordinates are introduced but without mention. The treatment of cylindrical coordinates is readable but not of the highest caliber. Author uses awkward and antiquated notation for unit vectors (like r, $\bar{E}$ instead of the almost universal notation $\bar{e}_n$, $\bar{e}_t$, $\bar{e}_r$, etc.). In addition notations like $\bar{r}$ are utilized for scalars that is confusing, since it is often used as a standard vector notation in script form. Author uses $\bar{V}$ to denote the velocity vector while the almost universal standard in *engineering* is to employ $\bar{v}$ for that purpose.

In the integration of Newton's equations of a particle author has failed to meet his standards of generality by refusing to integrate a simple vector relation directly to secure a general solution for the position vector $\bar{r}$ valid for rectilinear as well as three-dimensional problems. Instead author goes through a series of tedious and unnecessary motions to solve every particular problem separately by way of scalar equations and thus show the vector approach at something less than at its best.

The problem of quadric resistance of ballistics is covered, but the reviewer feels that the general Bernoulli's ballistic problem with the resistance proportional to any power of velocity should also have been included. This problem is of great historical interest since it was issued by John Keill of Oxford as a challenge to continental mathematicians in a vain attempt to prove that Newton's fluxions method was superior to Leibniz's calculus. Jean Bernoulli not only solved the quadratic resistance problem but also the more general problem, ending up with the celebrated Bernoulli's nonlinear differential equation. This affords the student another opportunity to realize the importance of mechanics as the best source of undergraduate mathematics. The ballistic problem with resistance directly proportional to velocity would have also been of considerable instructional value, since in this case the general three-dimensional vector differential equation can again be integrated directly. Tartaglia-Torricelli's ballistic trajectory and range problems are not discussed. The actual aspects of a real ballistic problem involving spin, etc., should have been forcefully pointed out.

The impact, central-force and space mechanics sections are interesting and ably presented. Unfortunately, rotational effects in impact problems were not included. The section on particle dynamics is adequate.

The inertia tensor is covered along well-treaded path, yet more variety could have been provided in this section. Reviewer laments the absence of the dyadic representation of the second-rank tensors, which is consistent with a vectorial treatment. Inertia dyadics should have been introduced, since by their use the inertia properties, the rotational motion and the kinetic energy of rotation can be dealt with in a concise symbolic fashion that is easy to remember. The section on energy methods is adequate, but not penetrating, in its formal development. Euler's equations and the motion of the body about a fixed point follow traditional pattern.

The part on balancing of rotating masses is, however, too sketchy and consequently lacks depth.

The weakest section, as is so common in most modern textbooks, is the D'Alembert's principle. In reviewer's opinion the D'Alembert principle finds its best philosophical expression in Mach's representation that incorporates Jacques Bernoulli's "lost force" concept. It must be remembered that this method as developed by Jacques Bernoulli, Hermann, Euler and D'Alembert represents the first relative motion approach in mechanics. It is incomprehensible to the reviewer why the author has practically ignored the important Lagrangean form of D'Alembert's principle, which serves as the very foundation for all advanced principles of mechanics. No reference is made to the *reversible* character of the virtual displacements for which the virtual work principle, as given in the text, is valid.

The rotational form of D'Alembert's principle, being only stated and not rigorously developed, is practically rendered meaningless. Yet a simple and most general vectorial derivation could have been easily effected!

The deformable medium has been quite capably handled. Again in this area dyadics and Taylor series expansions would have served a useful purpose. Hamilton ∇-operator has, unfortunately, not been introduced and thus concise and easily remembered symbolic results could not be given. Reviewer feels that dyadic analysis that is elementary for works like this, should be included in every *modern* text on *vector mechanics.*

The type of criticism brought out in this review bearing on the subject springs from the ambitious goal of this two-volume work which is, in general, on the same level, and certainly not worse, than other existing vector mechanics texts in English. Author has, in general, failed to represent vector analysis at its three-dimensional best, and because of this his text is not perfect. Author, however, has not committed the common mistake of similar textbook writers to try to cover too much subject matter and thus necessarily spread it too thin. One other unfortunate feature of this work is the author's tendency to use rather awkward notation and in addition, some illustrations are so heavily cross-hatched that it virtually hurts to read them. The illustrations, in general, are not of the same artistic quality and clarity as, for instance, in the books published by Addison-Wesley Company. In perusing this text meant for the education of undergraduates, one is virtually flabbergasted by the scarcity of references to the originators of the methods that are profusely used in the book. This omission certainly leaves an unfortunate impression on the reader that all the material covered in the text was either devised by the author himself or, at least, is of a contemporary vintage.

Nowhere can one see references to Boscovich's Hypothesis of the Rigid Body, Euler's Formula of Infinitesimal Displacements, Huygen's Theorem of Parallel Axis, Poinsot's Principal Axis and his Theory of Couples, Fourier's Inequality of Virtual Work, Leonardo da Vinci's laws of friction, Coulomb-Morin laws of kinetic friction, Wallis' solution of fully inelastic impact, Huygens' and Wren's solution on fully elastic impact, Newton's hypothesis of semi-elastic impact, König's Theorem on Kinetic Energy of a System, Tsiolkovski's formula on rockets, Poisson's formulas $\dot{\bar{k}} = \bar{\omega} \times \bar{k}$, etc., Stress Principle of Euler, Euler's vectorial equations of equilibrium, Euler's free-body method, Euler's equations of fluid flow, etc. On Euler's contributions alone one could fill a whole page. The Chasle's Theorem should have been properly called Bernoulli-Chasle's Theorem, since Daniel Bernoulli was the first to propose it.

The modern trend in English language textbook writing on engineering mechanics, mechanics of materials, fluid mechanics, etc., has been to strip the subject matter, which by and large constitutes a collection of highly singular achievements, from all the vestiges of the human element and the all-imperative role played by the comparatively few individuals in its conception. Just as an example, the elimination of Euler's contributions in mechanics is tantamount to excluding more than half of the material covered in this text with the inclusion of the *so-called* Newton's vectorial equations of motion, which are really due to Euler! Since most of the textbooks have been written with a materialistic indifference that does not honor individuals through whose efforts mechanics has taken such great strides toward its present useful form and technological excellence, the students are incapable of fully appreciating mechanics and lack in the depth of sound judgment. In order to be able to understand their own purpose in the society it is essential that they be familiar with the past and the present struggle that produces science. It is no wonder that our students are probably the most ignorant and poorly informed group in this essential aspect of education. In all other fields of higher learning such an uninformed person is plainly called *uneducated.*

It becomes necessary to go virtually through hundreds of textbooks before one can be found that is educational in this academic sense. To the author's knowledge only very few textbooks, like Kinney's "Indeterminate structural analysis," Houlton and Roller's, "Foundations of modern physical sciences" and very few other texts contain a section on historical survey of the subject matter treated. Recently the physicists have certainly assumed the leadership in writing educationally consciencious textbooks with a just consideration for the historical facts. In the field of engineering mechanics the best books in English, like the one reviewed here, run a distant second to such excellent tracts as the Russian two-volume work on mechanics by Loitsianskii and Lur'e and the German texts by Szabó, which cover the technological as well as the historical subject matter in a masterly and educationally conscious manner. It would be of no small value to any textbook writer to study these works, especially the Russian tract which, in reviewer's opinion, constitutes probably the best and also the most comprehensive textbook of mechanics on the market today. The illustrative material in these two works is a paragon for clarity and descriptiveness that is requisite for any vehicle of instruction in engineering mechanics.

G. A. Oravas, Canada

Book—6168. Gunder, D. F., and Stuart, D. A., Engineering mechanics, New York, John Wiley & Sons, Inc., 1959, xi + 391 pp. $7.75.

Book is written for undergraduate engineers who have previously studied mechanics only as a branch of mathematics or physics.

The chapter headings are:

1. Basic concepts and definitions; 2. Vectors; 3. Physical problems; 4. Methods of work and energy; 5. Friction; 6. Flexible cables; 7. Motion of a particle (kinematics); 8. Particle dynamics (kinetics); 9. Relative motion; 10. Rigid body dynamics; 11. Vibration.

The use of vector and scalar products from the most elementary stage onwards is a notable feature which may make the book particularly attractive to those who want their students to be familiar with vector methods before proceeding to advanced work.

All topics are treated from most elementary ideas possible and student is encouraged to think about his own experience of simple mechanical things in order to increase his understanding of principles.

Although book is essentially elementary, authors have introduced some topics not commonly included—for instance orbits in a gravitational field and relativistic variation of mass. These will surely stimulate the student's appetite.

Diagrams are clear and well designed. Style is intentionally colloquial and will please some and infuriate others.

D. C. Johnson, England

Book—6169. Housner, G. W., and Hudson, D. E., Applied mechanics, Vol. 2, Dynamics, 2nd ed., Princeton, N. J., D. Van Nostrand Co., Inc., 1959, viii + 392 pp. $5.25.

The second edition of this textbook shows some remarkable additions and modifications compared with the first. The dimensional analysis has been rewritten and accompanied by a short treatment of model theory. The impact theory has been extended to problems with variable mass. The section on systems of particles now includes a more general representation of the momentum equations, which are completely developed for rigid bodies, especially with regard to gyroscopic effects and motions about a fixed point and to stability problems of rolling motions.

Within the mechanics of continua the propagation of longitudinal waves in an elastic bar is considered in comparison with vibration methods. The application of generalized coordinates and Lagrange's equations is treated more intensively, including a discussion of the small vibrations of a conservative system; also the variational calculus is treated especially in connection with Hamilton's principle. The new edition contains a greater number of examples; some of the original examples are modified. Thus in its complete form this textbook fulfills its purpose to give a good methodical development of the fundamental principles of dynamics, together with a great number of very illustrative examples.

H. Neuber, Germany

Book—6170. Foppl, L., Higher point of view of elementary mechanics [Elementare Mechanik vom hoheren Standpunkt], Munchen, R. Oldenbourg Verlag GMBH, 1959, 174 pp. DM 20.

The book will give the engineer and physicist a broader insight into the fundamental ideas and methods of classical mechanics. It begins with a short review of the historical development of the foundation of the principles. Then author deals with spinning tops, impact, differential- and integral-principles in mechanics, and finally some elementary ideas on the theory of relativity. Well-chosen practical examples illustrate the use of the principles. The mathematical calculations and demonstrations are made as simple as possible in order to be satisfying to the advanced reader. The treatment is not as broad as is necessary in a textbook, and thus the average student does not fully learn how to use the tools presented.

Reviewer thinks the book to be very well written in general. A few usual but unnecessary simplifications are used, as for example, that bodies in impact are deformed only near the point of impact. The title of the book seems to promise a critical analysis of the fundamental concepts, but more of this is to be found in penetrating modern textbooks, see for example, Hamel.

H. O. Faxen, Sweden

6171. Rauch, L. M., Iterative solution of the *N*-body problem for real time, *ARS J.* **30**, 3, 284–286 (Tech. Notes), Mar. 1960.

Paper presents an iterative method for an approximate solution of the *N*-body problem based on the use of power series and not necessarily requiring large-scale digital computers for the solution. Partial sums of power series in time components of positions in inertial space, for squares of the distances between particles, and for cubes of the reciprocals of these distances, replace these quantities respectively in the $3N$ Newtonian equations of motion. Coefficients of these power series are obtained by recursion formulas given by W. Cheney, "Power series for the *N*-body problem," Mathematical Preprints, no. 16, Convair Astronautics. Author gives explicit formulas for successive partial sums of the power series which are independent of preceding partial sums. For practical application reviewer regrets lack of estimates of bounds on differences between approximate solutions and true solutions. There are also a few easily discovered misprints in the formulas.

M. L. Juncosa, USA

6172. Hochstadt, H., A special Hill's equation with discontinuous coefficients, AFOSR TN 60-322 (New York Univ., Inst. Math. Sci., Div. Electromagnetic Res. no. BR-32), 16 pp., Mar. 1960.

Paper gives a mathematical treatment of the equation

$$y'' + \lambda^2 Q(x) y = 0$$

where $Q(x)$ is an even periodic function of period $2L$ defined by

$$\begin{aligned} Q(x) &= 1 \quad (0 \leqq x < 1) \\ &= a^2 \quad (1 \leqq x \leqq L). \end{aligned}$$

$Q(x)$ is thus a piecewise constant function with two step discontinuities in the periodic interval.

R. E. D. Bishop, England

6173. Duncan, W. J., Some properties of the admittances of dynamical chains, *Aero. Quart.* **11**, 1, 99–104, Feb. 1960.

This paper is a sequel to AMR **12**(1959), Rev. 36. New relations between the coefficients appearing in the impulsive admittances of dynamical chains are obtained by considering the initial motion. A note by R. J. Fitzgerald is added in which it is shown that all coefficients occurring in the direct admittances are positive.

A. I. van de Vooren, Holland

6174. Kirgetov, V. I., On "virtual displacements" of material systems with linear differential constraints of the second order, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 4, 956–962, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

For a material system with linear differential constraints of the second order (according to the terminology of E. Delassus) a definition of virtual displacements is given. Author discusses some properties of this definition and shows that within this definition the d'Alembert-Lagrange principle yields the same equations of motion as Gauss' principle.

M. P. Bieniek, USA

6175. Kharlamova, E. I., On a particular solution of the Euler-Poisson equations, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 4, 975–988, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Assuming certain quadratic relations to be satisfied by the components of the angular velocity of an asymmetric heavy rigid body, a system of algebraic relations is derived for the determination of the required parameters. Solutions discussed in this paper correspond to those found by Grioli [*Ann. Mat. Pura Aplic.* (4) **26**, 1947] and the present author [*Dokladi Akad. Nauk SSSR* (*N.S.*) **125**, no. 3, 1959]. The second solution comes about when the principal moments of inertia satisfy the relations $C > 2A > 2B$. Though this is impossible for a pure rigid body, they can be satisfied if the body has cavities filled with an ideal incompressible fluid. Reviewer believes this situation may be related to the motion of a missile or rocket with liquid propellant. Anyway, for such an application to be of practical value, a more realistic formulation of the equations of motion is required.

W. W. Lubomirsky, Argentina

6176. Zhdanov, V. M., Some cases of motion of a pendulum the length of which changes in accordance with a given principle (in Russian), *Sb. Statei Ural'skogo Politekhn. In-ta* no. 56, 23–38, 1955; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2224.

An investigation is carried out on the problem given in the title. A solution is furnished for the linearized equation

$$\frac{d}{dt}\left(l^2 \frac{d\theta}{dt}\right) + gl\theta = 0 \qquad [1]$$

to cover the case where the length of the pendulum l is a quadratic function of time t. The solution is recorded with the aid of hypergeometrical series. In one special case an approximate solution of the problem is given in elementary functions. The concluding portion of the paper is given to finding an approximate solution for equation [1] on the assumption that the length of the pendulum changes slowly in accordance with an assigned principle. Simple approximate formulas are given for a few practical interesting cases of linear and quadratic dependence of the length of the pendulum l on the time (t).

N. N. Bautin

Courtesy Referativnyi Zhurnal, USSR

6177. Voronkov, I. M., Motion of a mechanical system when the external forces are the forces of gravity and there is resistance in the surrounding medium (in Russian), *Nauchn. Trudi Mosk. Gorn. In-ta* no. 19, 5–19, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2239.

Theorems are investigated on the motion of the center of gravity and on the kinetic moment for the case where the internal forces appear to be arbitrary, while the resistance forces are proportional to the velocities and masses of the points of application. The latter theorem is studied in the form of a vector equation of the motion of the end of the kinetic moment on a hodograph, which represents in itself a plane curve. If the sum of the work of the internal forces is equal to zero then the theorem is also investigated regarding the kinetic energy and it is shown that the motion of the system around the center of gravity proceeds analogously to the rotation of a body near an immovable center in the Euler-Poincare case. The work is intended for the use of students in mechanics.

G. N. Sveshnikov

Courtesy Referativnyi Zhurnal, USSR

6178. Rumyantsev, V. V., The stability of a motion in relation to the part played by the variables (in Russian), *Vestn. Mosk. In-ta Ser. Matem., Mekh., Astron., Fiz., Khimii* no. 4, 9–16, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2252.

This paper develops the Lyapunov method of functions to cover the case of the stability in relation to the part played by the variables entering into the equation of disturbed motion.

$$\frac{dx_i}{dt} = X_i(x_1, \ldots, x_n, t) \quad |x_i| < H(i \qquad ., n) \qquad [1]$$

The undisturbed motion is designated as stable in relation to the variables $x_1, \ldots, x_m$ ($m < n$), if for an arbitrary positive number $A < H$ it is possible to select a number $\lambda > 0$ in such a way that for all disturbances satisfying the condition

$$x_{10}^2 + \ldots + x_{n0}^2 \leqslant \lambda \qquad [2]$$

with every $t \geqslant t_0$ the inequality

$$x_{10}^2 + \ldots + x_{m0}^2 < A \qquad [3]$$

shall be fulfilled. The known concepts of Lyapunov's second method are applied to the problem being investigated here. For instance, function $V(x_1, \ldots, x_n)$ is given the name of sign-determining by $x_1, \ldots, x_m$, if it is sign-positive and changes to zero value only when $x_1 = \ldots = x_m = 0$. The theorems regarding stability and asymptotic stability are substantiated.

Let us consider the theorem of stability. If equations [1] are of such a nature that it is possible to find function V whose sign is determinable by relation to the variables $x_1, \ldots, x_m$, and whose derivative V' by virtue of these equations shall be the constant sign function of opposite sign to V or equal to zero, then the undisturbed motion is stable in relation to the variables $x_1, \ldots, x_m$. It should be noted that in the case of instability in relation to the parts played by the variables the theorem advanced by N. G. Chetaev retains its effectiveness ["Stability of motion," Moscow, Gostekhizdat, 1955]. As an example of the application of the theorems referred to, the problem is investigated on the stability of the motion of a solid body with an immovable point (the Lagrange case) in relation to the cosine of the angle of mutation y and the projection of the angular velocity η on to the axis of dynamic symmetry; also the problem on the stability of the position of equilibrium in relation to the generalized velocities q_i' in the cases of holonomous systems: (a) when only gyroscopic forces are acting, and (b) when potential, gyroscopic and dissipating forces are operating, where the force function U possesses, when in the position of equilibrium, an individual maximum, while the dissipative function f possesses an individual minimum.

N. N. Krasovskii

Courtesy Referativnyi Zhurnal, USSR

6179. Kappus, R., and Clerc, D., Determination of the inertial coefficients of elastically suspended airplanes from ground tests (in French), *Rech. Aéro.* no. 65, 51–57, July-Aug. 1958.

Relations between the inertia coefficients and the frequency and modes of a rigid body elastically suspended are deduced using matrices. These relations permit the determination of the complete inertia characteristics (mass, moments of inertia, center of gravity, directions of principal axis of inertia) of the body from oscillation measurements. The frequencies of the body on its elastic suspension must be much lower than the natural frequency of the elastic body to satisfy the assumptions made.

The effect of changes of the suspension stiffness and of an additional mass on the oscillation properties of the system is given in the appendix.

This method may be useful for experimental determination of the inertia characteristics of complicated bodies and has the advantage of permitting additional checks which help to determine the accuracy of the obtained results.

W. Fiszdon, USA

6180. Laricheva, V. V., Nonlinear damping of the natural vibrations of systems of arbitrary order, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **22**, 4, 745–749, 1958. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

Author considers the free oscillations of a dynamic system with nonlinear damping. The particular form of nonlinear damping used by author is a simple bi-linear hysteresis in which the stiffness of the system assumes different values depending on whether the product of velocity and displacement is positive or negative. Author analyzes the single-degree-of-freedom system in some detail and then applies the same concepts to the two-degree-of-

freedom system. The form of the equations is so chosen that the bi-linear damping mechanism exists separately in each normal mode. Reviewer considers this particular choice of the equations to be restrictive and somewhat unrealistic. Author concludes by generalizing the results to include a system with n degrees of freedom. T. K. Caughey, USA

6181. Gerasimov, I. S., The asymptotic integration of a differential equation arising in a self-preserving problem in dynamics, *Soviet Phys.-Doklady* **4**, 3, 582–584, Dec. 1959. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **126**, 4, 727–731, May/June 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

6182. Odier, J., On the influence of the general characteristics of a motorcar and of the speed factor on the stability while braking (in French), *C. R. Acad. Sci., Paris* **249**, 8, 878–880, Aug. 1959.

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(See also Revs. 6147, 6154, 6464, 6570)

6183. Bodner, V. A., Ovcharov, V. E., and Seleznev, V. P., On the synthesis of invariant damped inertial systems of arbitrary period, *Soviet Phys.-Doklady* **4**, 2, 303–305, Oct. 1959. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **125**, 5, 986–988, Mar./Apr. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Authors consider a horizontal stable platform with two acccelerometers and a gyroscopic vertical. They show how external (non-inertial) information can be used, if it is available, to damp the Schuler oscillations of the platform. In addition, it is shown that by using such information the period of oscillation can be reduced considerably, making possible the damping of initial errors in a reasonable amount of time. A short discussion of the effect of errors in the external information is presented.

A. Robinson, USA

6184. Wang, S.-M., Compensation of a continuous automatic control system by means of a delay element filter, *Automation and Remote Control* **20**, 4, 421–430, Jan. 1960. (Translation of *Avtomatika i Telemekanika* **20**, 4, 437–446, Apr. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

The use of a delay element filter to compensate a continuous automatic control system requires determining the order of the filter and the values of its parameters to meet a specified system response. The method of selecting parameters used by the author is based on the theory of the approximation of a function by the method of the expansion of the frequency characteristic of a control system into a Taylor series (Bruce's method). The parameters are considered optimum when the frequency characteristic of the compensated system deviates by the least amount from its value when $\omega = 0$. In determining the filter parameters other restrictions may be taken into account. Two examples of compensation of automatic control systems are given.

S. Z. Dushkes, USA

6185. Bendrikov, G. A., and Teodorchik, K. F., The analytic theory of constructing root loci, *Automation and Remote Control* **20**, 3, 340–344, Jan. 1960. (Translation of *Avtomatika i Telemekanika* **20**, 3, 355–358, Mar. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Authors present an analytic way of constructing root loci. In the characteristic equation of a linear closed system, given as

$$f_n(p) = 1 + W_0(p) = \sum_{i=0}^{n} a_i p^i = 0$$

where $W_0(p)$ is the open loop transfer function, and one or more of the coëfficients a_i are dependent on a parameter ρ, they substitute

$$p = \delta + i\omega$$

When the real and imaginary terms are separated, one gets

$$P(\delta, \omega, \rho) + i\omega Q(\delta, \omega, \rho) = 0$$

with $P(\delta, \omega, \rho) = 0$ and $\omega Q(\delta, \omega, \rho) = 0$ separately. From these equations the root loci for variable ρ can be found.

Functions P and Q can be represented as finite series in ω^2 with coëfficients containing derivatives of $f_n(\delta)$.

Authors consider the cases where only a_0 is dependent on ρ, and where the coëfficients $a_i (0 \leqslant i \leqslant m,\ m \leqslant n)$ are linearly dependent on ρ. An example is given of the root locus of a system with an open loop transfer function that has four poles and one zero.

P. DeWaard, Holland

6186. Gille, J.-C., Determination of free vibrations in relay servomechanisms (in Italian), *Automazione e Strumentazionie* **8**, 1, 3–10, Jan. 1959.

Paper presents a lucid mathematical comparison of the method of Hamel with that of Cypkin for approximating the free oscillations of relay servomechanisms. It is shown that although the two methods are theoretically equivalent they utilize different approximations. The practical advantages of each method are itemized and briefly discussed. Papers by both Hamel and Cypkin are referenced. W. C. Orthwein, USA

6187. Berezkin, E. N., The stability of the unperturbed motion of a certain mechanical system, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 3, 864–871, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

The equations for the perturbed motion of an airplane with autopilot have been stated and by using linear substitution have been transformed into canonical form suitable for stability analysis by means of the second method of Lyapunov. Ingenious transformation has been performed on the system which facilitated the construction of the suitable Lyapunov functions. The stability of the unperturbed motion of the system has been studied when parameters assume different sets of values and it is shown that, in the majority of cases considered, the system as described by the stated equations is unstable. M. Mesarovic, USA

6188. Troitskii, V. A., On the autovibrations in controlled systems possessing several control organs (in Russian), *Trudi Leningr. Politekhn. In-ta* no. 192, 201–219, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2280.

The following periodic solution of a system of differential equations is calculated by means of Poincaré's method of the small parameter.

$$\dot{x}_k = \sum_{\alpha=1}^{n} b_{k\alpha} x_\alpha + \sum_{\beta=1}^{n} b_{\beta k} f_\beta(\sigma\beta, \dot{\sigma}\beta)$$

$$(k = 1, \ldots, n) \qquad [1]$$

$$\sigma\beta \sum_{\alpha=1}^{n} j_{\beta\alpha} x_\alpha, \quad f_\beta(\sigma_\beta, \dot{\sigma}_\beta) = c_{\beta 1}\sigma_\beta + \mu\varphi_\beta(\sigma_\beta \dot{\sigma}_\beta)$$

This describes a system of automatic control when there are several control organs (μ being the small parameter). It is assumed that the characteristic equation for the generating linear system has a pair of purely imaginary roots $\pm\, i\omega$, with no zero and multiple i roots. Computations are furnished for making corrections for amplitude and frequency up to the order μ^2. Questions regarding the convergence of the series and the evaluation of the remainder of the series are not investigated. A description is given of the calculations of the roots of the characteristic equation of the system of equations for the disturbed motion, which has been derived

relative to the sought periodic solution. The author deduces the criteria for the stability of the periodic solution, limiting himself however to an examination of the terms of the first order of μ. As an example of the application of the results a description is given of the calculation for the periodic solution of the following system of equations

$$\dddot{x} + \nu_1^2 x + \alpha_1 \dddot{y} = 2(\delta_0 - \delta_2 x^2)\dot{x}$$

$$\dddot{y} + \nu_2^2 y + \alpha_2 \dddot{x} = 2(y_0 + y_2 y^2)\dot{y}$$

which describes the work of two connected generators.

N. N. Krasovskii
Courtesy Referativnyi Zhurnal, USSR

6189. Ryabov, B. A., Experimental determination of the dynamic characteristics (parameters) of the links in a system of automatic control (in Russian), Sb. Statei po Avtomatike i Elektrotekhn., Moscow, Akad. Nauk SSSR, 1956, 80–95; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2291.

The method proposed is based on the utilization of the time characteristics of corresponding links, taken at the moment of entry of a link with disturbances of three types: type of a single jump; type of a jump followed by a change of the disturbing reaction in conformity with the principle of the straight line, inclined to the axis of the abscissa; and type of a jump followed by a change in accordance with the quadratic principle. The method is applicable to the case where the material part of the roots of the characteristic equation of the corresponding link has a negative value. The proposed method is applicable as well to links with roots of any form, provided a new link is joined with the link being investigated in the prescribed way. A method is also put forward for the determination of the parameters of links for which the recording of the coordinates is not essential.

A. M. Popovskii
Courtesy Referativnyi Zhurnal, USSR

6190. Balchen, J. G., and Berre, A. G., A method for evaluating the accuracy in the time domain associated with approximation in the frequency domain, *Acta Polytech. Scandinavica* no. 262 (Mathematics and Computing Machinery Series no. 3), 10 pp., 1959.

Method is described to evaluate the response of a linear system to a known transient. The actual amplitude-phase-frequency plot is approximated by a known characteristic that can be easily synthesized. Paper derives the required accuracy of this approximation in the frequency domain to satisfy the conditions of a specified maximum error in the time domain. The error criterion employed here makes use of logarithmic frequency and amplitude plots.

P. P. Biringer, Canada

6191. Kuznetsov, S. M., The probability of defective elements in automatic control systems, *Automation and Remote Control* **19**, 11, 1023–1034, June 1959. (Translation of *Avtomatika i Telemekhanika, USSR* **19**, 11, 1048–1061, Nov. 1958 by Instrument Society of America, Pittsburgh, Pa.)

Existing methods of experimentally estimating element reliability under various conditions of storage and use do not attain the necessary accuracy, although entailing large expenditures of time and deterioration of elements and apparatus during testing. Consequently, analytical approaches and techniques such as presented by this author are needed.

An element is defined as a nondecomposable union of interacting components which perform the simplest transformations of physical quantities such as condensers, resistors, vacuum tubes, etc. On the premise that functioning of an element is characterized by several independent parameters representing the electrical, mechanical, and thermal reliability of the materials, the total probability of element malfunction as a function of all of its constituent parameters is first expressed. The real problem arises because various disturbing factors introduced in the processes of manufacture and use of elements can influence not only the initial deviations of the element parameters but also the speed of their variation. To avoid extreme complication, this analysis considers only the variations of element parameters when the disturbing factors are stationary.

On this basis, author develops mathematical expressions for the probable useful life of an element depending upon the assumption made in regard to probability distribution law which defines the disturbing factors. Simplified computational method is suggested to determine probable useful life of an element for the general case in which the distribution law is a function of several random variables.

R. L. Sutherland, USA

6192. Krutova, I. N., The dynamics of a vibration loop in an electrical servo during free oscillations, *Automation and Remote Control* **20**, 4, 409–420, Jan. 1960. (Translation of *Avtomatika i Telemekanika* **20**, 4, 422–436, Apr. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

The vibration loop has two parallel control channels, which have a common feedback. Three first-order equations describe the motion. The motion is studied in the phase plane, by point-transform methods. Bifurcation relations are found for the parameters; these relations define the bounds to the various forms of motion in the parameter space.

From author's summary by S. Z. Dushkes, USA

6193. Saunders, H., A further extension of the Routh-Hurwitz stability criteria for quintic and sextic equations, *J. Aero/Space Sci.* **27**, 5, 396–397 (Readers' Forum), May 1960.

Book—6194. Bowen, J. H., and Masters, E. F. O., Nuclear reactor control and instrumentation, New York, Simmons-Boardman Publishing Corp., (Nuclear Engineering Monographs), 1959, x + 78 pp. $2.75. (Paperbound)

This book will provide an elementary understanding of the general operating characteristics of the gas-cooled graphite-moderated reactor and the associated control and instrumentation problems. While it is not (and is not intended to be) a treatise on the subject matter, its coverage should be a useful introduction for operating personnel and others who may be coming in contact with the reactor control problem for the first time.

Chapters 1–4 deal with the effects on the reactor of changes in reactivity and cooling rate. The interactions between variables affecting reactor response are so complex that resort must normally be made to computer techniques for obtaining quantitative predictions. Despite this fact, much useful information about the main characteristics of the reactor system can be obtained through the use of simplified methods. The present text shows in a general way how typical real situations may be reduced to simplified analytical models through use of the proper assumptions. Emphasis is placed upon the limitations which apply to predictions based on the model.

Chapters 5 and 6 discuss some of the special problems connected with reactor control rods and reactor instruments and trip circuits.

The treatment is concerned principally with "open loop" characteristics. Because of this emphasis, system stability as normally associated with closed loop automatic control systems is not discussed in detail.

The book is well written and would be a good reference copy for libraries and others who have occasional need to orient new personnel with respect to the basic characteristics of the air-cooled graphite-moderated reactor. In reviewer's opinion, the basic analytical approach is a useful one which deserves wider application to the solution of other engineering problems.

E. L. Foster, USA

6195. Petow, W., Investigation of a steam turbine control system and experiments on the tripping behavior by means of a scale-

model control unit (in German), *VDI Forschungsheft* **26,** 478, 40 pp., 1960.

Modern turbine-generator units with their small inertia require fast and sensitive control of the steam turbine. In order to study and to develop such control systems a model unit was built to experimentally simulate the controlled turbine-generator unit. The test results allow for direct conclusions to be drawn on the behavior of the real turbo set. The effects of friction in the control system were revealed and a reduction of the permanent dead band was achieved. The dynamic performance of a hydraulic steam turbine control system at instantaneous loss of load may be accurately judged from oscillograms of the essential quantities. Interception of the maximal speed rise after tripping the unit at sudden load drop was accomplished by back feeding speed acceleration rate. Tachograms and oscillograms taken on the model unit prove the superiority of the accelerometer control system at maximal load tripping over other methods of control.

From author's summary

6196. Cohen, A. D., Use of the construction parameter in staging optimization, *ARS J.* **30,** 8, 769–770 (Tech. Notes), Aug. 1960.

In the recent literature on staging optimization, the solutions have required a knowledge of the hardware weight staged in order to obtain the construction parameter. This parameter has been assumed as a constant, but, as is shown here, the value is, in reality, a function of the stage size. A method of incorporating this factor into the solution is presented.

From author's summary

6197. Duffin, R. J., and Schmidt, T. W., Simple formula for prediction and automatic scrutation, *ARS J.* **30,** 4, 364–365 (Tech. Notes), Apr. 1960.

Prediction from data obtained at regular time intervals is discussed. Attention is restricted to the case in which the more recent data should be given more weight than past data. Thus a polynomial is fitted to the data by exponentially weighted least squares. It is found that the formula for predicting the next datum is quite simple. This formula needs to "remember" only a limited amount of data. The prediction formula can be used to automatically scrutinize data for gross error. A gross error or a gap in the data may be replaced by a predicted value, and the prediction and scrutation process can be continued.

From authors' summary

6198. Novoselov, V. S., On the motion of gyroscopic systems, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 242–246, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The stability of a gyroscopic system has been examined for the steady motion in the case where the coefficients in the equation describing the motion are variable. It has been shown that forcing vibration, indicated by the presence of corresponding generalized forces $f_i(t)$ on the right side of the Lagrangian equation, can be described as a particular solution of the nonhomogeneous problem with homogeneous initial conditions.

M. M. Stanisic, USA

6199. Goodstein, R., A perturbation solution of the equations of motion of a gyroscope, *ASME Trans.* **81** E (*J. Appl. Mech.*), 3, 349–352, Sept. 1959.

Paper uses the perturbation method for obtaining solutions of the simultaneous nonlinear equations of motion for three problems of a frictionless gyro, i.e. (1) motion due to an impulse on the outer gimbal with the gyro housing or case considered fixed, (2) motion due to a sinusoidal torque applied to the outer gimbal, with the gyro housing considered fixed, and (3) motion due to a sinusoidal displacement of the gyro housing.

By choosing the suitable small parameter for each case, author solves the free vibration problem in detail and then considers the resonance effect, reaching the following conclusions:

(1) The hypothesis of periodicity in a perturbation solution for nonlinear systems with neutral stability may hide some aspects of the results.

(2) If the gimbal carrying a gyro may vary from the perpendicular during the normal operation, the sinusoidal excitation at the gyro natural frequency is considered as a possible ruinous input.

(3) The natural frequency of the gyro varies with inner gimbal initial angle. The vibration isolators should not increase the possibility of the gyro to lose its reference or initial orientation through the amplification of the external excitations at possible gyro natural frequencies.

C. S. Pelecudi, Roumania

6200. Novoselov, V. S., Regular precession of a gyroscope with a variable mass (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 98–99, 1958.

A gyroscope of variable mass is understood to be a body of variable mass which has a fixed point, possesses kinetic symmetry and preserves its principal directions during the whole course of motion. It is assumed that the law of variation of the mass is of the Meshcherskii type [AMR **12**(1959), Rev. 3729]. In addition, it is assumed that the principal vector of reactive forces is equal to zero and the principal moment of reactive forces, calculated at the fixed point, is directed along the axis of symmetry of the gyroscope and is given in terms of the time by a certain function $K(t)$.

For a body of variable mass the Lagrangian equations of motion of the second kind can be obtained from those for a body of constant mass, if the masses of the particles which constitute the body are imagined to be subjected to an arbitrary functional dependence and generalized reactive forces are added to those acting on the body [AMR **12**(1959), Rev. 4845].

It is shown that the system of differential equations of the gyroscope with variable mass has the unique solution $\theta = \theta_0 = \text{const}$, $\dot{\psi} = \dot{\psi}_0 = \text{const}$, which is called regular precession, θ being the angle of nutation and ψ that of precession. The fact that this solution is stable is shown by means of the second method of Liapunov.

E. Leimanis, Canada

6201. Kuzovkov, N. T., Uniaxial force gyrostabilizer with an oblique mounting of the gimbal axis (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 70–74, 1958.

The motion of a gyrostabilizer is studied with respect to a rectangular co-ordinate system $\xi\eta\zeta$ rigidly connected with its base, the ξ-axis being directed along the axis of the outer ring. In order to determine the position of the gimbal with respect to the above system, a rectangular coordinate system xyz is attached to the outer ring with the y-axis along the axis of rotation of the gimbal. The position of the system xyz with respect to that of $\xi\eta\zeta$ is defined by the angles α and γ, γ being the angle between the axes ξ and x and α between those of ζ and z. The gimbal of the gyroscope is connected with a rectangular coordinate system, $x_1 y_1 z_1$, the y_1-axis ($= y$-axis) of which is directed along the axis of rotation of the gimbal. The position of the system $x_1 y_1 z_1$ with respect to that of xyz is defined by the angle β between the axes x and x_1. It is assumed that the angle between the axes y_1 and ξ is different from a right-angle. Using a method proposed by A. Yu. Ishlinskii [Mechanics of special gyroscopic systems, *Izdat. Akad. Nauk Ukrain. SSR*, 1952 (Russian); and AMR **11**(1958), Rev. 4365] moment equations around the axes ξ and y_1 are written. It is assumed that the moment M of the stabilizing motor is of the form $M = -K\beta - f\dot{\alpha}$, K being the coefficient of amplification of a circuit consisting of a potentiometer, amplifier and a stabilizing motor, and f that of viscous friction.

For small deviations of the variables and exterior perturbing moments from their equilibrium state values $\dot{\alpha}_0 = \ddot{\alpha}_0 = 0$, $\beta_0 = \dot{\beta}_0 = \ddot{\beta}_0 = 0$, $M^0_{y_1} = m_0 = 0$, $M'_{\xi_0} = 0$ the equations of motion of the gyrostabilizer assume the form $(A + A' \cos^2 \gamma)\ddot{\alpha} + f\dot{\alpha} - \frac{1}{2}H\dot{\alpha}\sin 2\gamma = -H\dot{\beta}\cos\gamma - K\beta + m\sin\gamma + M'_{\xi}$, $C'\ddot{\beta} = C''\ddot{\alpha}\sin\gamma + H\dot{\alpha}\cos\gamma + m$.

Here A', C' are moments of inertia of the gyroscope (rotor + gimbal) with respect to the principal axes x_1, z_1 respectively. A is the sum of the moment of inertia of the outer ring and that of the anchor of the motor and reductor calculated with respect to the ξ-axis, $H = I\dot{\varphi}$, I being the axial moment of inertia of the rotor and $\dot{\varphi}$ the velocity of rotation of the rotor with respect to the system $x_1^T y_1 z_1$.

The stability of the gyrostabilizer is studied in terms of the variables $\dot{\alpha}$, β and the parameters K, H, f but the results obtained are too detailed to be reproduced here.

E. Leimanis, Canada

The system characteristics of modern guidance techniques, *Control Engng.* **7,** 8, Aug. 1960. (Revs. 6202–6206).

6202. Sarett, D. P., System aspects of inertial navigator design, 86–92.

6203. Chatterton, E. L., System considerations for guidance radars, 93–96.

6204. Cole, B. R., Doppler radar techniques for aerospace navigation, 97–100.

6205. Nichinson, D. B., Modern techniques in celestial navigation, 101–104.

6206. Mundo, C. J., Recognition in perceptive guidance systems, 105–106.

End of Symposium

Tables, Charts, Dictionaries, etc.

(See Rev. 6627)

Elasticity

(See also Revs. 6140, 6146, 6159, 6165, 6221, 6231, 6243, 6244, 6245, 6247, 6250, 6251, 6255, 6259, 6268, 6278, 6293, 6294, 6296, 6298, 6348, 6387)

6207. Golecki, J., On the foundations of the theory of elasticity of plane incompressible non-homogeneous bodies (in English), *Arch. Mech. Stos.* **11,** 4, 383–398, 1959.

6208. Allison, I. M., The prediction of three-dimensional stress concentration factors, *J. Roy. Aero. Soc.* **63,** 585, 549–551 (Tech. Note), Sept. 1959.

It is shown that it is only possible to infer three-dimensional stress concentration factors from two-dimensional stress concentration factors in very restricted circumstances, i.e. the three-dimensional stress raiser must contain a plane of symmetry and this plane must specify the geometry of the two-dimensional problem and the midplane of the solid body must be a plane of symmetry of the equivalent loading system in two-dimensions. Although the method is limited in applicability, it may be useful where both three- and two-dimensional data are available and it is desired to extrapolate the information.

E. E. Sechler, USA

6209. Parr, C. H., Stress-strain equations for case-bonded solid propellant grains, *ARS J.* **30,** 8, 778–779 (Tech. Notes), Aug. 1960.

Equations expressing elastic stresses and strains in a three-material, composite, hollow, circular cylinder representing a case-bonded propellant grain, liner and motor case are developed for loading conditions consisting of internal pressure, arbitrary radial temperature gradient and volumetric changes.

From author's summary

6210. Spencer, A.J.M., On finite elastic deformations with a perturbed strain-energy function, *Quart. J. Mech. Appl. Math.* **12,** 2, 129–145, May 1959.

This highly mathematical paper shows that if the solution of a finite deformation problem corresponding to a W strain energy function is known, then the problem of an additional small deformation superimposed on the finite deformation, causing a perturbation of the strain energy function to $W + \varepsilon W'$, can be solved without the knowledge of the form of W'. The notation and mathematical formulation agree with that used by Green and Zerna in "Theoretical elasticity" (Oxford, 1954). Two examples are treated: simultaneous extension, inflation and shear of a cylindrical tube with perturbing shear deformation and the special case in which the finite deformation is shear only. A Mooney-type solid (idealized rubber) is assumed in the determination of W; the solutions of the finite deformation problems are due to Rivlin, and Green and Zerna.

G. Sved, Australia

6211. Seika, M., Stresses in a semi-infinite plate containing a U-type notch under uniform tension (in English), *Ing.-Arch.* **27,** 5, 285–294, 1960.

Author considers the effect of a single U-type notch on a semi-infinite plate under a uniform tension applied parallel to the straight boundary. The complex variable method of Muskhelishvili demands the determination of certain parameters which are found by a perturbation method. By varying the parameters other shapes may be included, particularly the semi-ellipse and the rectangle with rounded corners. There are useful tables, graphs and values of the stress concentration factor.

L. M. Milne-Thomson, USA

Book—6212. Nowacki, W., Problems of thermoelasticity [Zagadnienia termosprezystosci], Warszawa, Panstwowe Wydawnictwo Naukowe, 1960, 395 pp. Zl. 77.

A short Preface gives only an extremely modest idea of the real value of this excellent work. Strictly speaking, Nowacki's book represents, in common with the well-known two volumes by Melan and Parkus, without any doubt the best that has been written on the subject.

Based on the classical assumptions of linear elasticity, the present volume covers a vast field of thermoelasticity of homogeneous, perfectly elastic and viscoelastic media. A substantial part of the material involved comes from the original research work of the author.

Treated are three-dimensional cases as well as two-dimensional ones, each category containing both steady- and nonsteady-state problems. The uniform conception of the work results in that every solution consists of two parts, one having the character of a thermoelastic potential, the other corresponding to Galerkin's displacement vector. The study presupposes adequate knowledge of the theory of Green's functions and a familiarity with modern integral transformations.

A short survey of all 8 chapters will give a more detailed idea on the subject covered in the book.

Chapter I is of introductory character, deducing in its four subtopics all the fundamental equations of thermoelasticity and discusses the methods of their solution, as far as used in the following sections.

Chapter II contains nine subtopics devoted to three-dimensional steady-state problems. Special attention is given to the fundamental role of heat sources within the infinite elastic space and

semispace; treated are thermal stresses due to both continuous and discontinuous temperature fields.

The six subtopics of the following main part deal with nonstationary three-dimensional problems. Discussed are thermoelastic questions concerning an infinite and semi-infinite space, problems of thick elastic plates and layers.

Interesting problems from the two-dimensional thermoelasticity of thin plates and strips are the object of eight subtopics which form the fourth chapter of the book. Special attention is given, in the fifth main part (with four paragraphs), to steady thermal stresses in plates of moderate thickness. Interesting nonsteady two- and one-dimensional problems are solved in the five subtopics of chapter VI.

Seventh main part contains four paragraphs devoted to some questions from the thermoelasticity of viscoelastic media and the volume concludes with six subtopics concerning recent developments in the field of dynamical problems of thermoelasticity.

A detailed index of literature with 123 references and a brief author index facilitate the use of the book and provide a good survey on the present state of thermoelasticity.

It is reviewer's warm desire to emphasize the author's masterly knowledge of both mathematical and technical part of the subject. There is real intellectual enjoyment in reading this beautiful work, which is sure to be translated into other languages,

V. Vodicka, Czechoslovakia

6213. An IBM-650 program for the computation of thermal gradients in mass concrete structures, Wwys. Exp. Sta. Tech. Rep. 6-540, 24 pp., Apr. 1960.

6214. Ungar, E. E., and Shaffer, B. W., Thermally induced bond stresses in case-bonded propellant grains, *ARS J.* **30,** 4, 366–368 (Tech. Notes), Apr. 1960.

Equations for thermal stresses due to radial temperature gradients in assemblies of cylindrical propellant grains bonded to thin wall casings are developed. Both long and short assemblies are considered. The casings are assumed to be elastically stiff and thermally thin. The results show that under the same conditions the bond stresses in very long assemblies are always greater than those in very short assemblies. The effect of geometry and material properties on the maximum bond stress is discussed.

J. F. Lee, USA

6215. Derski, V., On some problems of thermoelasticity (in Roumanian), *Studii Si Cercetari Mecan. Apl.* **11,** 1, 131–151, 1960.

Following a brief history and some preliminary data, paper presents a solution for the unsteady thermic stresses occuring in a thick-walled tube of infinite length; a linear relation between the temperature function and its normal derivative is known on the boundary. A numerical application is included, the results of which are plotted in diagrams.

Introducing the influence of the inertia forces, author gives some results for (1) an elastic half-space and (2) a circular cylinder of infinite length; as regards the boundary conditions, Cauchy-type conditions are admitted for stresses (first problem) and displacements (second problem).

P. P. Teodorescu, Roumania

6216. Ignaczak, J., Dynamic thermoelastic problem of a spherical cavity (in English), *Arch. Mech. Stos.* **11,** 4, 399–408, 1959.

6217. Choudhury, P., Three-dimensional thermal stresses due to periodic supply of heat on the plane boundary of a semi-infinite solid (in English), *Atti Accad. Sci. Torino* **92,** 1, 3–9, 1957/58.

A solution of the title problem in the form of Fourier integrals is proposed. No numerical examples or applications to specific situations are given. The discussion is based on classical Neumann-Duhamel stress-strain-temperature relations.

D. Radenkovic, Yugoslavia

6218. Kovacs, A., Mechanics and analysis of articulated pipe lines, Parts 1, 2, and 3 (in French), *Chaleur et Industrie* **40,** 411, 301–316, Oct. 1959; **40,** 412, 347–361, Nov. 1959; **40,** 413, 381–400, Dec. 1959.

The thermal stresses induced in many pipe lines, e.g. those associated with atomic piles, are so severe that in practice it is often necessary to introduce articulations. Paper presents methods of stress analysis adequate to deal with all the possible practical cases. Numerical examples are given showing the efficacy of articulations. Methods for analyzing the flexibility of practical articulations are also presented.

W. S. Hemp, England

6219. Deev, V. M., Solution of the three-dimensional problem in the theory of elasticity for an anisotropic medium (in Ukrainian), *Dop. Akad. Nauk URSR* no. 7, 707–711, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6609.

This is an investigation of the elastic equilibrium of a homogeneous medium with rectilinear anisotropy of the most general form (the number of independent elastic constants being equal to 21). Starting from the equations for equilibrium in the transposition projections, analogous to the Navier-Lamé equations for an isotropic medium, the author is able to show that the transpositions and stresses can be expressed through three functions, satisfying one and the same differential equation with constant coefficients containing only derivatives of the sixth order. In a particular case the general formulas and equations derived by the author pass over to formulas and equations for an orthotropic medium (with nine independent constants) evolved by Mossakovska.

S. G. Lekhnitskii

Courtesy Referativnyi Zhurnal, USSR

6220. Rostovtsev, N. A., Remarks on the paper by V. S. Gubenko, "Some contact problems of the theory of elasticity and fractional differentiation," *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 4, 1143–1149, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

In a previous paper, V. S. Gubenko gave a solution to the problem of a rigid circular punch on an elastic half-space introducing concept of derivatives of fractional order. In present paper, N. A. Rostovtsev shows that the derivatives of fractional order may be more satisfactorily defined using an integral transformation. He also corrects the errors in Gubenko's derivations.

M. P. Bieniek, USA

Viscoelasticity

(See also Revs. 6300, 6336, 6337, 6349)

6221. Lianis, G., Analysis of deformations of viscoelastic bodies due to gravitational forces, Purdue Univ., School Aero. Engng. Rep. no. S-60-1, 29 pp., June 1960.

To study distortion of solid-fuel rocket grains during storage, thick-walled tube with axis horizontal is considered. General solution of equivalent elastic plane-strain problem is found by using complex potentials. Transformation into viscoelastic solution is carried out for case of incompressible material with four-element model shear response.

A. C. Pipkin, USA

6222. Distefano, N., Experimental basis for a theory on a behaviour of linear viscoelastic structures—applications (in Italian), *Ric. Scient.* **30,** 1, 114–127, Jan. 1960.

Elastic and creep properties of a phenol-formaldehyde material in tension were investigated experimentally and curves of creep and recovery shown. Linear relation between creep and applied stress was found and presented in known form of Volterra's inte-

gral equation for hereditary process. Author applied it generally to problem of bending of beam on foundation, both viscoelastic, but no concrete results are given.

Z. Bychawski, Poland

6223. Kleeman, P. W., Creep in aircraft structures, Aero. Res. Lab., Melbourne, Austral., Rep. SM 269, 32 pp., Sept. 1959.

In this report the following aspects of creep in aircraft structures are reviewed and discussed: the creep buckling of columns, plates and shells; the redistribution of stress under creep conditions; the estimation of total structural deformation, together with the more basic topics of stress-strain relations, methods of analysis, and material data, on which the above aspects depend.

From author's summary

6224. Meleka, A. H., and Evershed, A. V., The dependence of creep behavior on the duration of a superimposed fatigue stress, *J. Inst. Metals* **88,** 411–414, 1959/60.

The superposition of a fatigue stress in a creep experiment results in a sudden increase in the creep rate, which decreases with time, the strain following a $(\text{time})^{2/3}$ law. If the fatigue stress is removed, it is observed that the resulting creep rate is even less than the original static rate. This hardening, too, follows the $(\text{time})^{2/3}$ law. A model based on the increased mobility of an oscillating dislocation is proposed to explain the softening due to fatigue. The subsequent hardening is attributed to the effect of vacancies produced by the fatigue stress.

From authors' summary

6225. Davies, P. W., and Dennison, J. P., The role of voids in creep and fracture of a pure metal *J. Inst. Metals* **88,** 471–476, 1959/1960.

Voids have been introduced into high-purity nickel by sintering to various densities, and their influence on creep and fracture behavior has been studied by comparison with vacuum-melted metal. Their most important effect appears to be in draining off excess vacancies, which results in a change in activation energy for creep. This is shown to give an increase in creep rate of the order of magnitude required by observed results. On this basis a theory of the initiation of tertiary creep in void-free metals is proposed, which suggests that the behavior of these materials in compressive creep should be independent of pre-existing voids except for recrystallization effects. Confirmatory evidence has been obtained. The work indicates that vacancy absorption is a relatively unimportant mechanism in the growth of voids.

From authors' summary

6226. Dolmatov, E. G., and Sitnikov, I. I., A method for measuring the velocity of the plastic flow of steel when being subjected to tension to the destruction limit (in Russian), *Zavod. Lab.* **24,** 5, 629–631, 1958; *Ref. Zh. Mekh.* no. 2, 1959, Rev. 2039.

The deformation of tempered specimens of the Gagarin type was carried out by means of gas pressure produced by the burning of powder [in a special vessel representing the conditions of a closed space]. The velocity of the plastic flow was determined by high-speed cinephotography with camera SKS-1 at 4000 frames per second. The magnitude of the tension was taken to be the change of position of the upper face of the piston determinable from the cinephotographs; with these, curves were drawn of the plastic flow.

G. A. Tulyakov
Courtesy Referativnyi Zhurnal, USSR

6227. Derkach, V. F., Influence of creep and shrinkage of concrete on the stressed-deformed state of a compressed ferro-concrete element (in Russian), *Trudi Khar'kovsk. Inzh.-Stroit. In-ta* no. 5, 83–97, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6901.

An analytical solution is furnished for the problem of a symmetrically-reinforced ferro-concrete component, when there is no external load and when the load changes following an arbitrary principle. An investigation is carried out for the case where there is a linear linkage between the stress and the deformation of the concrete. The relations obtained enable the author to make some qualitative and quantitative evaluation by means of the separate factors influencing the stressed-deformation state of the ferro-concrete component. The author asserts that the principle of superposition is inapplicable when there is simultaneous influence from creep and shrinkage and that its mechanical utilization may lead to noticeable error.

M. M. Manukyan
Courtesy Referativnyi Zhurnal, USSR

6228. Rzhanitsyn, A. R., Determination of the stresses in a thin layer during drying, with consideration for changes in the characteristics of creep in the material (in Russian), The drying and humidifying of building materials and constructions (Sushka i Uvlazhnenie Stroit. Materialov i Konstruktsii), Moscow, Profizdat, 1958, 228–234; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 7026.

The problem is looked into of the determination of stresses (taking creep into account) appearing in a thin layer of material in the course of drying. A linkage is obtained between the stress σ and the deformation ε

$$H\frac{d(\varepsilon-\varepsilon_w)}{ds}+E(\varepsilon-\varepsilon_w)=\sigma+\frac{d\sigma}{ds}$$

$$s=\int_0^t\frac{dt}{n(t)},\quad \varepsilon_w=\alpha(W-W_0)$$

Here ε_w is the humidity component of the general deformation, W and W_0 are the humidities at time t and the initial humidity respectively, H and E the prolonged and momentary moduli of elasticity, s the conditional time, $n(t)$ the function of time, dependent on the elastic and viscous (K) properties of the material. The equation is derived from an investigation of the viscoelastic model. For the first approximation it is assumed that the relation of the moduli of elasticity to the humidity is negligibly small. As an example of obtaining a closed analytical solution the case is solved of the drying of a thin plate, rigidly fastened on the surface, on the assumption that the change of humidity of the material with time conforms to the equation $W = W_0 \exp(-\nu t)$, where $\nu = \text{const}$, and that there is a linkage between the viscosity and the humidity in the form of $K = K_0/W$, where K_0 is the viscosity corresponding to the complete saturation of the material with moisture.

B. I. Panshin
Courtesy Referativnyi Zhurnal, USSR

6229. Nikolaev, B. A., and Samarina, I. A., The relation of the elasto-viscous properties of dough to the basic elements of its structure, Communication 2: The elastic-viscous properties of "models" of the dough (in Russian), *Khlebopek. i Konditersk. Prom-st'* no. 8, 4–9, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 7078.

Results are published of investigations undertaken to study the effect of different substances (albumens, glutens, starch, bran, cellulose, moulds) on the properties of "models" of dough made from ordinary commercial brands of flour. The tests were carried out on an appliance for the measurement of shear deformation in a plane-parallel space on a sloping surface under conditions of constant stress. Calculations were made, based on the experimental results, for the elastico-elastic-viscous characteristics for all the dough "models." The authors make the deduction that natural dough has a more elastic structure than the "model" dough. The significant influence of the time factor on the properties of the models of the dough is the subject of comment. Tables are given of the test results and also graphs reproducing the relation of shear deformation to the quantity of the basic elements of the dough found in the models.

V. A. Sveshnikova
Courtesy Referativnyi Zhurnal, USSR

Plasticity

(See also Revs. 6275, 6304, 6323, 6332, 6383, 6495, 6735)

6230. Szczepinski, W., Solution of the plane problem of plasticity in hyperbolic series (in English), *Bull. Acad. Polonaise Sci.* **7**, 6, 359–364, 1959.

Author solves the plane problem of plasticity in closed form by using hyperbolic series. The method is applied to the determination of the stress distribution during the process of sheet forming by stretching. A. Phillips, USA

6231. Olszak, W., and Zahorski, S., A non-homogeneous orthotropic circular segment as an elastic-plastic problem (in English), *Arch. Mech. Stos.* **11**, 4, 409–419, 1959.

This is an extension of the work of Shaffer and House [*J. Appl. Mech.* **22**, no. 3, 1955; AMR **9**(1956), Rev. 693]. Principal directions of orthotropy are assumed to coincide with the principal directions of stress. Solutions are obtained first for elastic and entirely plastic zone. The cases of plastic zone beginning from interior surface and also from outer surface are then dealt with. Lastly the case of plasticity at both ends together is treated. The criteria for the appearance of plastic zones are discussed at the end. S. C. Das, India

6232. McComb, H. G., Jr., Some experiments concerning subsequent yield surfaces in plasticity, NASA TN D-396, 33 pp., June 1960.

Some experiments to obtain further knowledge concerning subsequent yield surfaces in plasticity have been performed. Combined load tests have been made on seven thin-walled tube specimens of 2014-T4 aluminum alloy. The secondary (that is, the first subsequent) yield surface is outlined in the normal-stress—shear-stress plane for an initial loading in pure tension. Discussion is presented of significant changes in the shape of the stress-strain curves obtained from the tests of the thin-walled tube specimens.

From author's summary

6233. Johnson, W., and McShane, I. E., A note on calculations concerning the plastic compression of thin material between smooth plates under conditions of plane strain, *Appl. Scient. Res.* (*A*) **9**, 2/3, 169–176, 1960.

The solution to this problem was originally given by Green and consists of two distinct slip-line field solutions, and these fields he determined in a relatively complex way using the velocity equations. This note gives a method of approach to the problem which is easy to understand, and gives results demonstrating almost all the features mentioned in Green's paper. Authors' method is very simple to apply, and though it falls short in one important respect, it is offered as an interesting method of attacking the problem.

From authors' summary

6234. Perov, N. P., Investigation of the elastic and plastic properties of peat (in Russian), *Trudi In-ta Torfa. Akad. Nauk BSSR* **6**, 339–343, 1957; *Ref. Zh. Mekh.* no. 2, 1959, Rev. 1829.

The elastic-plastic properties of peat in relation to its humidity were investigated by the method of compressing cylindrically-shaped test samples between two parallel surfaces with a gradually stepped-up increase in the loading. Ten minutes after each stepped-up loading, measurements were made of the samples' full deformation. Then the load was shed and after ten minutes the residual deformation was determined. Calculations were made for the modulus of elasticity E_y, the modulus of full deformation E_n and "the degree of elasticity" ψ using the formulas

$$E_y = \sigma/\varepsilon_y, \qquad E_n = \sigma/\varepsilon_n, \qquad \psi = E_n/E_y,$$

the calculations being based on the magnitudes of the stress σ, the elastic deformation ε_y and full deformation ε_n. It was shown that peat when deforming follows Hooke's principle and that the magnitude ψ increases with the reduction of humidity.

V. S. Namestnikov
Courtesy Referativnyi Zhurnal, USSR

6235. Il'yushin, A. A., and Ogibalov, P. M., On plastic deformations of a thick-walled tube under the action of heat impulses and pressure (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 12, 85–89, Dec. 1958.

Paper discusses the elasto-plastic behavior of thick-walled tubes under the short-duration action of the internal gas pressure and temperature.

Authors proceed from the differential equation of heat conductivity reduced to a finite difference equation, which is obtained by neglecting the variation of the conductivity coefficient λ and the terms affected by the ratio of the thickness δ of the intensely heated layer to the distance r up to the tube axis. A mean value of the heat quantity Q is taken for the equation of heat transfer and a temperature varying linearly up to the value ϑ_0 is considered in the intensely heated layer. The formulas $\delta = \sqrt{\frac{2\lambda T}{c\gamma}}$; $\vartheta_0 = Q_0 \sqrt{\frac{2T}{c\gamma\lambda}}$ are obtained, where T is the interval of the heat shock and c is the heat-capacity coefficient.

For estimating the plastic stresses and strains, a stress-strain law consisting of two linear portions is adopted. It is deduced that in the internal layer, plastic tractions occur initially due to gas pressure which are followed by plastic compressions due to heat impulses and finally by plastic tractions caused by cooling.

By neglecting the quasi-static or even dynamic viscous phenomena, some of the conclusions are rather qualitative (such as those concerning the fatigue) and are limited to a certain dimension or impulse times.

No references are given. Formula (10) should read $e_i = \phi^{-1}(\sigma_i)$ instead of $e_i\ \phi^{-1}(\sigma_i)$.

M. M. Misicu, Roumania

6236. Tomlenov, A. D., Theory of sheet metal hydrostatic testing (in Russian), *Vestnik Mash.* **38**, 10, 47–49, 1958.

Observations of the plastic bulging of a metal diaphragm subjected to gradually increasing lateral pressure (the so-called 'bulge test') are of much interest in permitting an assessment to be made of the physical properties of metals which effect their behavior in technological forming processes, such as drawing, extrusion and the like. In the present paper, a simple approximate analysis is given of the bulge test on the basis of plastic deformation theory. It is assumed that the shape of the diaphragm at any time (up to the onset of instability) is sufficiently well represented by a polynominal expression. The experimental data obtained from a bulge test may be used to permit the theoretical determination of the relationship between octahedral shear stress and strain.

A number of theoretical analyses have previously been made of the bulge test. An exact analysis made on the basis of plastic flow theory, assuming Tresca's yield condition and flow rule, is due to E. W. Ross, Jr. and W. Prager [see *Quart. Appl. Math.* **12**, 86–91, 1954].

H. G. Hopkins, England

6237. Ambartsumyan, S. A., and Zadoyan, M. A., On the problem of elasto-plastic bending of beams (in Russian), *Izv. Akad. Nauk SSSR Otd. Tekh. Nauk* no. 10, 130–132, 1958.

The object of the paper is to investigate the influence of the shear stresses on the deflection of the beam in the elastic-plastic stage. A rectangular beam symmetrically loaded is considered. The assumptions are as usual,

$$\sigma_y = \sigma_z = \tau_{yz} = \tau_{xy} = 0 \qquad \tau_{xz} = \tau(x,z) \qquad \sigma_x = \sigma(x,z)$$
$$W(x,z) = W(x)\cdot z$$

with the x-axis coinciding with the geometrical axis of the beam, the z-axis being directed downwards.

$$\tau(x,z) = \begin{cases} -fe(x)(1 - z/b^2) & \text{in the elastic region} \\ -fp(x)(1 - z^2/b^2) & \text{in the plastic region} \end{cases}$$

where $fe(x)$, $fp(x)$ are the sought-for functions and the sought-for distance of the boundary of the plastic region from the neutral plane.

Detailed calculation has been performed for a simply supported beam under continuous load. The formula for deflection is given.

The conclusions, of qualitative character, are: (1) The influence of the shear stresses on the deflection at the middle point increases proportionally to E/G (For transversal anisotropy, E/G may amount to 5, 10, etc.). (2) The role of the shear stresses becomes less important as the ultimate load-carrying capacity is approached.

J. Rychlewski, Poland

6238. Ivlev, D. D., On isotropic hardening of plastic bodies, *Soviet Phys.-Doklady* **4,** 4, 913–915, Feb. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **127,** 4, 777–779, July/Aug. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

In this brief note, the stress yield criterion is assumed to depend upon

$$\int \varepsilon_{ij}\, d\sigma_{ij}$$

where the ε_{ij} are the strain-rate components. An associated flow condition is also presented. Then, using the Tresca condition and the incompressibility assumption, several simple examples are considered, from which it follows that the maximum shearing stress is a function of the maximum shearing strain rate.

D. Frederick, USA

6239. Cristescu, N., On the Bauschinger effect (in French), *C. R. Acad. Sci., Paris* **249,** 5, 616–618, Aug. 1959.

6240. Pogodin-Alekseev, G. I., and Zhuravlev, S. B., The nature of the deformation on the yield surface (in Russian), The thermal treatment and strength of metals and alloys, Moscow, Mashgiz, 1958, 169–178; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6981.

The inconsistency of Kester's hypothesis is pointed out; this hypothesis explains the appearance of the physical limit of yield by the presence of a brittle skeleton on the grain boundaries. The deduction is made that a much more likely explanation is the assumption which merges the process of plastic deformation on the yield surface with a mainly mutual rotation of the grains by means of sliding along their boundaries. The claim is made that the appearance of the physical limit of yield is universal. The magnitude of the critical degree of deformation for various metals depends on the chemical composition, the temperature, the speed of deformation, the thermal treatment, etc.

A. D. Pospelov
Courtesy Referativnyi Zhurnal, USSR

6241. Davidenkov, N. N., On the classification and detection of residual stresses, *Indust. Lab.* **25,** 3, 335–336, Apr. 1960. (Translation of *Zavod. Lab., SSSR* **25,** 3, 318–319, Mar. 1959 by Instrument Society of America, Pittsburgh 22, Pa.

Rods, Beams and Strings

(See Revs. 6139, 6214, 6222, 6261, 6264, 6268, 6276, 6304, 6321)

Plates, Shells and Membranes

(See also Revs. 6138, 6139, 6145, 6159, 6211, 6214, 6230, 6270, 6271, 6272, 6274, 6277, 6279, 6357, 6612, 6613)

Book—6242. On the computation of thin cylindrical shells (Handbook no. 4 of the Institute for the Applications of Calculus of the Italian National Research Council) [Manuali per Applicazioni Tecniche del Calcolo. IV. Calcolo delle volte cilindriche circolari sottili], Roma, Edizioni Cremonese, 1960, xvii + 332 pp. Lire 3.500. (Paperbound)

The present volume is a very valuable addition to the collection of technical and engineering handbooks edited by the Italian Institute for the Applications of Calculus, directed by Professor Mauro Picone. It furnishes formulas and numerical data for the computation of stresses and deformations in cylindrical shells under various external loading conditions (such as the shell's own weight, the weight of snow, etc.) and under different boundary conditions.

This work, carried out by Professor Domenico Caligo of the Institute of Calculus under the supervision of Professor Giulio Krall, will be of great value to those interested in the subject, particularly to civil engineers and thin-shell designers. A list of contents follows: Preface by Professor G. Krall, general theory of thin shells, tables, numerical examples of how to use the tables.

E. Volterra, USA

Book—6243. Pogorelov, A. V., Infinitely small bending of general convex surfaces [Beskonechno malye izgibaniya obshchikh vypuklykh poverkhnostei], Khar'kov, Izdatel'stvo Khar'kovskogo Ordena Trudovogo Krasnogo Znameni Gosudarstvennogo Universiteta, 1959, 105 pp. 4r 15k.

The convex surface considered is one whose equation can be expressed in the form $z = z\ (x, y)$. The displacement components can be expressed in terms of the location of the point relative to the lines of curvature of the surface. Various theorems pertaining to the differential geometry of convex surfaces are established rigorously. The deformations of a general convex surface subject to applied vertical displacements around its edge are considered and established approximately in terms of analytic functions. Finally, the special case of bending of a convex surface in a space of constant curvature is considered.

W. A. Nash, USA

6244. Chernykh, K. F., Relation between dislocations and concentrated loadings in the theory of shells, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 2, 359–371, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

Paper discusses the analogous situation between dislocations (which according to Love are multiple-valued displacements corresponding to single-valued strains in a multiply-connected region) and concentrated loadings in the thin shell theory. The displacement vector is expressed in terms of a contour integral involving strain and rotation components. Illustrations discussed are the example of a shell of revolution from which a portion has been removed and the simple example of bending of a circular plate (which is a degenerate shell) under a concentrated force at its center.

Y.-Y. Yu, USA

6245. Chernin, V. S., On the system of differential equations of equilibrium of shells of revolution under bending loads, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 2, 372–382, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

The bending load under consideration is proportional to the sine or cosine of the angle ϕ that defines the location of the meridian, an example being wind pressure. The displacements and stress resultants of the shell are similarly proportional to sin ϕ or cos ϕ. The equations of a thin shell of revolution thus become

ordinary differential equations of the eighth order. This paper is to show how the order of the equations can be lowered to the fourth by determining explicitly four integrals of the original system of eighth-order equations. Y.-Y. Yu, USA

6246. Keefe, R. F., and Overby, J. A., An experimental investigation of effect of end conditions on strength of stiffened cylindrical shells, David W. Taylor Mod. Basin Rep. 1326, 23 pp., Dec. 1959.

Six pairs of stiffened cylinders, machined from thick tubes and identical except for the size and spacing of the frames at the ends, were subjected to external hydrostatic pressure to establish the adequacy of a design procedure for the end bays. All the cylinders failed by axisymmetric shell yielding, four in the end bay, four in the first full-length bay, and four in the second full-length bay. For each pair of identical cylinders, the locations of damage at failure were identical and the collapse pressures differed by less than 1% when adjusted to a common yield strength. Test results indicated that with the end conditions established by the end-bay theory of TMB Report 1065, the collapse pressure could be increased as much as 5% and failure could be shifted away from the ends of the cylinder. From authors' summary

6247. Cicala, P., On the theory of elastic orthotropic shells in the form of a surface of revolution or in cylindric form (in Italian), *Atti. Accad. Sci. Torino* **92,** 1, 30-64, 1957/1958.

General equations of an orthotropic shell in the form of a surface of revolution are obtained. By inspection of the resulting homogeneous equation the various types of characteristic solutions are identified and classified. A series development in terms of the thickness is used in the solution, which makes it possible to specify reasonable simplifications corresponding to the adopted degree of approximation. A general solution for the fundamental perturbations in the zone of the edge is indicated. For a spherical shell the equations are given for the calculation of the effects of generic constraints.

From author's summary by D. Radenkovic, Yugoslavia

6248. Soare, M., Application of the plurilocal method to the calculation of thin shells (in Roumanian), *Studii Si Cercetari Mecan. Apl.* **11,** 1, 239-259, 1959.

The principle of the plurilocal method is explained; it consists in improving the classical finite difference method by associating the development into Taylor series of the unknown function to the differential equation which is to be integrated. The method is applied to a Poisson-type equation and the manner in which the relaxation method can be used in this case is shown.

For a hyperbolical paraboloid the membrane stress state is expressed with the aid of a stress function which verifies such an equation. A complete study of this shell is presented and a numerical example of calculation is included, the corresponding stress state being plotted in diagrams.

P. P. Teodorescu, Roumania

6249. Kornishin, M. S., and Mushtari, Kh. M., A certain algorithm of the solution of nonlinear problems of the theory of shallow shells, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 211-218, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Authors discuss approximate methods of solutions of equations for shallow shells with pertinent computations.

J. L. Ericksen, USA

6250. Mader, F. W., Determination of stresses in torsion tubes with cut-outs (in German), *Bautechnik* **36,** 8, 301-306, Aug. 1959.

Author considers cylindrical tubes with cross section consisting of arcs of circles. Cut-outs are bounded by generatrices and orthogonals to them. By extending of boundaries tube is divided into partial shells to which shell theory is to apply.

Solution of homogeneous shell equation for unloaded shell is done by a Fourier representation; characteristic equation of eighth degree gives for each member eight solutions and therefore possibility to fulfill four boundary conditions at two opposite boundaries. With an analogous statement for the other direction for each partial shell and series member 16 constants are found from 16 equations. An exterior boundary gives 4 boundary conditions, and an interior 8 transition conditions between the two partial shells adjoining each other. If free boundaries are reinforced by members with certain stiffness against tension, bending or torsion, these conditions are modified. Equations may be too extensive even for some computers; for this case iterative method is presented that needs at each step calculation of 8p or 16 p unknowns, respectively. (p are partial shells).

Procedure is shown on cantilever tube with one cut out loaded by a torsional moment introduced by stiffening bulkhead, so that three partial shells occur, and way is shown to obtain the 48 constants for each member. There are no numerical results for the example. W. Mudrak, Austria

6251. Saito, H., Stresses in rotating disks with noncentral circular holes, Parts 1 and 2, *Technol. Rep. Tohoku Univ.* **21,** 2, 217-245, 1957; **22,** 1, 63-72, 1957.

The disks treated here contain a number of eccentric holes of equal radii, the centers of which are arranged symmetrically on a circle of arbitrary radius. Assuming an elastic material, the stresses due to centrifugal forces in the rotating disk are calculated. In the first part of the investgations the disk is not affected by tractions at the boundaries. It turns out that the concentration of the stresses is remarkable along the edges of the holes, and these tangential stresses are calculated in some detail. Numerical examples are given for disks with 4 and 6 holes. They show that as long as the diameters of the holes are comparatively small the largest of these stresses appear at those points of the hole boundaries which are nearest to the center of the disk. However, as the diameters of the holes become larger the stress-maximum moves towards those points which connect the narrowest section between adjacent holes.

The second part deals with rotating disks with a rim or blades or disks submitted to uniform tension p on the outer boundary. The author summarizes that the stress distribution in the case of a rim and blades may be regarded as consisting of two parts. The one is the stress produced in the freely rotating disk without the rim and blades, the other is the stress produced in the disk of the same form submitted to uniform tension p at the outer boundary.

The first part is followed by a report about measurement of the stresses in the rotating disk by bonded wire strain gages.

F. Chmelka, Austria

6252. Eriksson, E., Effect of deflections on moment distribution in gable roofs (in Swedish), *Nordisk Betong* **3,** 1, 31-46, 1959.

Author considers a symmetrical and elastic gable roof placed on vertically rigid side walls. The ends of the ridge are allowed to move only horizontally. The roof is made subject to a continuous line load along the ridge, it is assumed that a load distribution over the roof may be reduced to such a line force. Author considers specially the case of a uniform line load along a hinged ridge. It turns out that for a certain value of load the deflection curve of the ridge becomes unstable, bending, if originally hanging downwards, upwards. As an application the design of a concrete roof is considered. The results are shown to be applicable also in the case where the failure begins with longitudinal plastic joints. O. B. Hellman, Denmark

6253. Werren, F., and Ethington, R. L., Deflection characteristics of two 20-foot-diameter laminated wood rings subjected to compressive loading along a diameter, For. Prod. Lab., U. S. Dept. Agric. Rep. 1877, 34 pp., Aug. 1960.

Tests were made to determine if the existing theory for curved members applied to a large glued-laminated wood ring loaded along a diameter. A trial southern pine ring and a white oak ring were fabricated. The rings were tested by compressive loading along a diameter, and the vertical deflection, the horizontal deflection, and the strain in the outer fiber at various locations were measured. The compressive loads imposed bending stresses to approximately the proportional limit.

From authors' summary

6254. Czerny, F., Tables for hydrostatically loaded rectangular plates (in German), *Bautech.-Arch.* no. 14, 115 pp., 1959.

This extremely useful work for design treats the rectangular plate of constant thickness under hydrostatic loading. The computations are executed within the framework of Kirchoffian theory, allowing the consideration of the influence of Poisson's ratio. The plate material is assumed to be homogeneous, isotropic, and Hookean. Detailed tables and diagrams are presented and these contain the characteristic numerical quantities indispensable to a rapid analysis of the following six boundary-value problems:

(1) all edges simply supported;
(2) three edges simply supported and one edge clamped;
(3) two opposite edges simply supported—the other two clamped;
(4) two adjacent edges simply supported—the other two clamped;
(5) three edges clamped and one edge simply supported;
(6) all edges clamped.

A. W. Coutris, USA

6255. Sherman, D. I., On the transverse bending of a plate supported along the edges and consisting of several closed curves, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23,** 1, 145-164, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

This important paper by a leading Russian elastician is additive to his already extensive and authoritative writings on the solution of problems in plane stress, using complex variable theory. A plate of finite area with a finite number of cut-outs is represented by a finite multiply-connected region S, bounded by a totality of sufficiently smooth closed curves L_j ($j = 1, 2, \ldots m + 1$) where L_{m+1} is the external contour, and L is the entire contour. The unknown deflections $w_1(x, y) = w(x, y) + w_0(w, y)$. w_0 is a particular solution of the plate equation; it is shown that w_0 is constant everywhere in S. w is given by the well-known Goursat formula

$$2\,w(x, y) = \bar{z}\phi_1(z) + z\overline{\phi_1(z)} + \chi_1(z) + \overline{\chi_1(z)} \text{ where } \psi_1(z) = \chi_1{}'(z).$$

ϕ_1 and ψ_1 are analytic multi-valued functions to be determined on L.

In the first part of the paper, appropriate forms of ϕ_1 and ψ_1 in terms of complex variable t (local complex co-ordinate on L) and loading functions f, are developed, and a Fredholm integral equation is obtained, the kernels being continuous (elementary) functions of t. It is next proved that this equation is solvable, uniquely. Author stresses repeatedly that his analysis is intended to be quantitative. In particular, author treats of plate in form of eccentric ring. The case where S is infinite (but with finite area of loading) is discussed.

Paper (in translation) is clearly written; groundwork of analysis involved is to be found in treatises by Muskhelishvili and Sokolnikoff.

F. A. Gerard, Canada

6256. Brotchie, J. F., General elastic analysis of flat slabs and plates, *J. Amer. Concr. Inst.* **31,** 2, 127-152, Aug. 1959.

By introducing an imaginary liquid support, author develops a general method for analyzing flat slabs and thin plates. The liquid provides a reaction proportional to the deflection; its density is chosen arbitrarily to meet the condition that when all loads and support reactions are applied together the individual liquid reactions effectively cancel each other and boundary conditions of the problem are satisfied. A parameter s having the dimension of length, and termed "radius of relative stiffness" of the floating plate, is introduced as a measure of the density of the supporting plate. Calculations are based on influence charts and tables for moments, deflections and shears against the dimensionless ratio of the radial distance to the parameter s of an infinitely long floating plate under unit loadings.

Examples presented show the applicability of the method to a wide range of problems, in particular to cases of point loadings and irregular supports, continuous plates on columns, panels of varying span, drop panels, column settlement and flat slab raft foundations. The method is suitable for both accurate analysis and design.

Z. Karni, Israel

6257. Phillips, H. B., and Allen, I. E., Stresses around rectangular openings in a plate, *Proc. Amer. Soc. Civ. Engrs.* **86,** EM 3 (*J. Engng. Mech. Div.*), 165-174, June 1960.

Curves have been developed for determination of the normal stress around rectangular openings when subjected to uniaxial or biaxial stress fields. Height-to-width ratios between 0.4 and 2.5 have been studied. The study was made experimentally utilizing the photoelastic interferometer method of stress analysis.

From authors' summary

6258. Benthem, J. P., and van de Vooren, J., On the diffusion of a load from an edge-stiffener into a wedge-shaped plate, Nat. LuchtLab. Amsterdam TN S. 541, 47 pp., July 1959.

A wedge-shaped plate with one or two edge stiffeners, whose cross-sectinal areas may be variable, is considered under the condition that a single force is introduced at the edge. The method of calculation developed in this paper uses the so-called Mellin transform of the shear flow. For a wedge with one stiffener only the method yields a functional equation. Inverse Mellin transform of the functional equation delivers a singular integral equation of the Cauchy type. Numerical iteration methods are applied and show, in the case of an edge angle of 180°, good agreement with the known exact solutions.

H. Neuber, Germany

6259. Higashi, Y., and Komori, K., Uniformly loaded rectangular plates with clamped edges and free edges, *Mem. Fac. Technol., Tokyo Metrop. Univ.* no. 9, 690-702, 1959.

Solution of the problem in the form of double Fourier series is given. Numerical examples are calculated of plates: with two adjacent edges clamped others free; three edges clamped, one free; and four edges clamped.

Z. Olesiak, Poland

6260. Uspenskii, M. M., and Flavianov, V. P., The application of the theory of deflection of anisotropic plates in the designing of interstory coverings (in Russian), *Sb. Trudi Voronezhsk. Inzh.-Stroit. In-ta* no. 4, 138-146, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6873.

Results are given of the tests carried out on coverings consisting of ferro-concrete beams with the spaces in between filled with blocks made of slag concrete. The experimental data are compared with the theoretical calculations of the covering viewed as an anisotropic plate. The differential equation for the deflection is integrated by the Kantorovich method. The divergence between the experimental data and the theory comprised 30%. Some conclusions are published on the calculations and construction.

E. F. Burmistrov

Courtesy Referativnyi Zhurnal, USSR

6261. Rivlin, R. S., The deformation of a membrane formed by inextensible cords, *Arch. Rational Mech. Anal.* **2,** 5, 447-476, Feb. 1959.

Author derives general equations for the deformation of a net, formed by two families of inextensible cords, which forms a curved surface. The cords of the two families form a curvilinear net on the surface. Particular cases of cylindrically symmetric deformation, the superposition of small deformation on cylindrically symmetric deformation, and small deformation of cylindrical membrane with given edge displacements are discussed.

B. E. Gatewood, USA

Buckling

(See also Rev. 6366)

6262. Boyce, W. E., Buckling of a column with random initial displacements, AFOSR TN 60-150 (Rensselaer Polytech. Inst., Dept. Math. Rep. 32), 16 pp., Feb. 1960.

Using mathematical methods of statistics author discusses the relation between the load and the transverse displacement of a column when the initial configuration of the column is random in nature.

S. E. Kindem, Norway

6263. Lebelle, P., Elastic stability of prestressed concrete beams with regard to lateral buckling (in French), *Ann. Inst. Tech. Bât. Trav. Publics* **12,** 141, 779-831, Sept. 1959.

After revising the theory of lateral buckling of deep section double-T beams including effects of constrained warping, author applies it to a number of technical problems. Among those considered are: lateral stability of beams suspended by cables during the placement; effects of initial imperfections and wind loads; bracing by horizontal guys; influence of conditions of constraint against torsion and bending at ends of beam; continuous beams; effect of axial loads.

Although written especially for civil engineers this important contribution should also be of interest to others.

P. C. Dunne, Brazil

6264. Levin, G., Influence of thé position load location upon the buckling stability of a fork-supported simple beam with symmetrical cross section, which is loaded with a concentrated load at the middle of the beam (in German), Dtsch. Forschungsanstalt Luftfahrt Ber. 102, 50 pp., 1959.

Title problem of elastic stability in bending and torsion is solved by series expansion. Critical loads for various cross sections are presented extensively in graphs, the dependence on distance of the point of loading from the centroidal axis being shown. Results differ in some respects from previous work, which is found to have overestimated the buckling load appreciably in a certain range of the variables. The paper is said to be particularly relevant to crane structures.

R. Hill, England

6265. Huber, A. W., and Ketter, R. L., The influence of residual stress on the carrying capacity of eccentrically loaded columns in English), *Publ. Int. Assn. Bridge Struct. Engng.* **18,** 37-62, 1958.

The general method of solution for the maximum carrying capacity of columns containing residual stresses is first presented. Application is then made to a specific example of an H-column. The influence of an asymmetric residual stress distribution is considered and the results are presented in the usual forms as column curves for axially and eccentrically applied loads. Next, approximate solutions are presented for beam-columns having an idealized elastic-plastic stress-strain relationship and which contain residual stresses whose patterns have at least one axis of symmetry. The results of these studies are also shown in the form of column curves. Finally, the theory is compared with test results which have been reported in various publications.

From authors' summary by J. Heyman, England

6266. Lee, S. L., and Clough, R. W., Jr., Stability of pony-truss bridges (in English), *Publ. Int. Assn. Bridge Struct. Engng.* **18,** 91-112, 1958.

Method deals not only with buckling, but also determination of secondary stresses under working loads. Simultaneous linear equations are derived from equilibrium of top chord joints; solution of equations gives displacements of joints and hence secondary stresses. Buckling conditions are treated as eigenvalue problem. Numerical examples are given.

J. Heyman, England

6267. Horne, M. R., The elastic lateral stability of trusses, *Struct. Engr.* **38,** 5, 147-155, May 1960.

Author gives an approximate analysis, using the energy method, for an elastic plane truss with two parallel chords and with an arbitrary arrangement of vertical and diagonal web members. The joints are assumed to be rigid and the members prismatic between panel points. No account is taken of intersections within the web. The truss is subjected to any combination of uniform bending moment and over-all axial load. The compression chord is of uniform section, carrying a uniform thrust, but the use of some of the results for nonuniform conditions is also discussed. It will be assumed that the compression chord buckles in sinusoidal half-waves, the tension chord being held in position. Account is taken of the flexural and torsional rigidities of all the members, and of partial restraint of the tension chord against twisting. All restraints to the chord from the web members are considered to be distributed continuously as in a special sort of elastic medium. General solution is not explicit, but closed formulas are given for some special cases. A numerical example is presented.

Reviewer believes that the present solution is of particular interest in relation to trusses with chords of tubular section. In such cases the torsional rigidity cannot be neglected and the well-known Engesser formula underestimates the critical load.

C. Hoschl, Czechoslovakia

6268. Stein, M., The phenomenon of change in buckle pattern in elastic structures, NASA TR R-39, 22 pp., 1959.

A lumped parameter model consisting of three bars, linear torsion springs and nonlinear deflection springs, corresponding to a column on a nonlinear elastic foundation, is analyzed in its post-buckling behavior for both controlled end loading and end shortening. The model is studied to gain insight into the more difficult problem of the postbuckling behavior of plate structures. It is shown that various equilibrium patterns exist which correspond to a symmetrical, antisymmetrical and continuously mixed deflection mode. The actual path under loading consists of segments of the symmetrical and antisymmetrical path and the intercepted segment of the mixed mode. Transition occurs at the intersections of these paths and the equilibrium may be stable or unstable, depending on the type of loading and the structural parameters.

J. E. Duberg, USA

6269. Lowery, T. M., How to predict buckling and unseating of coil springs, *Prod. Engng.* **31,** 29, 62-66, July 1960.

Recent equations are given for predicting the behavior of a coil spring when compressed between parallel or nonparallel plates. The theory is confirmed by test results, and a sample problem is discussed.

From author's summary

6270. Suhara, J., Snapping of shallow spherical shells under static and dynamic loadings, AFOSR TN 60-831 (Mass. Inst. Technol., Aeroelastic and Structures Res. Lab. TR 76-4), 72 pp., June 1960.

A study of finite deformations of clamped shallow spherical shells subjected to static and dynamic loadings is presented in this report. The solutions of the static problem are obtained by means of both the variational method and the method of expansion in the series of combined Legendre polynomials.

The results are compared with those obtained by Keller and Reiss. Fairly good agreements of both results are obtained for the unbuckled branch in the static cases.

Dynamic behavior of shallow spherical shells which undergo suddenly applied uniform pressure of infinite duration are investigated using two different methods. In the first a simple trial function for the deformation mode is assumed and the dynamic equation is obtained by variational method; the equation is then solved by means of an electronic analog computer. Other trials are made to estimate the dynamic maximum deformations due to a given dynamic load. Principle of conservation of energy is used in this case. The analyses involve two sets of solutions. In the first a simple trial function is used while in the other the Legendre polynomials solutions that were obtained previously for the static problem are used. Estimations of the dynamic snapping loads are given for shells of two kinds of shallowness.

From author's summary

6271. Singer, J., and Hoff, N. J., Buckling of conical shells under external pressure and thermal stresses, AFOSR TR 59-203 (Techn. Res. Develop. Foundation, Haifa, Israel), 82 pp., Dec. 1959.

Simplified differential equations governing the deformations of deep thin circular conical shells subjected to arbitrary loads and arbitrary temperature distributions are derived by the principle of minimum total potential energy. The equations reduce to Donnell's equations (extended to include an arbitrary temperature distribution) when the cone angle approaches zero. An asymptotic solution of these simplified equations is then obtained for a truncated conical shell subjected to an axisymmetrical temperature distribution. A very simple particular solution is obtained for temperature distributions that can be expressed in a certain form, which however has fairly general application. The thermal stresses are computed for a typical temperature distribution.

The equations are then extended to include problems of elastic stability. The stability equations are solved for loading under lateral and under hydrostatic pressure in the presence of slightly relaxed boundary conditions for the u and v displacements; for the w (radial) displacements the usual conditions of simple support are enforced rigorously. The results are presented in a parametric form analogous to that used by Batdorf for circular cylindrical shells, which yields convenient design curves, and are compared with results by earlier methods of solution (Pfluger, Niordson, Bijlaard, Mushtari and Sachenkov). The solution is valid for thin shells having small cone angles which buckle with a large number of circumferential waves.

The same method of solution, with the slightly relaxed boundary conditions for the u and v displacements, is then applied to the thermal buckling problem of truncated conical shells subjected to axisymmetrical temperature distributions. A typical example is analyzed in detail.

From authors' summary by H. G. Hopkins, England

6272. Grigolyuk, E. I., On taking into account the compressibility of the material in determining the lower critical loads (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 5, 104-105, May 1958.

The critical (buckling) loads are determined for cylindrical shells of an elastic-plastic material with an arbitrary Poisson ratio. Longitudinal, lateral, and simultaneous longitudinal and lateral loadings are considered. Author compares his results with those obtained by G. Gerard [*J. Aero. Sci.* **24**, 4, 269-274, 1957; AMR **11**(1958), Rev. 2537]. M. P. Bieniek, USA

6273. Sharvashidze, D. A., The causes of buckling of strata in the mine workings of the Akhaltsikhskikh pits (in Russian), *Trudi Gruz. Politekhn. In-ta* no. 2 (59), 3-22, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6736.

Results are given for an actual example in the Akhaltsikhskikh coal fields of field investigations on the break down of the "upper sandstone" stratum. The elastoplastic stressed state and the deformation of the strata are examined in the vicinity of the horizontal unsupported working [see A. Salustowicz, *Pszegl. Gorniczy* **2**, no. 11-12, 1946; **8**, no. 1-2, 3, 1947]. I. V. Fedorov

Courtesy Referativnyi Zhurnal, USSR

6274. Erickson, B., French, F. W., Patel, S. A., Hoff, N. J., and Kempner, J., Creep bending and buckling of thin circular cylindrical shells, NASA TN D-429, 49 pp., June 1960.

A number of circular cylindrical shells manufactured of 5052-0 aluminum alloy were tested in pure bending, most at a temperature of 500°F, but a few at room temperature. Fifty-four specimens were unreinforced, seventeen had longitudinal, fifteen had circumferential, and sixteen had longitudinal and circumferential reinforcing elements. All the specimens not subjected to static tests failed by creep buckling. The results of the experiments are presented in tables and diagrams. A few theoretical considerations are given in an appendix.

From authors' summary

6275. Mellor, P. B., Plastic instability in tension, *Engineer, Lond.* **209**, 5435, 517-521, Mar. 1960.

Author considers the condition for plastic instability under biaxial tension for materials whose strain-hardening characteristics can be fitted by the empirical equation $\bar{\sigma} = A(B + \bar{\varepsilon})^n$ suggested by H. W. Swift [*J. Mech. Phys. Solids* **1**, 1, 1952]; in the foregoing formula $\bar{\sigma}$ and $\bar{\varepsilon}$ denote generalized stress and strain while A, B, n are constants depending on the material. To locate the point of instability, corresponding to the maximum load, a simple construction based on the equation:

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\bar{\sigma}}{z}$$

is used. Author shows that (a) for the case of balanced biaxial tension applied through two equal loads the value of subtangent in plane $(\bar{\varepsilon}, \bar{\sigma})$ is $z = 2$; (b) for thin-walled cylinder subjected to internal pressure, with closed or open ends, $z = 1/3$ and $z = 2/3$, respectively; (c) for spherical shell $z = 2/3$. Also the case of a circular membrane clamped at its periphery and subjected on one side to fluid pressure is examined; experimental values of instability are shown to be in good agreement with the theoretical treatment by Hill [*Phil. Mag.* (7) **41**, p. 1133, 1950; AMR **4**(1951), Rev. 4108].

It must be noticed that in all the cases treated in this paper, but of course not in general, the stress ratio can be shown to remain constant, so that the Levy-Mises relationship between stress and strain increment may be integrated to give the total strains.

E. Storchi, Italy

Vibrations of Solids

(See also Revs. 6159, 6172, 6179, 6298, 6300, 6604, 6608, 6611, 6612, 6613, 6614)

6276. Svetlitskii, V. A., String vibrations, taking changes of tension into account (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 31-42, 1958.

Author derives the general equation of motion of a string under tension, correcting errors made by A. P. Minakov, and considers an example of small, planar oscillations. He finds in the example that small deviations from a constant tension have a small effect

on the amplitude but not necessarily a small effect on the kinetic energy of a string element. E. E. Zajac, USA

6277. Trubert, M., and Nash, W. A., Effect of membrane forces in lateral vibrations of rectangular plates, AFOSR TN 60-437 (Univ. Florida, Dept. Engng. Mech., Engng., Indust. Experiment Sta. TN-2), 44 pp., May 1960.

Paper analyzes the effect of membrane forces by a perturbation method, expanding frequencies and modes in terms of a suitable nondimensional parameter. Method agrees with exact solutions for plates with simply supported edges and beams under tension. Good agreement with known solutions for a square clamped edged plate is also obtained. Method is limited by the finite convergence range for the parameter, but is satisfactory for square plates whose thickness is greater than one hundredth of the side. Mathematical techniques used are not limited to problems treated.

W. S. Hemp, England

6278. Hu, H. -C., On two variational principles about the natural frequencies of elastic bodies, *Scientia Sinica* **7,** 3, 298-312, Mar. 1958.

Two variational principles are proposed for the natural frequencies of elastic bodies. Author demonstrates that the true natural frequencies are all extrema of a certain functional. It is in general not true, however, that all of the extrema are natural frequencies and, in particular, the lowest natural frequency is usually not the minimum extreme value of the functional.

For example, the eigenvalues defined by the first principle often include the zero eigenvalue with infinite multiplicity. This is obviously the case when only displacement boundary conditions are prescribed. L. E. Payne, USA

6279. Knowles, J. K., Large amplitude oscillations of a tube of incompressible elastic material, *Quart. Appl. Math.* **18,** 1, 71-77, Apr. 1960.

Problem treated is that in which only radial deflection occurs and only varies with radius. Theory described in AMR **8**(1955), Rev. 2296 is used to derive the differential equation governing this displacement. A first integral is obtained and the condition on the form of the strain energy function for the existence of periodic motion is found.

The limiting case of the thin cylinder is considered and it is shown that the limiting values of the maximum and minimum magnitudes of the radius during oscillation are reciprocal fractions of the static radius. The expression for the period is also given. Explicit expressions for these quantities are given for the Mooney-Rivlin material, and the period is shown to be independent of the amplitude. K. H. Griffin, England

6280. Galaka, P. I., Some characteristics of the vibrating motion of a system with two mutually impacting masses when there is an elastic bond between them (in Ukrainian), *Dop. Akad. Nauk URSR* no. 7, 712-715, 1958; *Ref. Zh. Mekh.* no. 6, Rev. 5999.

Data are produced to show the influence exerted by the elastic bond between the mutually impacting masses on the character of the change of position of these masses and on the amplitude of the vibrations of the system being investigated.

M. E. Temchenko
Courtesy Referativnyi Zhurnal, USSR

6281. Vol'vich, S. I., Determination of the frequencies of vibrations of some systems (in Russian), *Trudí Saratovsk. Avtomob.-Dor. In-ta* **15,** 1, 97-107, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6678.

The application of the equations for finite differences for the determination of the frequencies of natural vibrations is demonstrated in two examples: a beam with evenly distributed masses and a plane cantilever-type double-lattice framework with parallel flanges carrying a concentrated mass on its end. The expediency is indicated in some cases of the substitution of a system with distributed parameters by some system with a simple formation structure, with the object of determining the approximate value of the natural frequencies from the equations of finite differences.

V. V. Bolotin
Courtesy Referativnyi Zhurnal, USSR

6282. Corbetta, G., Computation of higher critical speeds (in Italian), *Ingegnere* **32,** 9, 775-784, Nov. 1958.

Method is given for computing higher critical speeds of shafts. Starting from a system with one degree of freedom, author finds effect of a second mass. On this basis an approximate method of iteration is worked out. Numerical examples show good accuracy.

H. Fernandez Long, Argentina

6283. Plander, I., Theoretical and experimental study on the oscillations of linear systems when passing over the critical state (in Slovenian), *Strojnicky Casopis, Slovenskey Akad. Vied, Bratislava* **10,** 1, 5-30, 1959.

Paper concerns the study of a system with n degrees of freedom, represented by n weights which produce torsional oscillations of a rotating shaft, the dampings being proportional to the rotating speeds. The exciting moment is considered as varying according to a law given by the function $\cos\ \varepsilon \frac{t^2}{2} + \psi$.

By applying the Laplace transform and the convolution theorem to the system of equations which describe the motion, author obtains the functions of the rotation angles. Detailed calculations are performed for systems with two degrees of freedom in a manner similar to that employed by A. P. Filippov for a system with one degree of freedom. Theoretical results are completed by electrical and mechanical model tests.

In the last line of formula (2.16), the constant ψ_{ik} should be of minus sign and expression $(y_k + i\mu_k)$ should be read $(y_k - i\mu_k)^2$.

M. Misicu, Roumania

6284. McLaughlin, W., Vibration problems in marine engineering and their solution, *Trans. Inst. Engrs. Ship. Scotland* **103,** 5, 136-242; **103,** 6, 243-251, 1959/60.

A number of typical vibration problems in turbo-geared and diesel machinery are discussed; electronic instruments for the location of the sources of vibration are described.

From author's summary

6285. Chuvikovskii, V. S., Free vibrations of the sheathing of ships' decks, taking into account the torsion of the assembly (in Russian), *Sudostroenie* no. 12, 12-14, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6815.

Adjacent portions of the sheathing of the deck are investigated as plates elastically fastened on two sides; the other two sides are considered to be freely supported. The frequencies of the antisymmetrical forms of the natural vibrations of the sheathing are determined. The reinforcing rib of T-form is represented as consisting of a flange-plate, freely supported on the end edges (the other two edges being connected with the sheathing and a flange), and a flange-beam, freely supported at the ends. By making his starting points the differential equations for the vibrations of each of the elements and the condition of co-stressing operating, the author succeeded in getting an equation for the frequencies. A graphical method for solving this equation is proposed. A numerical example is given. It is shown that a rib of T-profile can have an appreciable influence on the vibrations of the plate.

A. S. Vol'mir
Courtesy Referativnyi Zhurnal, USSR

6286. Byllo, G. I., The influence of resonance on the major frequencies during vibrations of metal beam bridges on the forces

acting on them (in Russian), *Trudi Rostovsk. In-ta Inzh. Zh.-d. Transp.* no. 24, 76–91, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6827.

The differential equation for the vibrations of a heavy beam of constant stiffness is integrated; the beam is subjected to a system of disturbing forces produced by the unbalanced masses of moving locomotives. The inertia effect of the moving loads is disregarded. The following items in the calculations are considered: (1) the amplitude of the disturbing forces is constant, is proportional to the frequency, is proportional to the square of the frequency; (2) the coefficient of damping is constant, is proportional to the frequencies of the natural vibrations, is proportional to the square of the last. The calculations follow the customary procedure; trigonometrical series are employed. The calculations show that for a bridge with a length of $l = 158.4$m there may be a resonance at the third frequency (the critical speed being 225 km/hr). A. A. Pikovskii

Courtesy Referativnyi Zhurnal, USSR

6287. Stanisic, M. M., An approximate method of determining the vibration angle of turbo-blading with lashing wire (in English), *Forsch. Geb. Ing.-Wes.* **24,** 5, 154–160, 1958.

A method, based on the principle that during vibration the stored energy in the system remains a minimum, is presented for finding the direction of the eigenvector of the first vibratory mode. Numerical examples are carried out for several different blades interconnected with lashing wire and the results for the fundamental frequency agree satisfactorily with experimental values for propellers and turbine blades. H. H. Hilton, USA

6288. Hirsch, G., Investigation of vibrations in bending of rotating turbine blades in nonrigid supports (in German), Jahrbuch Wissenschaft. Gesellsch. Luftfahrt, 1958, 174–183.

A theoretical analysis of free bending vibrations of an untwisted turbine blade is given, in which the blade is considered as a cantilever beam supported by an elastic support with different stiffnesses in two perpendicular directions. The blade chord is allowed to be inclined with respect to the axis of rotation. The rotational inertia of the beam is included. Solutions are given by Galerkin's and by Rosard's methods; use of digital computers is discussed. Experimental techniques are described.

B. A. Boley, USA

6289. Ovrutskaya, N. B., and Kheifets, M. Z., On the axial stability of the rotor of a turbomachine when a load-shedding device is in operation (in Russian), *Trudi Leningr. Metallich. Z-da* no. 5, 345–350, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2318.

In machines fitted with blades, loading-shedding devices are frequently employed; these balance the axial force acting on the rotor and ensure the necessary axial disposition of the rotor when support of a bearing is not available. When the rotor shifts from the position of equilibrium there is a change in the section of the space for the flow of the working liquid in order that, after elimination of the cause of the displacement, a place would be found for the "returning force." Equations are derived for the motion of the rotor on the assumption that all the parameters of the system are centered, while the axial force of the rotor is constant after a certain moment of time. Analysis of the derived equations indicates that the introduction of the reverse linkage leads to rather complicated conditions for stability, while absence of any reverse linkage appears to be an adequate condition of stability. In the appliance with complete load-shedding the existence of a negative reverse link is essential, but is insufficient for the axial stability of the rotor and an additional requirement is a sufficiently small volume of the working chamber of the load-shedding piston. A numerical example is given for the device for complete load-shedding. V. Kh. Abiants

Courtesy Referativnyi Zhurnal, USSR

6290. Lur'e, A. I., and Osorin, V. I., The adoption of Chebyshev's extremal polynome for the synthesis of the mechanical system of a vibro-transmitter working in conditions of slowly-changing overloads (in Russian), *Trudi Leningr. Politekh. In-ta* no. 192, 109–127, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2276.

Evidence is furnished that a vibro-transmitter with one step of freedom is unsuitable in principle for the recording of vibrations which are accompanied by secondary vibrations of markedly smaller frequency and greater amplitude. For such a case a transmitter can be used which is described in the paper; this consists of a system of two masses, joined together and with the body of the vibro-transmitter by means of springs and damping components. The body is firmly braced to the detail whose vibrations are being measured; as the outgoing signal, the relative displacement of the mass of the transmitter is used. The transmitter shows optimum results when the relation of the amplitude of the outgoing signal to the amplitude of the harmonic excitation, with the motion steady ("the transmitter's scale") in the given frequency range $0 \leqslant \omega \leqslant \omega_1$, is sufficiently close to zero, while with $\omega_1 < \omega < \infty$ the least divergence is shown from the constant value corresponding to $\omega \to \infty$.

Having made the calculation for the amplitude-frequency characteristic of the transmitter the authors proceed to merge the problem of the determination of the optimum parameters of the apparatus, as conceived and described in the paper, with the delection of a polynome of the fourth degree which, in the range $0 \leqslant x \leqslant 1$, shows the least divergence from zero, while with $1 < x < \infty$ it grows rapidly with increase of x. The corresponding Chebyshev polynome is selected to represent such a polynome. The finding of the parameters of the system by means of direct comparison of the coefficients of the two polynomes led to a complex system of algebraical equations. This obstacle was overcome by transferring from Chebyshev's polynome to the corresponding characteristic polynome. A comparison of the coefficients of this last polynome with the coefficients of the characteristic polynome, expressed through the initial parameters of the transmitter, leads to a system of four comparatively simple algebraical equations with six unknown dimensionless parameters.

Formulas are obtained for the calculation of the scale of the transmitter and the precision of the apparatus in the working range of frequencies; the question is gone into in detail regarding the extent of filtration of the low-frequency overloads. A concrete example is given of the procedure adopted for the calculation of the parameters of the sensitive element of the transmitter with due regard to the assigned technological requirements and also of the effectiveness of the proposed method of selection of the parameters. The transmitter, as calculated, retains constancy of scale with a precision of ±5%, over the whole range of frequencies of the vibrations being measured ($\nu \geqslant 10$ Hertz). The transmitter shows no reaction to constant readings; acceleration with an amplitude of 30g and frequencies of up to 1.75 Hertz produces a reaction not exceeding 5% on the scale. It was demonstrated that a supplementary investigation of the phase characteristics of the apparatus indicated appreciable dependence of the phase recording of the vibrations on their frequency. Consequently, when polyharmonic vibrations are being measured it is essential to have a harmonic analysis of the recording or to insert a harmonic analyzer in the signal's registering circuit.

I. I. Blekhman

Courtesy Referativnyi Zhurnal, USSR

6291. Lukomskii, S. I., and Vartanyan, R. A., An electromagnetic vibrator of percussion action viewed as a vibrating system with a delimitor (arrestor) (in Russian), Investigations of the kinematics and dynamics of the structure of construction machines (Issledovanie kinematiki i dinamiki construktsii stroit. mashin), Moscow, Mashgiz, 1957, 29–34; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2279.

An electromagnetic vibrator of percussion action is demonstrated in the form of a system consisting of two masses joined together by a spring. Forces of electromagnetic interaction are in being between the masses, which change in accordance with the harmonic principle with the frequency of the industrial current. Vibrators of the type being investigated normally correspond to correlations between the parameters of the system which result in periodic motion with a single impact of the masses for each period of the disturbing force. Two cases of the work of the vibrator are studied. In the first case one of the masses is joined to the immovable foundation by means of a very ductile spring acting as the elastic member compared with the initially used spring. In the second case this mass is firmly attached to some body (the working organ) of undelimited large mass. The latter variant, actually, is a special case of the first. The problem of finding the periodic motions of the type referred to, after making the customary assumptions, merges with the problem investigated by M. G. Rusakov and A. A. Kharkevich [*Zh. Tekh. Fiziki* **12**, 11-12, 1942]. The experimentally determined values of the amplitudes of the vibrations and the velocities of one of the masses up to impact agree well with those calculated in accordance with the theoretical formulas.

I. I. Blekhman
Courtesy Referativnyi Zhurnal, USSR

Wave Motion and Impact in Solids

(See also Revs. 6144, 6169, 6172, 6355, 6387, 6405, 6658)

6292. Mindlin, R. D., and McNiven, H. D., Axially symmetric waves in elastic rods, *ASME Trans.* **82** E (*J. Appl. Mech.*), 1, 145-151, Mar. 1960.

Interesting paper uses variational method to investigate axially symmetric modes of vibration of elastic rods, differing from authors' previous work in that it includes effect of an axial shear mode with displacement parallel to rod axis and one circular mode; compensating factors are introduced to diminish error caused by omission of higher-order terms in the Jacobi polynomial expansions that form the basis of the method. Although reviewer finds it difficult to evaluate accuracy of approximation, resulting modes are pictorially intriguing. C. E. Pearson, USA

6293. Misicu, M., Dynamic torsion of elastic bars of constant cross-section (in Roumanian), *Studii Si Certari Mecan. Apl.* **10**, 4, 1171-1208, 1959.

Author gives a general formulation of the problem of cylindrical elastic bars subjected to dynamic torsion moments; corrections are determined, with respect to the static case, proportional to $(\omega L/c_t)^n (n = 2,4, \ldots.)$, where ω is a fixed frequency of a monochromatic vibration component, L is the transverse maximum dimension and c_t is the propagation velocity of transverse waves. Examination is confined to the determination of transverse waves; assumption is made that, on the end surfaces, the boundary conditions correspond to the reflection of the longitudinal waves on these surfaces. Complex variable functions are used for calculating the stress and the rigidity expressions. The case of bars with multi-connected cross section is also considered. Author finds an analogy of the Prandtl stress functions and generalizes Bredt's theorem for tangential stress circulation.

An extension of Greenhill's analogy to viscous fluids and of Prandtl's membrane analogy are also included.

P. P. Teodorescu, Roumania

6294. Misicu, M., Dynamic problem of torsion (in Roumanian), *Studii Si Cercetari de Mecan. Apl.* **11**, 1, 153-171, 1960.

Paper presents applications of the problems discussed by the author in a previous paper [see preceding review] to a reinforced bar of circular cross section, to a bar of epitrochoidal cross section and to a bar whose cross section is bounded by Booth's lemniscate. P. P. Teodorescu, Roumania

6295. Mindlin, R. D., and Fox, E. A., Vibrations and waves in elastic bars of rectangular cross section, *ASME Trans.* **82** E (*J. Appl. Mech.*), 1, 152-158, Mar. 1960.

Authors combine classical solutions of elastic wave equation for plates so as to obtain exact solution of wave equation for rectangular bars of infinite length, for certain frequencies and depth-width ratios. Paper discusses coupled dilatational and equivoluminal modes and so is valuable extension of early work by Lamé on purely equivoluminal modes. C. E. Pearson, USA

6296. Brekhovskikh, L. M., Propagation of surface Rayleigh waves along the uneven boundary of an elastic body, *Soviet Phys.-Acoustics* **5**, 3, 288-295, Feb. 1960 (Translation of *Akust. Zh., USSR* **5**, 3, 282-289, July/Sept. 1959 by Amer. Inst. Phys., New York, N.Y.)

Paper considers scattering of Rayleigh waves, caused by surface roughness, using method of successive approximations. Results show that dispersion due to surface roughness can be great, even when due to fine-grained nonuniformity, if the wavelengths thereof are close to certain definite values. Two-dimensional unevenness is expressed as a double Fourier series, subject to the limitations that the depth of unevenness be small compared with the length of the Rayleigh wave and that slopes of the surface relative to the medium surface also be small. Author gives example of cylindrical sinusoidal unevenness, graphically presenting results for aluminum, steel and soil media.

E. Rosenblueth, Mexico

6297. Sluridin, G. A., and Gvozdev, A. A., The boundary conditions for rapid changes of discontinuous solutions for the dynamic equations in the theory of elasticity (in Russian), *Izv. Akad. Nauk SSSR Ser. Geofiz.* no. 2, 145-156, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6671.

An investigation is carried out on the solution of equations for the dynamics of a heterogeneous elastic body undergoing disruptions on the surface $t - \Psi = 0$. The coincidence of results is noted with

$$u = u_1 V(t - \Psi) + u_2 V(\Psi - t) \qquad [1]$$

(V is Khevisaid's function) and with

$$u = A \exp [i\omega(t - \Psi)] \qquad [2]$$

(ω is a sufficiently large frequency). The asymptotic conditions are recorded for the longitudinal and transverse waves; these conditions are then converted to a convenient form with the aid of an investigation of an infinitely fine radiation tube. By substitution of [1] the boundary conditions were obtained for the wave reflected from the boundary of the semispace. Using the same method, the boundary conditions for a leading wave were derived. It should be noted that when recording the boundary conditions for a reflected wave it is essential to consider two waves, while for recording the boundary conditions for a reflected and a refracted wave, four waves have to be taken into account. I. S. Arzhanykh

Courtesy Referativnyi Zhurnal, USSR

6298. Baltrukonis, J. H., and Gottenberg, W. G., Transverse wave propagation in a solid, elastic mass contained by an infinitely long, rigid, circular-cylindrical tank, Proc. Fourth Midwest. Conf. Solid Mech., Austin, Texas, Sept. 1959; Austin, Tex., Univ. Press, 1959, 396-414.

The solution of the problem is obtained in terms of three displacement potential functions. The dispersion equations are determined for the three cases defined by the relative magnitudes of the circular frequency, the wave number and the dilatational and

shear wave velocities. Solutions of the dispersion equations for values of Poisson's ratio of 0.3 and 0.5 and dispersion curves for the first four modes for both thickness-shear and transverse vibrations are presented. E. Saleme, Argentina

6299. Gordeev, N. I., Calculations for the strength of machine parts under cyclic stresses (in Russian), *Trudi Leningrad Tekhnol. In-ta Kholodil'n. Prom-sti* **15**, 156–172, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6915.

This paper contains a brief but at the same time sufficiently precise and fully up to date description of methods for the design of machine parts which are loaded periodically. These methods are based on the method of a generalized function and proceed from it to special cases which at the same time appear to be themselves generalized. From author's summary

Courtesy Referativnyi Zhurnal, USSR

6300. Datta, S., A note on stress waves in visco-elastic rod, *J. Technol., Calcutta* **4**, 1, 41–46, June 1959.

Author treats problems of longitudinal impact in semi-infinite and finite rods of Maxwellian material. Method employs Laplace transform (with simple inversions), solutions being obtained for both end velocity step and rectangular pulse inputs. In the case of the semi-infinite rod, and pulse input, author shows stress at a station experiences an exponential decay and a change in sign as soon as the pulse time has elapsed. Lee and Kanter [*J. Appl. Phys.* **24**, 1953; AMR **7**(1954), Rev. 724] treated the step input basic to this case earlier. J. Miklowitz, USA

6301. Kukudzhanov, V. N., Stress waves in elasto-visco-plastic and visco-plastic media (in Russian), Avotref. Diss. Kand. Fiz.-Matem. Nauk, Moscow, Fiz.-Tekhn. In-ta, Moscow, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6694.

6302. Bagdoev, A. G., Some transient problems on the propagation of waves in a semi-space (in Russian), Avtoref. Diss. Kand. Fiz.-Matem. Nauk, MGU, Moscow, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6674.

6303. Andriankin, E. I., and Koryavov, V. P., Shock waves in a variably compacting plastic medium, *Soviet Phys.-Doklady* **4**, 5, 966–969, Mar./Apr. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **128**, 2, 257–260, Sept./Oct. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Paper examines the problem of a spherically symmetric explosion in a medium which, at the wave front, has a compaction depending on the amplitude of the pressure. The medium is plastic in back of the wave front (satisfies the Prandtl plasticity condition) and is incompressible (retains its density at each point). From authors' summary

6304. Symonds, P. S., and Neal, B. G., Travelling loads on rigid-plastic beams, *Proc. Amer. Soc. Civ. Engrs.* **86**, EM 1 (*J. Engng. Mech. Div.*), 79–89, Jan. 1960.

The problem is considered of a rigid plastic beam crossed by a load which is too large for the beam to support statically. The main question considered is whether or not there is a critical magnitude of the traveling load, above which the crossing cannot be made at any finite speed without causing total plastic collapse. Such a critical magnitude was predicted in an analysis made by E. W. Parkes. The present study indicates that a critical load will not be obtained if the mass of the beam is taken into account. From authors' summary

6305. Kaliski, S., and Osiecki, J., Unloading wave for a body with rigid unloading characteristic (in English), Proceedings of Vibration Problems, no. 1, 49–66, 1959.

The problem of the unloading wave is solved for a semi-infinite bar of a material with the following stress-strain relation: for increasing stress the relation is linear with a constant modulus; for decreasing stress the modulus is infinite (no recovery of deformation). An increasing-decreasing stress pulse at the end of the bar is applied; special attention is given to the stress pulse which suddenly increases and decreases monotonically. Due to the assumed simplifications, the solution is obtained in a relatively simple form. M. P. Bieniek, USA

6306. Dorofeev, V. M., and Levin, V. Ya., A device for the recording of the impulse of a pulsating current (impulsometer) (in Russian), *Trudi Kubyshevsk. Aviats. In-ta* no. 6, 49–55, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6605.

The plans of the device are studied and a description is given of the construction of the apparatus—an impulsometer for the recording of momentary values of reactive pull. The principle of the apparatus consists of the active method used for the measurement of the reactive pull. The trap (a plate), suspended cantilever-fashion on the bar and rigidly fixed in an immovable socket, vibrates because of the action of the pressure of a nonstationary flow impinging on it from a nozzle. The bar on which the trap is suspended is subjected to deformation by bending. Two identical wire strain gages (tensometers) are glued on the bar, on parallel surfaces, at equal distances from the axis of the trap, which form the two arms of a Wheatstone bridge. The presence of two gages operating in identical conditions, but under different forms of deformation (tension and compression) guarantees adequate thermal compensation,that is to say, the influence of the temperature of the surrounding medium and of the bar is neutralized in regard to any effect on the readings of the apparatus. The bridge is supplied with current with a frequency of 4000 Herz obtained from a sonic generator. The changes of current by time taken off the bridge are transmitted through a special amplifier onto an oscillograph and there recorded. The apparatus enables measurements to be made of the impulses of transient gas currents, the recording of the pull by time and the investigation of the pulsating flow when the latter is emerging from a nozzle. If an investigation of the working pulsating chamber with the aid of the apparatus is concerned it is quite possible to obtain supplementary relationships which help to simplify a scrutiny of the working process of the chamber. The main designs for the apparatus are given and also an example of an oscillogram showing the changes in position of the trap.

I. A. Lukashevich

Courtesy Referativnyi Zhurnal, USSR

Soil Mechanics: Fundamental

(See Rev. 6691)

Soil Mechanics: Applied

(See Revs. 6388, 6691)

Processing of Metals and Other Materials

(See also Revs. 6236, 6271)

6307. Dovnorovich, V. I., Regarding a single, axially symmetrical problem on the contact of a hard die with the surface of rotation of an elastic semi-space (in Russian), *Uch. Zap.*

Belorussk. In-ta Inzh. Zh.-d Transp. no. 2, 19–30, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6620.

By using the method of resolution in series by spherical functions a solution is found for the axially symmetrical problem of the pressure exercised by a hard die on an elastic semi-space, to meet the case where the surface of the die is assigned by the equation $z = A\rho^{2n}$ (n being a natural number). The numerical coefficients in the solution are found by calculation of some small series of recurring correlations, containing different intermediate magnitudes. This procedure, however, is unnecessary, as the solution of the problem has previously been solved and the coefficients are clearly expressed through an indicator $2n$. The solution is to be found in I. Ya. Shtaerman's book "Contact problem in the theory of elasticity" [Moscow-Leningrad, Gostekhizdat, 1949, pp. 184–187). [see I. Ya. Shtaerman, *Dokladi Akad. Nauk SSSR (N.S.)* **25,** no. 5, 1939] N. A. Rostovtsev

Courtesy Referativnyi Zhurnal, USSR

6308. Wood, W. W., High-energy forming methods—a critical review, *Tool Engr.* **44,** 7 (Suppliers Directory Issue), 93–105, June 1960.

High-energy forming methods include high-mass systems, high-velocity systems and high-temperature systems. The author shows how each method works and suggests the materials and types of parts suitable for each method.

From author's summary

6309. Belousov, A. I., The role of thermophysical properties of materials in the deformation process (in Russian), *Inzhenerno-Fizicheskii Zh.* **3,** 4, 90–94, Apr. 1960.

Results are given of an investigation of the effect of heat conductivity and critical heat content on the tangential resolution of cutting forces, heat resistance, and wear resistance of hard alloys. Analytical expressions are derived for the coefficient of relative wear resistance and hardness of the metalloceramic materials. From author's summary

Fracture (Including Fatigue)

(See also Revs. 6224, 6329, 6340, 6343, 6370)

6310. Shteinberg, M. M., Sokolov, E. N., and Varaksina, M. N., The propensity of metals to brittle fracture (in Russian), *Trudi Ural'skogo Politekhn. In-ta* no. 68, 54–58, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6993.

This is a paper devoted to the substantiation of the efficiency of the method of obtaining the quantitative relation of the resistance to fracture (S) to the preliminary plastic deformation. The magnitude of S is determined for a number of materials (silicon ferrite, molybdenum ferrite, steel mark 30 × GSA worked up to a brittle state) during the static fracture of smooth test samples at $-196°$ after plastic deformations of various magnitudes, obtained at $+20°$. In all cases S increases to different degrees in relation to the composition and the initial structure. The smallest increase of S (2 kg/mm² to 1% of reduction in the transverse section) was observed in the case of silicon steel and the largest (4–5 kg/mm² to 1% of reduction in the transverse section) for steel mark 30 × GSA. In the latter case, after 11% deformation and subsequent fracture at $-196°$ the destruction along the grain boundaries was replaced by destruction in the grain, which the authors attribute to the natural annealing brittleness. E. M. Shevandin

Courtesy Referativnyi Zhurnal, USSR

6311. Mayer, E., The surface stability to thermal stress cracking of materials for unbalanced mechanical and circumferential seals *(in German), ZVDI* **102,** 18, 728–732, June 1960.

A "thermal stress crack stability factor" for mechanical seals can be arrived at through simple calculations. By selecting contact materials having a stability factor not lower than a minimum value, failure of seals as a result of thermal stress cracks can be largely avoided. From author's summary

6312. Lubahn, J. D., Correlation of tests using the congruency principles, *Proc. Amer. Soc. Test. Mat.* **58,** 678–684, 1958.

The congruency principle states that fracture will occur at the same nominal stress in two different objects if the nominal stress gradient at the notch root is the same in both objects and if the notches are congruent in the two objects. This concept was applied to the results of notch bend tests and notched disk bursting tests. The strengths in the two types of tests could be correlated within about 8%.

The congruency principle and another fracture concept, the Griffith-Irwin theory, were also compared as to accuracy of correlation and breadth of applicability.

From author's summary by F. Garofalo, USA

6313. Kvapil, R., The theory of mine collapses (in Russian), *Ugol'* no. 5, 46–48, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6734.

Information is furnished on the causes of sudden breakdowns of rock in mine workings, the so-called mine "impacts." It is shown that these mine "impacts" are produced in the majority of cases as the result of the diminishing capacity of the rock stratum to absorb the potential energy of deformation due to increased loading of the rock. Yu. P. Zheltov

Courtesy Referativnyi Zhurnal, USSR

6314. Frid, A. M., Influence of the surface condition of an eroding body on the formation of cavitation bubbles (in Russian), *Trudi Khar'kovsk. Aviats. In-ta* no. 17, 143–146. 1957; *Ref. Zh. Mekh.* no. 2, 1959, Rev. 1465.

Author assumes that the role of the air when cavitation makes its appearance can be described as a filler of the microscopic cracks and hollows on the surface of the body being washed over, the body being assumed to be nonwettable; with a drop in pressure these air bubbles begin to grow because of diffusion of the surrounding air or formation of vapor. A series of experiments was carried out to substantiate this hypothesis. Two graduated cylinders filled with water were taken; one of these was subjected to a pressure of 700 to 1000 atm for 5 to 10 minutes. Then the cylinders were heated. The water in the cylinder subjected to pressure boiled at 118°, the water in the other, untreated, boiled at 100°. In the second series of experiments the water from the pressure-treated cylinder was transferred to an ordinary cylinder, while the emptied cylinder was refilled with ordinary water. The cylinders were heated at atmospheric pressure. The ordinary water in the pressure-treated cylinder boiled at 118°, while the water which had been pressure treated and transferred to an ordinary cylinder boiled at a 100°. On the basis of the experimental data the deduction is put forward that meticulous treatment of the surface of the body and also its impregnation at high pressure with some substance having small porosity impedes the genesis of cavitation.

B. S. Kogarko

Courtesy Referativnyi Zhurnal, USSR

6315. Gladshtein, L. I., Sokolovskii, P. I., and Rudchenko, A. V., An investigation on the mechanical ageing of steel by the method of superposition of actual graphs of the tension (in Russian), *Zavod. Lab.* **24,** 10, 1236–1239, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6985.

By means of superposition of actual graphs of tension of riveted (up to 10%) and aged and unriveted test samples, proposals are advanced which enable conclusions to be reached regarding the influence of ageing on the toughening effect produced by riveting. The authors assert that for a steel of type MSt3 with artificially in-

duced ageing the toughening effect extends over the whole tension zone, while with natural ageing this disappears rapidly.

V. S. Namestnikov

Courtesy Referativnyi Zhurnal, USSR

6316. Hyler, W. S., Tarasov, L. P., and Favor, R. J., Distribution of fatigue failures in flat hardened steel test bars, *Proc. Amer. Soc. Test. Mat.* **58,** 540–551, 1958.

Authors determined location of fatigue nuclei in flat hardened steel bars tested in cantilever bending. Bars had been surfaced by various grinding and abrasive tumbling methods. Some grinding and all tumbling conditions resulted in improved fatigue strength, with fractures then nucleating predominantly subsurface, generally at inclusions. Kind, size, and depth of inclusions were determined for bars with predominantly subsurface initiation of failure. Inclusions were 55% complex oxide type, remainder titanium cyanonitride. No positive correlations were found between mean diameter of inclusions and either relative stress level or fatigue lifetime, or between depth of inclusions and either relative stress level or fatigue lifetime, or between depth of inclusions and either relative stress level or fatigue lifetime. I. Cornet, USA

6317. Vitovec, F. H., Effect of static prestrain on the fatigue properties under uniformly increasing stress amplitude, *Proc. Amer. Soc. Test. Mat.* **58,** 552–569, 1958.

A study was conducted on the effect of short-time and creep prestrain on the fatigue properties of unnotched and notched specimens of alloys 7075-T6 and 2024-T4 extruded, 16-25-6 hot-cold-worked, and S-816 solution treated and aged. Fatigue properties were determined using a progressive load fatigue test with a single loading rate of 0.01 psi per cycle. The unnotched and notched specimens were prestrained at test temperature and then immediately subjected to a fatigue test at the same temperature and in the same testing machine. The test temperatures for the short-time prestrain and creep prestrain, respectively, were 75 and 300°F for 7075-T6, 300 and 500°F for 2024-T4, 1200°F for 16-25-6, and 1500°F for S-816. Short-time prestrains up to 10% were applied. Stresses used for the creep prestrains corresponded to those which produced rupture in 100 to 200 hr at test temperature.

The main factors influencing the fatigue failure stress were residual stresses, metallurgical reactions, and cracks produced by tensile prestraining. Strain hardening and relaxation of residual stress appeared to be of secondary importance for the materials and the range of testing variables investigated.

From author's summary by I. Cornet, USA

6318. Mann, J. Y., The fatigue notch sensitivity of annealed copper, Aero. Res. Lab., Melbourne, Austral., Rep. SM 272, 18 pp., Sept. 1959.

Rotating cantilever fatigue tests are reported on notched annealed copper specimens having K_t values of 1.5, 2.5, 4.0, 5.5 and > 10.

Contrary to the commonly accepted view, this investigation has shown that annealed copper is not particularly fatigue notch sensitive—the maximum K_f value found being less than 1.5.

There is also evidence that this material is unlikely to exhibit the phenomenon of "non-propagating cracks."

From author's summary

6319. Muratov, L. V., An analytical expression for the fatigue curve (in Russian), *Sb. Nauchn. Trudi Kuibyshevsk. Industr. In-ta. Mekhanika* no. 7, 221–232, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 7011.

An equation for the curve for fatigue is put forward

$$N = k\,\frac{a-\sigma}{\sigma-\sigma_r}$$

where N is the number of cycles up to fracture, σ the maximum stress producing fracture in N cycles, σ_r the limit of fatigue during symmetrical and asymmetrical cycles, a a magnitude with dimensions of σ and having the characteristic of the material, k is a coefficient having the dimensions of N. The possibility is indicated of calculating the limit of fatigue at any point in the curve of different steels in different test conditions by means of the corresponding condition of the cyclic testing and of two arbitrary experimentally obtained points on the fatigue curve; to do so, the magnitude a is assumed to be equal to the limit of yield of the material. Comparison of the experimental values for σ_{-1}, taken from data cited in literature, with the calculation results, as worked out by the author's method, gave adequate convergence.

G. A. Tulyakov

Courtesy Referativnyi Zhurnal, USSR

6320. Glenny, E., and Taylor, T. A., A study of the thermal-fatigue behaviour of metals: The effect of test conditions on nickel-base high-temperature alloys, *J. Inst. Metals* **88,** 449–461, 1959/1960.

An attempt has been made to identify the significant factors governing the thermal-fatigue behavior of nickel-base high-temperature alloys, mainly by using a laboratory technique with hot and cold fluidized beds as the heating and cooling media. A succession of heating shocks is generally more damaging than a succession of cooling shocks between the same temperature limits. The duration of the heating shock and the upper temperature of the cycle are dominant factors. The thermal-fatigue cracks are initiated at the surface and are intercrystalline in origin and propagation. Surface oxidation, which is intergranular in nature for nickel-base alloys, has a significant effect on thermal-fatigue life.

From authors' summary

Experimental Stress Analysis

(See also Revs. 6208, 6214, 6288, 6328, 6373)

6321. Walter, H., The application of lines of constant shear stress for the determination of direct stresses in two-dimensional photoelasticity (in German), *Forsch. Geb. Ing.-Wes.* **26,** 1, 29–34, 1960.

σ_x and σ_y are to be separated along given y line. Superposition on transparent paper of isoclinics $\varphi(\varphi = 0$ for y direction) and isochromatics n leads to $\tau_{xy} = (n/2) \sin 2\varphi$, and lines of constant τ_{xy} are drawn (use different colours!). Simple nomogram based on Mohr's circle speeds work and also gives $\sigma_x - \sigma_y$. From τ_{xy} lines the slope of τ_{xy} in x direction is extrapolated in sections across y line. Since $(\sigma_y)_y = -(\tau_{xy})_x$, the curve of σ_y along y line can be built up from slope and known boundary value. σ_x follows point-by-point from $\sigma_x - \sigma_y$ obtained from τ_{xy} and isochromatics with nomogram.

In second method, believed especially useful near free boundary, plot τ_{xy} from boundary up to given y line along x lines small distances Δy apart, using τ_{xy} contours from above; planimeter areas of plots to find shear forces T_1, T_2 on neighboring x lines giving $(\sigma_x)_{mean} = (T_1 - T_2)/\Delta y$ on y line at end of strip formed by neighboring x lines.

Author gives detailed practical example of these methods known in principle and states that not every method is suitable for every application. M. L. Meyer, England

6322. Hetenyi, M., Photoelasticity and photoplasticity, First Symposium on Naval Structural Mechanics, Stanford Univ., Calif., Aug. 11–14, 1958, 38 pp.

This valuable paper provides a critical review of the present state of development of stress analysis by means of polarized

light. Included are those contributions which effectively advanced techniques, materials and analysis, with little space devoted to routine applications. The broad coverage of this review can be gleaned from the contents: birefringent materials; dynamic photoelasticity including streak photography, intermittent illumination, image-dissector camera; photoplasticity based on crystalline materials, synthetic resins with ductile behavior, birefringent coatings; interferometric methods; measurement of fractional fringe orders; laminated models; explorations using holes and cavities; photothermoelasticity; analysis of three-dimensional stress models; other miscellaneous topics.

The bibliography consists of 138 selected references, almost all published within the period 1950–1958.

W. Shelson, Canada

6323. Seregin, I. N., Observations of plastic deformations of concrete with the aid of induction strain gauges (in Russian), *Avtomob. Dorogi* no. 10, 12–13, 1958; *Ref. Zh. Mekh.* no. 6, 1959; Rev. 7117.

A description is given of the construction of an induction strain gage which enables measurements to be made of the deformation of the structural grain of concrete, produced by the shrinkage and creep of the material. The gage consists of a cylindrical body in which two coils are situated. Within the coils is a movable core the end of which rests on a steel rod specially fixed in the concrete. The working principle of the gage is based on the fact that when the core moves inside the coils (due to deformations of the concrete by shrinkage and creep) their induction resistance changes; it is possible to judge from these changes the magnitude of the transposition of the core, that is to say, of the deformations. The gage readings are taken by means of a special induction measurer of the deformations. The gage is capable of measuring deformations of from 5 to 20 mm with an accuracy of 0.02–0.05 mm. With the aid of the described gages observations are being carried on in the Soviet institutes concerned with the loss of tension in the reinforcement of concrete beams in automobile road bridges.

I. I. Ulitskii

Courtesy Referativnyi Zhurnal, USSR

6324. Prigorovskii, N. I., Experimental determination of deformations, stresses and forces in machines (in Russian), Theoretical bases in the construction of machines, Moscow, Mashgiz. 1957, 182–209; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6912.

The development and present state of experimental methods used in the U.S.S.R. for the determination of deformations, stresses and forces in machine parts and constructions is reviewed. Most attention is devoted to studies dealing with investigations regarding the actual work done by the constructions and parts, with observations on the distribution of stresses, the residual and temperature stresses, the apparatus used for strain-gage readings, and with methods of analogy used.

A. D. Pospelov

Courtesy Referativnyi Zhurnal, USSR

6325. Forrester, C. L., and Gee, S. W., Strain measurements on an aircraft propeller during ground running, Aero. Res. Lab., Melbourne, Austral., Note SM 261, 15 pp., Oct. 1959.

This note describes the electric resistance strain gage instrumentation used to measure vibratory strain on an aircraft propeller during ground tests of a new engine propeller combination.

From authors' summary

6326. Nosov, S. S., The static and dynamic strength of the components of a connecting (piston) rod (in Russian), Internal combustion motors (Dvigateli Vnutrennegò Sgoraniya (TsNIDI no. 31), Moscow, 1958, 69–108; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6940.

A description is given of a strain-gage device for measuring stresses in a connecting rod of a working motor; a comparison is made of the experiments with existing methods of calculation and a procedure is proposed which agrees with the tests for the determination of the static and dynamic stresses.

From author's summary

Courtesy Referativnyi Zhurnal, USSR

Material Test Techniques

(See also Revs. 6270, 6312, 6320, 6342, 6343, 6384, 6509, 6660)

6327. Basri, S. A., A method for the dynamic determination of the elastic, dielectric, and piezoelectric constants of quartz, Nat. Bur. Stands. Mono. 9, 22 pp., June 1960.

6328. Kalbfleisch, J., Dynamic calibration of a fatigue testing machine (in French), *Rev. Métall.* **55,** 2, 166–178, Feb. 1958.

Author discusses such methods and procedures for "resonance" and "hydraulic pulsator" types of fatigue testing machines. Using a strain-gage force transducer and circuitry developed by M. H. Roberts, author obtained extensive data on a 35-ton hydraulic pulsator. After making corrections for inertia effects, the resulting accuracy is proven to be well within the 2% requirement established in 1955 by the Institut International de la Soudure.

Reviewer notes omission of extensively used "inertia-force-compensated" fatigue machines, also subject to dynamic calibration, as well as "chopper" circuitry used with strain-gage transducers to perform similar function as Roberts circuits.

A. Yorgiadis, USA

6329. Rowland, R. M., and Cronin, J. L., Dynamic effects in fatigue testing, *J. Aero/Space Sci.* **27,** 5, p. 390 (Readers' Forum), May 1960.

6330. Ermakov, S. S., A method of testing steel for abrasive wear under varying shock loads, *Indust. Lab.* **25,** 3, 353–355, Apr. 1960. (Translation of *Zavod. Lab.. SSSR* **25,** 3, 337–338, Mar. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

6331. Shams El Din, A. M., and Ibl, N., Studies on the microhardness of metals. The dependence of the indentation hardness on the load (in English), *Schweiz. Arch.* **26,** 6, 246–250, June 1960.

6332. Hirsch, P. B., Mitchell, T. E., and Thornton, P. R., Effect of strain-rate on the flow-stress of f.c.c. metals, AFOSR TN 60–152 (Univ. Cambridge, England, Cavendish Lab., Dept. Phys.), 41 pp., Oct. 1959.

In attempting to distinguish between the dislocation pile up and dislocation forest models of work-hardening, authors extend measurements of rate sensitivity to polycrystalline brass and irradiated copper. Basic theory is that of Thornton and Hirsch [*Phil. Mag.* **3**, p. 738, 1958] correlating $\Delta\sigma/\sigma$ with strain, stress, and temperature which determines variations in density of short range and long range barriers to dislocation motion. Report does not cover completed contract. Results are somewhat ambiguous. Authors suggest more information on dislocation structure would be valuable. They will carry out transmission electron microscopy. Results show use of rate effect and existence of yield points and "jerky flow" over extended temperature range.

R. W. Guard, USA

6333. Manning, C. R., Jr., and Heimerl, G. J., An evaluation of some current practices for short-time elevated-temperature tensile tests of metals, NASA TN D–420, 44 pp., Sept. 1960.

An evaluation was made of the effect of the testing practice on the short-time elevated-temperature tensile properties of 2024-T3

aluminum-alloy, HM21A-T8 and HK31A-H24 magnesium-alloy, and 12 MoV stainless-steel sheet. Constant strain-rate, constant head-speed, and dual strain-rate tests were made. The need for uniform testing practices is shown. From authors' summary

6334. Koppelmann, J., Frielinghaus, R., and Meyer, F. J., The generation of very short ultrasonic impulses with piezoelectric vibrators (in German), *Acustica* **8**, 4, 181–187, 1958.

Nondestructive material testing by ultrasonic echo method applied on specimen of some few wavelengths thickness must consequently be done with short pulses. Theoretical conception about generating suitable shapes of pulses leads to the deadbeat transducer. Piezoelectric types and electronic circuits are discussed and experimentally demonstrated by some instructive oscillograms.

P. P. Heusinger, Germany

6335. Chernyi, Yu. F., An approximate method of assessing the strained condition of a metal, *Indust. Lab.* **25**, 3, 351–353, Apr. 1960. (Translation of *Zavod. Lab., SSSR* **25**, 3, 335–336, Mar. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

6336. Popov, N. I., A method of testing prestressed coil springs for relaxation, *Indust. Lab.* **25**, 3, 348–351, Apr. 1960. (Translation of *Zavod. Lab., SSSR* **25**, 3, 332–334, Mar. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Properties of Engineering Materials

(See also Revs. 6214, 6222, 6240, 6253, 6312, 6315, 6332, 6370, 6371, 6509)

Book—6337. Lucas, G., and Pollock, J. F., Gas turbine materials, New York, MacMillan Company, 1958, xi + 163 pp. $5.75.

This book presents an over-all view of the several specialized fields of knowledge required in the design and manufacture of gas turbines. In the preface, the authors state, "It is hoped that it will fill to some extent, the gap between the metallurgical textbooks and those works concerned with the technology of design and performance of gas turbines."

Chapter titles indicate the scope of topics considered. Chapter 1, An outline of the development of high temperature materials. Chapter 2, Section 1, The influence of performance requirements and upriding conditions of the gas turbine. Chapter 2, Section 2, The influence of the fuel on operating conditions in the gas turbine. Chapter 3, Metals under stress and temperature. Chapter 4, The application of high temperature materials. Chapter 5, Special temperature materials. Chapter 6, The manufacture of gas turbine components.

The material is presented at an easily readable elementary level, using simple concepts and theory to develop or illustrate important considerations of the thermodynamic process, the creep behavior of metals under stress at elevated temperature, and oxidation rates.

The authors have surveyed the field and put important aspects of the problem in proper perspective. Where topics are controversial, the authors have noted this.

The major contribution of this 150-page volume consists in presenting a simple, readable discussion of the many interrelated problems with which a designer must cope in arriving at a suitable design compromise for a particular application. The book would be particularly suitable for a young engineer entering this field or for an individual with limited technical background.

H. T. Corten, USA

6338. Jones, E. S., Rocket materials: a challenge for metallurgists, *J. Metals* **12**, 6, 449–454, June 1960.

6339. Dugdale, D. S., Yielding of steel sheets containing slits, *J. Mech. Phys. Solids* **8**, 2, 100–104, May 1960.

Yielding at the end of a slit in a sheet is investigated, and a relation is obtained between extent of plastic yielding and external load applied. To verify this relation, panels containing internal and edge slits were loaded in tension, and lengths of plastic zones were measured. From author's summary

6340. Kori, T., and Sasaki, S., A study on the fatigue strength of carbon steels under multiple repeated stresses, *Bull. JSME* **3**, 10, 281–286, May 1960.

6341. Lihl, F., and Stickler, R., Temperature dependence of iron-rich iron-aluminum alloys (in German), *Arch. Eisenbuttenw.* **31**, 1, 47–57, Jan. 1960.

Alloys investigated were basically iron containing 0.04–0.09% carbon and 0.23–17.2% aluminum plus Si, Mn, P, and S. Tensile strength was not a function of temperature below 300 °F except alloys containing over 13% Al had maximum strength at this point. All alloys showed brittle fracture below −100 °C, elongation thereafter remaining relatively unchanged to 300 °C, after which it increased rapidly. Impact strength maximized for (nearly) pure iron at 0 °C, others not developing comparable values until approx 300–400 °C. Minimum impact value of 15 kg/sq cm was usually observed about 500–600 °C. As there is a phase boundary at 13% Al, the behavior of alloy series above and below this value are expected to be different. Below 13%, electrical resistance drops off rapidly below 900 °C. Above 13%, this critical value is reduced to 500 °C. Electrical resistance measurements also show great evidence of ordering in the neighborhood of 14.5% Al.

C. E. Balleisen, USA

6342. Taira, S., Koterazawa, R., and Inoue, M., Fatigue and dynamic creep properties of 13% Cr steel at elevated temperature under combined stresses of static tension and alternating bending *Bull. JSME* **2**, 8, 508–513, Nov. 1959.

The strength of 13% Cr steel was evaluated at 450 °C under conditions simulating service of turbines blades, i.e. combined static tension and alternating bending stresses at 780 cpm and measurements were made of creep strain and cycles or time to failure.

Authors found that combined stress reduces fatigue life. It also increases creep strain except for small ratios of alternating to static stress for which a decrease in creep strain occurs. This apparent strengthening by alternating stress was attributed to hardening of the material as evidenced by hardness measurements on specimen cross section.

The results of this investigation are similar to earlier ones employing axial dynamic stress superimposed on static tensile stress and indicate that either bending or axial dynamic stresses produce equivalent effects on elevated-temperature strength.

P. Shahinian, USA

6343. Uryu, T., On the hardness change in mild steels due to reversed stresses and the plastic region around tip of fatigue crack, *Rep. Res. Inst. Appl. Mech. Kyushu Univ.* **6**, 22, 49–62, 1958.

In mild steels subjected to rotary bending stressing, the mean microhardness of the ferrite does not increase for stresses up to the fatigue limits of the steels but does increase abruptly by about 7% for stresses beyond the fatigue limits; on the other hand, the mean microhardness of the pearlite does not change for stresses near the fatigue limits (below the lower yield points of the steels).

From the results of micro-Vickers hardness tests on the ferrite around nonpropagating fatigue cracks in notched mild steel specimens, it will be expected that the magnitude of plastic (fatigued) region at the tip of the fatigue crack in carbon steels is below 0.1 mm. From author's summary

6344. Migaud, B., and Talbot, J., Influence of the purity of steel on the discontinuity of the elastic limit, concluded from stress-strain curves (in French), *C. R. Acad. Sci., Paris* **249**, 20, 2071–2072, Nov. 1959.

6345. Ginsburg, Ts. G., and Litvinova, R. E., Variation with time of tensile and compressive strength of hydraulic concrete (in Russian), *Gidrotekh. Stroit.* **28**, 2, 30–33, Feb. 1959.

Authors present results of tests conducted at the BNIIG concrete laboratory. Tensile specimens had a critical cross section of 5 × 5 cm; compressive specimens were 7 × 7 × 21 cm or 10 × 10 × 10 cm. Portland cement was used. Water/cement ratio ranged from 0.5 to 0.7. Age of test was 3, 7, 28, 180, and 360 days.

Authors investigate the relation existing between tensile and compressive strength of mortar and of concrete for various W/C ratios and ages. Strength increases with time and decreases as W/C becomes larger. For concrete the ratio of tensile strength to compressive strength is not a constant. For ages from 3 to 28 days it decreases markedly with time. In general the higher W/C ratios correspond with the higher values of the tensile/compressive strength ratio.

The ratio of tensile strength of mortar to that of concrete varies also with time. At early ages ratio is > 1.0 (up to 1.6). As age increases, ratio decreases and at a year is < 1.0 (about 0.9).

Finally authors call for standardization of specimens and tests and for consideration of test results in design.

G. H. Beguin, Switzerland

6346. Lewicki, E., Recent tests on the steam-curing of concrete to accelerate the increase of its strength (in German), *Acta Techn., Acad. Sci. Hungaricae, Budapest* **26**, 1/2, 45–55, 1959.

Author deals with some properties of steam-cured concretes, cured at a temperature of 70° C. Tests proved that steam-curing increases the initial strength of the concrete, but only slightly influences its later strength attained in about a week's time. The strength at an age of three years is higher than that after 28 days. Referring to the question if steam-curing causes detrimental stresses in the concrete, tests gave a negative answer. Author points out with regard to the dimensions of test specimens, that in the case of a low surface-volume ratio a longer steam-curing was required to reach the same initial strength. Further, the optimum storage-time is dealt with, referring to cements of different setting times. Finally, data are obtained for the effect of steam-curing upon Young modulus for different kinds of cement, for different ages of the concrete, and as a function of stress.

From author's summary

6347. Youngs, R. L., Effect of thickness on the mechanical properties of glass-fabric-base plastic laminates, For. Prod. Lab., U. S. Dept. Agric. Rep. 1873, 30 pp., May 1960.

6348. Vol'kenshtein, M. V., On the structure and mechanical properties of amorphous polymers, *Soviet Phys.-Doklady* **4**, 2, 351–354, Oct. 1959. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **125**, 3, 523–525, Mar./Apr. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Paper examines the consequences of the possible existence of domains consisting of partly ordered bundles of chains in high polymers, as contrasted with the completely disordered, gas-like structure assumed in the past. Author emphasizes the analogy between this model of high polymers and the usual models of ferromagnetism. He derives an approximate force-deformation relation based on his model, and shows that it fits experimental results for rubber somewhat better than the older random model. Author neglects to mention the very pertinent theory of Flory on the effect of chain stiffness [*Proc. Roy. Soc. Lond. (A)*, **234**, 1196, 60–73, Jan. 1956].

S. Gratch, USA

Structures: Simple

(See also Revs. 6162, 6208, 6227, 6255, 6256, 6265, 6266, 6267, 6346, 6363, 6366, 6388, 6608)

6349. Mastachenko, V. N., Determination of the stresses and forces in a ferro-concrete slab joined to a steel beam due to the reaction of the shrinkage of the concrete and to the temperature (in Russian), *Trudi Mosk. In-ta Inzh. Zh.-d. Transp.* no. 101, 81–114, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6867.

Results are given for the calculations of the problems given in the title, the method used being that of the plane problem in the theory of elasticity, while creep is taken into consideration where the shrinkage of the concrete is concerned. The calculations are carried out by a procedure previously developed by the author [*Trudi Mosk. In-ta Inzh. Zh.-dor. Transp.* 1957, (91), 184–220]. Methods are indicated which improve the convergence of the series. Comparing the results of the calculations by the proposed procedure with those obtained by methods used by other investigators, the author comes to the conclusion that the latter give somewhat high values for the corresponding calculated magnitudes.

N. M. Figurnov

Courtesy Referativnyi Zhurnal, USSR

6350. Berman, F. R., Some basic concepts in matrix structural analysis, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 8 (*J. Struct. Div.*), 59-85, Aug. 1960.

The development of the electronic digital computer and the application of matrix algebra has made it possible for the structural engineer to analyze complex or highly redundant structures which previously had been prohibitive to analyze by the hand method of calculation. Papers recently written have utilized either the "flexibility" or "stiffness" matrix approach. The purpose of this paper is to point out the common origin of these methods in energy and to discuss procedures for calculating the total energy of a structure. The concept of the "total" structure is considered. Applications include analysis of a truss with elastic supports and a continuous beam.

From author's summary

6351. Kazimi, M. I., Sandwich cylinders; Part 1, State of the art and advantages of sandwich construction; Part 2, Uniformity of the mechanical properties of the core, *Aero/Space Engng.* **19**, 8, 32–37, 46, Aug. 1960; **19**, 9, 34–45, Sept. 1960.

In Part I, various conventional types of cylindrical construction are compared for reliability and efficiency under several types of loading. The growth and nature of fatigue failures are discussed and correlated to hoop stress and stiffening ratio when using honeycomb as a core. Improved results are obtained with honeycomb construction in structures exposed to vibration and shock.

In Part II, data are presented to illustrate the importance of uniform mechanical properties in the core by use of normal-expanded and overexpanded core construction. A solid shell was also tested for comparative purposes. Theoretical and test results are correlated.

Further improvements could be afforded by a radial core that would readily assume a cylindrical shape and retain uniform mechanical properties through the entire structure.

From author's summary

6352. Renskii, A. B., The deforming capacity and strength of flange-co-stressed (jointed) frameworks possessing columns in the steel skeletons of industrial buildings (in Russian), *Sb. Trudi Mosk. Inzh.-Stroit. In-ta* no. 22, 16–58, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6845.

Experimental investigations were carried out on a large-scale model of a two-span industrial building; also on models of the

individual nodal points of the transverse frameworks. The theoretical investigation set itself the problem of finding the relation between the external forces factors acting on the flange co-stressing (joints), and the total deformation of the co-stressing (joints). The work of the joints was examined both within the limits of elasticity and in the elasto-plastic stage. Comparison of the experimental and theoretical curves for deformation showed good convergence. The investigations made showed that the customary construction of the so-called hinge-jointed frames with columns actually guarantees the attainment of a moment equal to 40 to 80% of the moment of stiff fastenings. The failure to take into account the actual stiffness of such joints might well be reflected to a marked degree on the correctness of the static calculations for transverse frames. G. A. Geniev

Courtesy Referativnyi Zhurnal, USSR

6353. Mesarovic, M. D., Dynamic response of large complex systems, *J. Franklin Inst* **269,** 4, 274–298, Apr. 1960.

Large systems are studied by dividing them into *n* independent subnetworks and an extra "interaction" network, that represents all the possible interactions between the *n* isolated subnetworks. The interaction network is so defined that its reactions upon the subnetworks can not be separated from the inputs to the subnetworks. Consequently the interaction network can not be determined solely by measurements on the available subnetworks and over-all network. It is also necessary to make additional measurements by changing the *n* subnetworks. The unknown transfer functions of the interaction network are solved for in terms of correlation functions and spectral densities, in order to take care of random inputs. G. Kron, USA

6354. Schumpich, G., Contribution to the kinetics and statics of frames composed of curved bars (in German), *Öst. Ing.-Arch.* **11,** 3, 194–225, Nov. 1957.

Paper develops matrix methods for treating the problem of unrestrained vibrations of plane curved beams. Schumpich develops the differential equations governing the vibrations of the curved beam and proceeds to solve the system of equations by matrix methods. Several excellent examples and illustrations are presented which serve to amplify and clarify the method of application to particular problems of frames composed of curved bars. Forced vibrations of systems are also included in his presentation.

W. H. Sellers, USA

6355. Matarov, I. A., The work of deflecting ferro-concrete structures under repeated loading (in Russian), Theory and calculations for constructing ferro-concrete structures (Teoriya Rascheta i Konstruir. Zhelezobeton. Konstruktsii), Moscow, Gosstroiizdat, 1958, 85–107; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6855.

The conclusions arrived at show that: (1) repeated loads not producing stresses higher than the limits of endurance have practically no effect on the magnitude of the static destructive load; (2) after a large number of repeated loadings (sometimes after several hundreds of thousands) the process of crack formation practically ceases; (3) repeated loadings, though they lower the stiffness of construction, do so to an insignificant extent.

From author's summary

Courtesy Referativnyi Zhurnal, USSR

6356. Selig, E. T., Response of an elastic structure to blast-type loading, *J. Boston Soc. Civ. Engrs.* **47,** 2, 169–182, Apr. 1960.

A method is provided whereby a structural designer or analyst may rapidly investigate the effects of structural and blast loading parameters on maximum displacements, stress and strains. The structures considered are those which may be adequately represented by a linear elastic, single-degree-of-freedom system. Blast-type loads considered are those represented by an initial impulse together with a suddenly applied, linearly decreasing force. The designer is still faced with the problem of determining the values of the parameters which fit his particular case.

From author's summary

Structures: Composite

(See also Revs. 6213, 6214, 6223, 6285, 6286, 6325, 6352, 6388, 6613)

6357. Naruoka, M., and Ohmura, H., On the analyses of a skew girder bridge by the theory of orthotropic parallelogram plates (in English), *Publ. Int. Assn. Bridge Struct. Engng.* **19,** 231–256, 1959.

The classical equation of flexure of an orthotropic plate is applied to the analysis of skew girder bridges, by the finite-difference method. Influence coefficients of the deflection and bending moment are tabulated for several numerical cases. Experimental verifications were carried out through the use of model girder bridges. Y. Y. Yu, USA

6358. Scrivner, L. R., Digital computers for trial-load analysis of arch dams, *Proc. Amer. Soc. Civ. Engrs.* **86,** PO 4 (*J. Power Div.*), 1–12, Aug. 1960.

Stress analysis of concrete dams can be expedited by the use of electronic computers. The use of the "trial-load method" has been restricted due to the time-consuming, laborious computations that were required. The programming of many of these operations for an electronic computer results in a saving of time and costs, making this method of analysis more effective in the design of arch dams. From author's summary

6359. Copen, M. D., Design of arch dams by trial load method of analysis, *Proc. Amer. Soc. Civ. Engrs.* **86,** PO 4 (*J. Power Div.*), 13–29, Aug. 1960.

A brief resume of the basic principles involved in the trial-load method of stress analysis is presented. The various types of analyses, and their advantages and limitations, are examined. The application of trial-load methods to design problems and practical examples resulting from these procedures are illustrated and described in detail. It is also shown that the use of the trial-load method of stress analysis makes possible economy not only in the design of arch dams, but also in the time and expense required to produce such designs. From author's summary

6360. Benjamin, J. R., and Williams, H. A., Reinforced concrete shear wall assemblies, *Proc. Amer. Soc. Civ. Engrs.* **86,** ST 8 (*J. Struct. Div.*), 1–32, Aug. 1960.

This paper gives the results of an investigation of one-story and two-story reinforced-concrete shear wall assemblies. Tests were made on four one-story models with parallel shear walls connected by a reinforced-concrete diaphragm. Loads were applied to the diaphragm, and three separate diaphragm tests were then made. Following these tests, two one-story specimens with face walls were tested. Finally, two two-story models were tested.

Theoretical studies were made for all tests and practical design approximations are suggested. From authors' summary

6361. McGuire, W., and Fisher, G. P., Containment studies for an atomic power plant, *Proc. Amer. Soc. Civ. Engrs.* **86,** PO 3 (*J. Power Div.*), 27–53, June 1960.

A summary of the structural aspects of insuring complete containment for the Enrico Fermi Atomic Power Plant in the improbable event of an accident is given. The nature of the critical loading, as well as methods of investigation of structural behavior as a consequence of the loading, is discussed for such emergency conditions. From authors' summary

6362. Au, L. L. T., and Tsui, E. Y. W., Structural engineering aspects of nuclear power plants, *Proc. Amer. Soc. Civ. Engrs.* **86,** ST 6 (*J. Struct. Div.*), 23–46, June 1960.

A comprehensive description and discussion of the structural features of nuclear power plants is presented. Emphasis is placed on the structural engineering aspects which depart from conventional power systems. Stress analyses in reactor pressure vessels with supports under various loading conditions as well as temperature gradients are critically reviewed. The effect of dynamic forces and wind on structures is also examined.

From authors' summary

6363. Halliwell, A. R., Elementary stress analysis of circular frames in a cylinder partially filled with fluid, *J. Roy. Aero. Soc.* **64,** 593, 294–297 (Tech. Notes), May 1960.

Equations and curves are given for bending moment, direct force, and shear force in a horizontal cylinder partially filled with fluid. The maximum bending moment in a frame occurs when cylinder is half full and occurs at the fluid surface. Analysis is based on customary bending theory of thin circular rings loaded in a plane.

R. B. McCalley, Jr., USA

6364. Markelov, M. Ya., An approximate calculation for plane ships' decking by the method of selection of loading of cross connections (in Russian), *Sudostroenie* no. 7, 4–10, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6814.

The essence of the method consists in assigning an approximate form of deflection for the cross connections and the subsequent determination of reactions in the joints of the structure, starting from the parity of the flexures of the beams of both directions. The disclosure of the static indeterminability of the construction merges with the solution of a system of linear algebraical equations, whose number equals the number of cross connections. Supplementary tables are given which reduce the amount of work to be done in deriving equations for various forms of loading and different conditions for fastening the beams. The limits for application of the method are indicated and an evaluation given of the probable error. The main idea of the method has been demonstrated previously in articles by P. F. Papkovich "Constructional mechanics of a ship," **2;** part 1 [Leningrad, Morskoi Transport, 1947].

V. S. Chuvikovskii
Courtesy Referativnyi Zhurnal, USSR

6365. Nabikanova, M. V., Calculations for ships' coverings (in Russian), *Sudostroenie* no. 5, 6–11, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6816.

Graphs and tables are furnished, which simplify the calculations for coverings by the method of principal loads proposed by A. I. Segal. The author, considering the method of principal loads the more progressive, sets off this method against the method of principal deflections evolved by P. F. Papkovich, although both the methods, as regards essential calculations, are identical. The assertion is not true that the method of principal loads enables the problem to be solved which up to now has remained outside the possibilities of computational practice [see, *Trudî Tsentr. N.-i. In-ta Morsk. Flota*, (*A*), Leningrad, Morskoi transport, 1958]. It should be noted that, in general, the field of application of the calculation methods in which the coverings are investigated as systems of cross beams working only under deflection is sufficiently limited and that not one of the computation methods based on the investigation of a scheme of this nature can be viewed today as being progressive.

A. A. Kurdyumov
Courtesy Referativnyi Zhurnal, USSR

6366. Hoff, N. J., High temperature structures, AFOSR TN-60-383 (Stanford Univ., Dept. Aero. Engng. no. 79), 42 pp., June 1958.

Lecture surveys new problems confronting structural designer and analyst in consequence of aerodynamic heating. Distribution of heat in structure by conduction and radiation, thermal stresses, thermal buckling, creep under varying stress and temperature conditions, creep buckling, and structural safety are briefly discussed.

F. J. Plantema, Holland

6367. Vakhitov, M. B., Calculations for the strength of a tapered wing of monolithic construction (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Aviats. Tekhn.* no. 1, 61–68, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6804.

Formulas are derived for the computation of the deformations and stresses of a multi-walled wing with oblique ribs, fastened along one of these ribs, the following assumptions being made: (1) the flexural axis coincides with the line of the centers of loading of the wing 's section and is a straight line up to the actual point of the join; (2) the number of walls of the longerons and ribs is sufficiently great to enable the monoblock to be looked upon as an orthotropic plate; (3) the walls of the longerons and ribs only work under shear; (4) the transverse sections during deformation of the wing do not alter. A series of unusual deductions are obtained: (1) the wing with oblique ribs differs, in the conditions prevailing for the work, from the wing with straight ribs because the influence of the fastening with oblique ribs is felt not only in the Shukhov region but spreads along the whole length of the wing; (2) an increase in the section of the longitudinal joist assembly neutralizes the sagittal effect and with a pure caisson section present this effect becomes equal to zero; (3) in a caisson section of an oblique wing the stresses are distributed in the same way as in a straight wing. These deductions do not agree with the deductions published in S. N. Kan's and I. A. Sverdloy's book "Calculations for the strength of an airplane," [Moscow, Oborongiz, 1958].

S. Ya. Makarov
Courtesy Referativnyi Zhurnal, USSR

6368. Burt, M. E., Effects of design speed and normal acceleration on aircraft structure weight, Aero. Res. Counc. Lond. Curr. Pap. 490, 20 pp., 1960.

6369. Osgood, C. C., Structural design of Tiros I, *Astronautics* **5,** 6, 42–43, 90–91, June 1960.

6370. Heath, W. G., The structural effects of kinetic heating—Some factors affecting the choice of materials, *J. Roy. Aero. Soc.* **63,** 587, 615–619, Nov. 1959.

A general discussion is given of the effect of structural efficiency, stiffness, creep and fatigue on the choice of materials suitable for structures operating at temperatures up to 500°C, primarily as concerns aluminum and titanium alloys and steels. Problems of flight at temperatures up to 1000°C are briefly considered. The merits of various types of construction (e.g. honeycomb and corrugated sandwich) are also discussed.

B. A. Boley, USA

6371. Przemieniecki, J. S., The structural effects of kinetic heating—Design of transparencies, *J. Roy. Aero. Soc.* **63,** 587, 620–636, Nov. 1959.

A comprehensive, though somewhat concise, study of the design problems in transparencies for supersonic aircraft. Topics considered are: (1) Design requirements; (2) aerodynamic heating and cooling of transparencies, including description of heat-conduction calculations by analytical, numerical, seminumerical methods, and analogues; (3) thermal deformations and thermal stresses, including the interaction with pressure loadings, and also in the laminated panels; (4) effect of the design parameters; speed, altitude, acceleration and deceleration; (5) choice of materials; (6) practical design considerations, including fail-safe design; (7) testing of transparencies. Emphasis throughout is on engineering and physical aspects of the problem, and the paper gives in a clear way information of direct use to the designer.

B. A. Boley, USA

6372. Bore, C. L., The structural effects of kinetic heating—Some practical aspects of kinetic heating calculations, *J. Roy. Aero. Soc.* **63,** 587, 637–645, Nov. 1959.

Attention is concentrated upon methods of estimating temperature and heat-transfer rates for practical aircraft up to Mach numbers of five. Account is taken of the variation of temperature, and therefore viscosity, through the boundary layer, and of the variation with increasing speed of thermodynamic properties of air (e.g. the ratio of specific heats). The turbulent and laminar heat-transfer coefficients are studied. The equation governing transient heating for the structure is shown, and a step-by-step method of solution of the combined fluid-solid problem is developed. Some typical results conclude the paper. B. A. Boley, USA

6373. Sobey, A. J., The structural effects of kinetic heating—Advantages and limitations of models, *J. Roy. Aero. Soc.* **63,** 587, 646–656, Nov. 1959.

The use of models for structural tests under kinetic heating conditions is examined. The similarity criteria pertaining to external air flow, internal heat transfer, and elastic response are treated. Of these, the second is found to influence most model design. An analysis of a typical aircraft structure is given to illustrate the various effects of internal heat transfer.

B. A. Boley, USA

6374. Michael, Maureen E., and Bruce, Joan, The structural effects of kinetic heating—Summary of the discussion, *J. Roy. Aero. Soc.* **63,** 587, 656–658, Nov. 1959.

Discussions and authors' replies on the papers of the preceding four reviews are summarized. Emphasis was on the need for further experiments and closer liaison between theory and experiment, and on some speculation on the probable nature of future development. Critical analysis of the premises of some of the papers is also present. B. A. Boley, USA

Machine Elements and Machine Design

(See also Revs. 6208, 6269, 6565, 6567, 6570)

6375. Raven, F. H., Analytical design of disk cams and three-dimensional cams by independent position equations, *ASME Trans.* **81** E (*J. Appl. Mech.*), 1, 18–24, Mar. 1959.

Author applies independent vector equations, expressing the position of a point of cam contact with the follower, to the design of two- and three-dimensional cams. The desired kinematic quantities are obtained, by simultaneous solution of the independent equations. Expressions are also obtained for the cam contour, the radius of curvature, the pressure angle, and, by differentiation, the velocity and acceleration at a point on the cam surface are found. The method is direct and simpler than other methods and provides an easier approach to the analysis of complex cams. M. Martellotti, USA

6376. Wydler, R., Application of non-dimensional parameters in gear tooth design (in German), ZVDI **102,** 9, 349–352, Mar. 1960.

This abstract by H. Kreisel is an abbreviated version of the title paper presented by R. Wydler at the International Conference on Gearing, London, 23–25 Sept. 1958.

Technical calculations can often be simplified by means of a suitable parameter. The profile displacement factor is a characteristic magnitude of this kind that simplifies the design and calculation of gearwheels (including applications where there are abnormal distances between centers). A nomograph permits the anticipated flexural load-carrying capacity of the tooth to be established, which serves as an indication for the selection of a favorable form of tooth through profile displacement. Characteristics that are important for good gearwheel mating can also be assessed with the aid of a new non-dimensional engagement parameter. As a result, calculation of the specific slip, meshing period, Hertzian pressure, sliding load and wear data is simplified. The working face conditions can be displayed by plotting the numerical values of new parameters for the top of tooth of the pinion and wheel in restricted areas of a multiple nomograph. The alignment chart produced provides the possibility of assessing solutions and will furnish design directives.

Abstract by H. Kreisel, Germany

6377. Meyer zur Capellen, W., Seven popular types of three-dimensional drives, *Prod. Engng.* **31,** 25, 76–80, June 1960.

6378. Lang, H., Hydraulic couplings (in German), *ZVDI* **102,** 12, 447–456, Apr. 1960.

Hydraulic couplings can be grouped according to their mode of action into first, the hydrodynamic (reaction and Föttinger) and hydrostatic types and, second, into the starting varieties with constant filling and clutch couplings with random variation of the filling fluid transmitting the power. Föttinger couplings enable the thrust and flow losses of the fluid within the pump-turbine circuit to be estimated, in addition to the transmitted power, obtained by simple means from the pertinent dimensions. Widely varying types of hydrodynamic and hydrostatic couplings are available, for duties covering a wide range, to suit the respective applications.

From author's summary

6379. Frost, M. D., The inertial effects of springs in oscillation experiments to determine damping, *J. Roy. Aero. Soc.* **63,** 586, 602–606 (Tech. Notes), Oct. 1959.

6380. Keler, M. L., Analysis and synthesis of space linkages through the use of the geometry of line space and dual numbers, Parts 1 and 2 (in German), *Forsch. Geb. Ing. Wes.* **25,** 1, 26–32, 1959; **25,** 2, 55–63, 1959.

The analysis of space mechanisms and the concepts of spatial kinematics based on line and screw quantities can be simplified through the use of higher complex dual numbers. Since this results in an analogy with spherical and plane kinematics, it becomes possible to carr over the theorems of plane kinematics into spatial kinematics, both numerically and descriptively. By a technique of mapping line space into the plane of dual points, author succeeds in developing the synthesis of actual and virtual space mechanisms, with prescribed conditions up to three discreet and three infinitesimally adjacent link positions.

From author's summary by G. A. Nothmann, USA

6381. Obrosov, I. I., Principles governing the motions of mechanisms when taking into account elastic deformations (in Russian), *Trudî Taganrogsk. Radiotekhn. In-ta* **3,** 2, 253–260, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6021.

The errors are investigated which are introduced into the law of motion and the magnitude of the dynamic loads of the links of ideally stiff mechanisms due to the elastic deformation of the links caused by inertia loading. As an example an analysis is carried out of the mechanism of control of the motion of a calculating machine equipped with a proportional movement of the bars. It was shown that in such a mechanism the elastic deformation of the links results in an increase in the acceleration by several times.

V. N. Geminov

Courtesy Referativnyi Zhurnal, USSR

6382. Lambert, T. H., Study of the clamping conditions in a connecting-rod big end, *J. Mech. Engng. Sci.* **1,** 3, 223–234, Dec. 1959.

The connecting rods of reciprocating engines are required cyclically to carry a direct tensile load which is transmitted by bearing-cap bolts and which produces in these members both direct and bending stresses. These stresses are dependent upon the stiffness of the bearing cap. The authors present the results of an analysis, corroborated by experimental strain measurements, in which the combined direct stress and bending problem is analyzed with consideration of the important relevant factors of bearing-cap stiffness and the increased bending stiffness resulting from direct tension in the bolt. Earlier studies cited by the authors had neglected this latter effect. B. G. Johnston, USA

Fastening and Joining Methods

6383. Kolev, K. S., and Aseev, I. V., The influence of riveting and residual stresses on the strength of treated articles (in Russian), *Sb. Nauchn. Trudi Severo-Kavkazsk. Gorno-Metallurg. In-ta* no. 14, 267-276, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6977.

Data are submitted characterizing the influence of riveting and residual stresses, produced by the mechanical treatment of the surface of the test samples (by cutting, due to the geometry of the sample, by grinding), on the magnitude of the limit of yield when testing the samples for tension, and the angle of twist when carrying out experiments for torsion. I. I. Trapezin

Courtesy Referativnyi Zhurnal, USSR

6384. Brungraber, R. J., and Clark, J. W., Strength of welded aluminum columns, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 8 (*J. Struct. Div.*), 33-57, Aug. 1960.

Welding heat-treated or cold-worked aluminum alloys causes partial annealing of the material in the vicinity of the welds so that the strength of the material near the welds is lower than the strength of the material in the rest of the structure. An experimental and analytical investigation was conducted to determine the effect of the varying mechanical properties on column strength.

The strength of aluminum columns with longitudinal welds can be computed by means of the same techniques used in determining the effect of residual stresses on column strength. Columns with transverse or other localized welds can be analyzed as stepped columns. From authors' summary

6385. Visyashchev, V. S., Investigation of the distribution of the forces and the stresses in a "fir tree" locking device of a working blade of an aeroplane gas turbine when the material is in an elastic state (in Russian), *Sb. Statei Chelyab. Politekh. In-ta* no. 11, 59-79, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6807.

The basic conceptions for the investigations are incorporated in the following assumptions: (1) only a tension force is acting on the locking device by the blade, (2) the temperature of heating and the elastic constants of the material vary by staggered steps from one part to another, (3) the inertia forces are substituted by concentrated forces applied to the centers of the loads of the parts, (4) the concentration of stresses at the places of co-stresses of the projections and the body of the tail-piece is disregarded. The equations for the plane problem in the theory of elasticity are utilized; the distribution of the working forces along the length of the caudal part of the blade is determined on the basis of the conditions of the joint nature of the deformation of the tail end of the blade and the projections of the disk. S. B. Boyarshinov

Courtesy Referativnyi Zhurnal, USSR

Rheology

(See also Revs. 6238, 6433)

6386. Rathna, Miss S. L., Superposability of steady axisymmetrical flows in a non-Newtonian fluid, *Proc. Indian Acad. Sci. (A)* **51**, 3, 155-163, Mar. 1960.

The question of superposability of two flows of a viscous fluid studied earlier by R. Ballabh [*Proc. Benares Math. Soc.* **2**, 69-79, 1940], J. A. Strange [AMR **4**(1951), Rev. 1643] and others is here examined in the wider field of non-Newtonian fluids, for which the coefficients of viscosity and cross-viscosity are constant. A typical result is as follows: Any two axisymmetrical irrotational motions are superposable on each other.

G. Capriz, England

6387. Takizawa, E. I., On the thermoelastic waves in liquids (in English), *Acustica* **10**, 1, 25-29, 1960.

X-ray analysis indicated that liquids are composed of randomly oriented submicroscopic crystals; hyperfine structure in the spectrum of monochromatic-beam scattering further established that thermoelastic waves exist in liquid. Based on this evidence author attempted to extend Debye's theory of specific heat, and Brillouin's theory of heat conduction of solids, to the liquid state.

Generalization was achieved by introducing a viscoelastic model of liquid, i.e., adopting a generalized tensorial stress-strain relation, in terms of elastic moduli and relaxation times, in accordance with Maxwell's relaxation equation. Internal energy of liquid was then expressed as the sum of thermal energies of longitudinal and transverse waves, integrated over the frequency spectrum. The upper limits of energy integrations, i.e., cut-off frequencies, are obtained as definite functions of relaxation times τ. As τ tends to infinity, liquid approaches solid state. In this case it was shown that the thermal energy of longitudinal waves indeed corresponds to that given by Debye's theory of specific heat of solid, whereas thermal energy of transverse waves corresponds, on the one hand, to Luca's specific heat of viscous liquid derived from Stokes' expression; and on the other hand, to the contribution to internal energy of solid state by transverse waves.

It was also shown that the limiting case of Brillouin's formula for heat conduction, when transverse (shear) waves alone are left, also reduces to Luca's result for viscous liquid at elevated temperature. H. S. Tan, USA

Hydraulics

(See also Revs. 6149, 6314, 6412, 6432, 6687, 6688, 6706)

Book—6388. Izbash, S. V., and Khaldre, Kh. Iu., Hydraulics of damming river channels [Gidravlika perekrytiia rusel rek], Moskva, Gosenergoizdat, 1959, 208 pp. $1.40.

Book deals with hydraulic backgrounds of the complex problem of river closure by filling rock and gravel into running water. It contains: hydraulics of contracted channel, hydraulic resistance of filling material, general characteristics of various methods of damming, interaction between stream overflow and rock fill, turbulent filtration through the fill, framework fill, design of dammings. Book is based on large experience of Russian hydraulic engineers in successful damming of great rivers.

S. Kolupaila, USA

6389. Jones, I. R., and Edwards, D. H., An experimental study of the forces generated by the collapse of transient cavities in water, *J. Fluid Mech.* **7**, 4, 596-609, Apr. 1960.

Paper describes an experimental investigation of the pressures developed at the seat of collapse of cavities in water. Single transient cavities, generated by a spark discharge, are allowed to collapse on the end of a piezoelectric pressure-bar gage which measures the variation of thrust with time. It is shown that both the peak force and duration of the cavity collapse pulse are functions of the cavity lifetime. From an estimate of the minimum radius attained by the cavity and the peak force, the peak pressure on collapse is found to be at least 10,000 atm. Streak schlieren

photographs of the collapse process show that a shock wave is radiated into the water at the moment of collapse and that the cavity rebounds. At the collapse of the rebound cavities the pressures developed are comparable with those developed by the collapse of the initial cavity, and these probably contribute materially to cavitational surface damage.

From authors' summary

6390. Cumberbatch, E., and Wu, T. Y., Cavity flow past a slender pointed hydrofoil, Calif. Inst. Technol., Guggenheim Aero. Lab. Rep. 47-11 (Contract Nonr-220 (12)), 36 pp., Mar. 1960.

A slender-body theory for the flow past a slender, pointed hydrofoil held at a small angle of attack to the flow, with a cavity on the upper surface, has been worked out. The approximate solution valid near the body is seen to be the sum of two components. The first consists of a distribution of two-dimensional sources located along the centroid line of the cavity to represent the variation of the cross-sectional area of the cavity. The second component represents the cross-flow perpendicular to the centroid line. It is found that over the cavity boundary which envelops a constant pressure region, the magnitude of the cross-flow velocity is not constant, but varies to a moderate extent. With this variation neglected only in the neighborhood of the hydrofoil, the cross-flow is solved by adopting the Riabouchinsky model for the two-dimensional flow. The lift is then calculated by integrating the pressure along the chord; the dependence of the lift on cavitation number and angle of attack is of a nonlinear form.

From authors' summary

6391. Chepaikin, G. A., Determination of the losses when the flow enters the turbine chamber (in Russian), *Izv. Vyssh. Uchebn. Zavedenii-Energetika* no. 2, 103-113, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2625.

This is an investigation of the determination of loss of pressure at the inlet of river hydroelectric stations. To find a formula for the resistance of the inlet portion in a general form, that is at the inlet of the distorted flow at any angle and at any relationship of velocities up to and beyond the inlet, an equation is derived for the quantity of motion for two control sections separated out of the part of the flow which has penetrated the farthest into the inlet portion. A general formula is obtained which in boundary cases changes into known formulas for sudden expansion and sudden constriction. For intermediate cases simplified formulas are given with the introduction of modifying coefficients, the values for which for various cases are put forward according to data of the abstractor [Hydraulic resistances, (Physico-mechanical bases), Moscow-Leningrad, Gosenergoizdat, 1954]. Experiments with a special model of one span of the upper part of a block of a combined hydroelectric station yielded good convergence between the calculation data and the experimental. I. E. Idel'chik

Courtesy Referativnyi Zhurnal, USSR

6392. Agasieva, S. I., Hydraulic calculations for the trench in a side-weir (in Russian), *Trudi Vses. N.-i. In-ta Gidrotekhn. i Melior.* **29**, 78-87, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2636.

A description is given of the laboratory experiments on a model of a weir in the Cherkizovskii irrigation system; this was an unsubmerged weir with a lateral carry-off for the surplus water; the experiments were carried out in the All-Soviet scientific research institute for hydroengineering and its development. It is noted that on the so-called carry-off waterway (that is that part of the carry-off section which is situated in the limits of the crest of the weir-dam) a spiral type of motion of the water takes place, the directional axis of the motion being along the carry-off waterway. It is also noted that when the carry-off waterway is of constant width a zone of dead water is formed in its initial portion and here eddies are to be found periodically. A conditional method of calculation is proposed for the carry-off waterway, in conformity with which the mean velocity in the live sections of the trench along its whole length becomes a constant and equal to the mean velocity in the so-called carry-off channel (that is that part of the carry-off section which is situated beyond the limits of the crest of the weir-dam); the width of the carry-off waterway is assumed to change, in accordance with the linear principle, from the magnitude (0.20 to 0.25) B_{ch} at the beginning up to the magnitude B_{ch} at the end of the carry-off section, where B_{ch} is the width of the carry-off channel; the depths of the water in the live sections of the carry-off waterway are calculated by means of known values for live sections of discharge, velocity and width of the trench. The slope of the bottom of the trench is fixed arbitrarily; because of this the trench which has had its computations made by the described method will be characterized by the absence of the essential link between the mean velocity in the given live section and the hydraulic slope in that section. R. R. Chugaev

Courtesy Referativnyi Zhurnal, USSR

6393. Khomyak, Ya. V., An investigation of the conditions governing the flowing of water in submerged below-bridge currents (in Russian), *Trudi Kievsk. Avtomob.-dor. In-ta, Sb.* 3, 145-153, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2649.

Results are given of the investigations on the conditions governing the flow of water in the submerged below-bridge openings of rectangular transverse section, together with proposals for an approximate method for the hydraulic calculations, based on data of laboratory experiments carried out by the author. Data of the experiments (curves of the free surface, coefficients of discharge, etc.) are furnished for one of the series: a model of a bridge with openings of 10.1 cm with a length for the buttresses along the flow of 20 cm. The gist of the proposed method can be stated as follows: the author proceeds to investigate the relative depth of the below-the-bridge current K and the degree of submergence n; the first of these magnitudes is the relation of the depth in the below-the-bridge current at the end of the inlet portion to the full specific energy of the flow in the section in front of the bridge (E_0), calculated relative to the plane of comparison coinciding with the bottom at the end of the inlet portion; the degree of submergence is the relation of the full specific energies in the lower water and in front of the bridge calculated relative to the same plane of comparison; in the initial moment of submergence the author adopts

$$k = k_0 = h_k/E_0 = \sqrt[3]{2m^2}$$

where h_k is the critical depth, m is the coefficient of discharge of the submerged opening; the value of n at the given value for k the author takes to be N and designates it as the characteristic of the criterion of submergence. The relation between k and n is assumed to be linear (the possibility of this assumption is confirmed by the experimental data), so that with $n = N$, k is equal to k_0, while with $n = 1$, k is equal to 1. A fractionally-linear empirical relation to k_0 is found for the magnitude N. This relation does not take into account the characteristics of the outlet portion of the structure and of the lead-off current; because of this it cannot be held applicable to a wide range of different cases. This appears to be a defect in the whole method of calculation being investigated. For the coefficient of discharge of the submerged orifice $m = q/\sqrt{2}\, g E_0^{3/2}$ (here q is the specific discharge) the following empirical formula is proposed

$$m_n = m(1 + n - N)\sqrt{\frac{1-2}{1-N}}$$

A formula is also derived for the coefficient of velocity. In order to accelerate the calculations for the recommended relationships a number of graphs are given. N. P. Rozanov

Courtesy Referativnyi Zhurnal, USSR

Incompressible Flow

(See also Revs. 6159, 6165, 6441, 6444, 6449, 6459, 6471, 6474, 6512, 6517, 6519, 6522, 6523, 6524, 6525, 6560, 6578, 6589, 6602, 6691, 6695)

6394. Sleicher, C. A., Jr., Stern, R. A., Scriven, L. E., and Oppenheim, A. K., Fluid dynamics, *Indust. Engng. Chem.* **52,** 4, 347-358, Apr. 1960.

6395. Ol'khovskii, I. I., The moment-approximations method in generalized hydrodynamics, *Soviet Phys.-Doklady* **4,** 3, 624-627, Dec. 1959. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **126,** 4, 748-751, May/June 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

This paper is too condensed to be followed in detail. The author studies the infinite system of exact equations for the Hermite coefficients first derived from the Boltzmann equation by Grad [*Commun. Pure Appl. Math.* **2,** p. 331, 1949]. He gives some approximations for the collision integrals, for the case of an inverse $\nu - 1$ power law of interaction, and he proposes a truncation scheme somewhat different from Grad's.

C. Truesdell, Italy

6396. Bouligand, G., Searching for axioms on fluid mechanics (in French), *C. R. Acad. Sci., Paris* **249,** 5, 601-603, Aug. 1959.

6397. Isay, W.-H., Theory of a submerged hydrofoil moving near the water surface (in German), *Ing.-Arch.* **27,** 5, 295-313, 1960.

A complete (linearized) theory is given for a single submerged foil (or a pair in tandem) moving steadily and parallel to the equilibrium free surface. The motion is assumed to be two-dimensional. The feature of the paper is the determination of the distribution of circulation on the surface of the foil. This is achieved by considering the perturbation velocity field of the steady stream to consist of the sum of a velocity field induced by the circulation, a field necessary to satisfy the equation of motion, and a field which makes the total perturbation tend to zero at infinity. The resulting integral equations are solved by methods already known from turbine blade theory. Tables and graphs illustrate generously the effect and the free surface profile in several cases for the single foil and for a tandem arrangement.

L. M. Milne-Thomson, USA

6398. Sarpkaya, T., Added mass of lenses and parallel plates, *Proc. Amer. Soc. Civ. Engrs.* **86,** EM **3** (*J. Engng. Mech. Div.*), 141-152, June 1960.

Contained herein is a study of the added mass of lens-shaped bodies, two parallel square plates, and two parallel, infinitely long rectangular plates of various thicknesses. The added masses were determined from a mass-frequency relationship obtained by immersing the objects in water and accelerating them in oscillatory motion. The experimental results are in good agreement with the analytical studies of corresponding potential flows.

From author's summary

6399. Watson, J., Three-dimensional disturbances in flow between parallel planes, *Proc. Roy. Soc. Lond.* (*A*) **254,** 1279, 562-569, Mar. 1960.

The problem which has been investigated is whether the most highly amplifying disturbance at any given Reynolds number above the minimum critical Reynolds number is a two-dimensional or a three-dimensional disturbance. It has been shown that the most unstable disturbance is a two-dimensional one for a certain definite range of Reynolds number above the critical. For Reynolds numbers greater than this no definite general answer has been found; each basic undisturbed flow must be treated separately and a simple procedure has been given which, in principle, determines the type of disturbance which is most unstable. Difficulty arises in following this procedure because it requires knowledge of the two-dimensional stability curves in a certain region where this knowledge is very scanty at the moment. Although this difficulty arises, in Poiseuille flow the calculations available indicate very strongly that the most unstable disturbance at any given Reynolds number above the critical is two-dimensional. Further, it is believed that this result holds for all other basic flows. A second result is that if the wave number (α) in the flow direction is specified, as well as the Reynolds number, then for α in a certain range, the most unstable disturbance is three-dimensional.

From author's summary by S. B. Berndt, Sweden

6400. Graffi, D., On the theorem of uniformity of flow of a viscous fluid in a non-confined space (in French), *C. R. Acad. Sci., Paris* **249,** 18, 1741-1743, Nov. 1959.

6401. Carstoiu, J., On the slow motion of a viscous fluid between two parallel walls (in French), *C. R. Acad. Sci., Paris* **249,** 14, 1191-1193, Oct. 1959.

6402. Bianco, E., Cabannes, H., and Kuntzmann, J., On the Crocco number for the flow along a revolving body (in French), *C. R. Acad. Sci., Paris* **249,** 15, 1312-1314, Oct. 1959.

6403. Barua, S. N., Some aspects of rotational flow, *J. Sci. Engng. Res., India* **3,** 2, 351-356, July 1959.

6404. Aslanov, S. K., and Legkova, V. A., Outflow of a gas jet from a vessel of finite size, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 266-272, 1959. (Pergamon Press, 122 E. 55th Street, New York 22, N.Y.)

6405. Parkhomovskii, S. I., Virtual masses of some curvilinear contours immersed in detached flow, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 3, 827-833, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

Application of Pykhteev's [AMR **12**(1959), Rev. 1362] family of two parametric functions to the linearized two-dimensional, incompressible hydrodynamic impact problem. The parabola-shaped cross sections are allowed to rotate and translate (with arbitrary velocity vector). Limiting cases related to wedge and plate impact are shown to agree with classical results. Via the added (virtual) mass coefficients the impact forces and moments are computed.

V. G. Szebehely, USA

6406. Polubarinova-Kochina, P. Ia., A point source and a vortex filament in a helical current, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 4, 1122-1125, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author shows that the integral solutions of a two-dimensional helical motion given in Vasil'ev's book ["Fundamentals of the mechanics of vortex and irrotational flows," 1958] can be expressed in simpler forms. In special cases they are readily reduced to that of a point source in potential flow and that of an infinitesimally thin vortex line along the axis.

L. N. Tao, USA

6407. Fail, R., Lawford, J. A., and Eyre, R. C. W., Low-speed experiments on the wake characteristics of flat plates normal to an air stream, *Aero. Res. Counc. Lond. Rep. Mem.* 3120, 21 pp., 1959.

Authors give results of experiments performed in tunnel with 140 feet/sec speed on the aerodynamic characteristics of flat plates perpendicular to the wind. Tested plates were circular, equilateral-triangular and rectangular of different aspect ratios. Tests also were performed on a circular plate with tabs.

Measurements were made not only of drag, but also total head, static pressure and velocity fluctuations in wake. Spectra of fluctuations show that almost all plates shed turbulent eddies at particular frequencies. Drag, pressures in the wake and flow pattern are little different for rectangular plates with aspect ratios less than 10: there are two shedding frequencies for each rectangular plate, associated with the two dimensions of the plate. The shape of low-aspect-ratio plates has little effect on aerodynamic characteristics.

Reviewer observes that the accuracy of the test is not comparable with that of the experimental work.

M. Viparelli, Italy

6408. Arzhanykh, I. S., Equations for the vortex motion of a polytropic gas (in Russian), *Trudi In-ta Matem. i Mekhan. Akad. Nauk UzSSR* no. 21, 39–34, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2411.

The equations for the two-dimensional steady vortex motion of a polytropic gas are brought into a system of two equations (sufficiently unwieldy) to describe the plane q and the function of the current ψ, one of which is algebraical (of the 4th degree relative to q). An equation of the second order is obtained for ψ after the elimination of q. As an example of the potentialities of the method an investigation is made of a motion which is symmetrical relative to the beginning of the coordinates, during the absence of mass forces. Analogous investigations are carried out for the three-dimensional case.

R. G. Barantsev
Courtesy Referativnyi Zhurnal, USSR

6409. Nikitin, V. S., Some questions on the aerodynamics of the air flow in an open mine (in Russian), *Ugol'* no. 6, 33–36, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2503.

Results are given of an experimental investigation on the ventilation of open pits, the wind velocities varying from 0.8 to 8.6 m/sec, and the temperature from −25 to +25°. The experiments were made on models of the pits. They showed that the motion of the air in the pits with a wind operating on the surface conforms to the principles of turbulent jets; in particular, to the principles inherent in plane-parallel currents flowing into an unbounded space in a direct-flow system of ventilation, while the principles of secondary turbulent currents would apply in recirculation systems. On the basis of the obtained results the author puts forward recommendations on the forms for the pits which would ensure the maximum use of the wind energy to take the gases and dust out of the pits.

I. S. Simonov
Courtesy Referativnyi Zhurnal, USSR

6410. Dement'ev, M. A., The transport of a single solid body by a heterogeneous flow of a liquid (in Russian), *Izv. Vses. N.-i. In-ta Gidrotekhn.* **54,** 3–26, 1955; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6252.

Author examines the problem of flow about a cylinder by a stream with a constant amount of vorticity. This problem was actually solved by N. E. Zhukovskii [Full colln. of papers, Moscow-Leningrad, 19326, **3,** 439–540] before Taylor and Proudman, cited by the author. Some qualitative concepts are advanced on the influence exerted by the length of the cylinder, borrowed from the theory of the wing of finite span. The coefficient of lateral force is introduced as a function of two dimensionless parameters and a description is given of the experimental apparatus; curves are produced of the results of the experiments. Finally, an expression is put forward to demonstrate the forces of reaction of a liquid on a sphere. There are solutions for the approximate dynamic equations for the motion of the sphere and on the basis of these solutions three regimes for the motion of the sphere in a heterogeneous flow are discussed.

N. A. Slezkin
Courtesy Referativnyi Zhurnal, USSR

6411. Figovskii, A. Ya., Conversion of a complex function of a flow about a round grid to a form more suitable for differential treatment (in Russian), *Trudi Kuibyshevsk. Inzh.-Stroit. In-ta* no. 5, 223–230, 1958; *Ref. Zh. Mekh.* no. 6, 1959; Rev. 6253.

An expression is obtained for a complex potential of the flow, due to vortex sources, which flows about a region, more suitable in the author's view for the calculation of the velocities on the contours of the profiles of a round grid.

E. L. Blokh
Courtesy Referativnyi Zhurnal, USSR

6412. Vulis, L. A., and Isataev, S. L., Turbulent motion of an incompressible liquid in the wake of a buff body (in Russian), Investigations of the physical principles for the working processes of furnaces and ovens, (Issled. Fiz. Vsnov. Rabochego Protsessa Topok i Pechei) Alma-Ata, Akad. Nauk KazSSR, 1957, 112–124; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6257.

Experimental investigations were carried out to study the mean turbulent flow of an incompressible liquid in the wake of bodies of different geometrical forms (plate, disk, sphere, round and elliptical cylinders) in the range of Reynolds numbers of the order 10^4. The studies centered on the motion near the body, which was characterized by the presence of a circulation zone with return currents of the liquid. The self-modelling nature of the mean flow in the wake, in the range of Reynolds figures used, was established. On the basis of the measurements of the fields of velocities in the wake, determinations were made of the dimensions and form of the circulation zone for different bodies. The data on the length of the circulation zone agreed satisfactorily with the results of calculations made by the approximate method evolved by the author [*Uch. Zap. Kazab. Gos. In-ta. Im. S.M. Kirova* **23**, no. 4, 1956], enabling the relative length of the region of return currents to be determined in relation to the resistance coefficient of the body.

O. V. Yakovlevskii
Courtesy Referativnyi Zhurnal, USSR

Compressible Flow (Continuum and Noncontinuum Flow)

(See also Revs. 6143, 5156, 6159, 6372, 6434, 6437, 6442, 6445, 6451, 6453, 6455, 6456, 6461, 6476, 6492, 6530, 6548, 6572, 6578, 6590, 6595, 6625, 6658, 6659, 6661)

6413. Tayler, A. B., A rapidly convergent procedure for solving the equations of subsonic potential flow: Part 1, Numerical solutions; Part 2, Analytic solutions, *Proc. Roy. Soc. Lond. (A)* **255,** 1280, 101–123, Mar. 1960.

In Part I the subsonic potential flow equations for a perfect gas are transformed by means of dependent variables $\overline{s} = (\rho/\rho_0)^n(\overline{q}/a_0)$ and $\sigma = \frac{1}{2}\ln(\rho_0/\rho)$ where $\overline{q}$ is the local velocity, ρ and a the local density and speed of sound, and the suffix 0 indicates stagnation conditions; n is a parameter which is chosen to optimize the approximations. A family of the first-approximation solutions in terms of the incompressible solution is obtained on linearizing. It is shown that, for two-dimensional flow, the choice $n = 0.5$ gives results as accurate as those obtained with the Karman-Tsien solution. For the axially symmetric case, the best accuracy is generally obtained for smaller values of n, but the best value of n seems to be quite sensitive to the body shape. The exact equations are then transformed into the plane of the incompressible velocity potential and stream function and the first-approximation results substituted in the nonlinear terms. The resulting second-approximation equations can then be solved by a relaxation method and the error in this approximation estimated by carrying out the third-approximation solution. Results are given for a circular cylinder at a

free-stream Mach number, $M_\infty = 0.4$, and a sphere at $M_\infty = 0.5$. The error in the velocity distribution is shown to be less than ±1% in the two-dimensional case.

Part II is an extension of Part I, describing an approximate analytical solution of the problem. A further approximation is made in the equations for the second-order approximation consisting of an expansion of one of the equations in powers of a parameter related to the density ratio. In this manner, an approximate solution of the second-approximation equations is obtained valid for all two-dimensional bodies for which the conformal transformation of their cross sections into a circle is known. Results are obtained for a circular cylinder with circulation and for a Kaplan airfoil. The velocity distributions are obtained to within one per cent accuracy. Good convergence of the power series solution seems to be obtained even at low transonic Mach numbers. However, convergence of the method has not been proven.

From author's summary by M. Epstein, USA

6414. **Lutz, O., Aerodynamic mixing; thrust increase by jet mixing** (in German), Jahrbuch Wissenschaft. Gesellsch. Luftfahrt, 1957, 374–380.

Extending his own previous work [O. Lutz, "Diagram representation of the processes in combustion chambers and ramjets," WGL Jahrbuch 1955, pp. 252–256], author essentially presents and discusses a numerical graphical solution of the basic equations involved in mixing of two gases in a constant area duct, namely continuity, momentun and energy equations. Since velocity of mixed gases in such mixing duct cannot exceed sound velocity, there is an area on $p - w$ mixing diagram for which basic equations are incompatible. Properties of this area are extensively described and used in determining mixing regions with resulting static pressure decrease and increase, the latter case yielding possibility of increasing jet propulsion thrust. Such an example is calculated for subsonic flight conditions, showing a maximum theoretical thrust increase of 80%. P. G. Schwaar, USA

6415. **De Sendagorta, J. M., Diabatical flow of a gas in a rotating conduit** (in Spanish), Advances in Aeronautical Sciences, Vol. 1 (Proc. of the First International Congress in the Aeronautical Sciences, Madrid, Sept. 8–13, 1958), Pergamon Press, 1959, 343–363.

When heat is provided to a gas flow, velocity increases and may reach a maximum equal to velocity of sound. This imposes a limit to flow through exchanger chamber. When, in addition, heat exchanger rotates, some modifications result on heat transmission. Author uses following assumptions to establish equations of motion: cylindrical conduit, rotation axis intersects conduit, conduit length is much larger than any other characteristic length dimension. Coriolis acceleration is neglected, unidimensional motion, perfect gas, specific heat at constant pressure is constant. Mach number variation is studied and compared with that for steady motion. Ideal and real gas solution are given, the latter for application to flow in a helicopter rotor blade with tip propulsion.

A. Balloffet, USA

6416. **Woolston, D. S., Gibson, F. W., and Cunningham, H. J., Some divergence characteristics of low-aspect-ratio wings at transonic and supersonic speeds,** NASA TN D-461, 43 pp., Sept. 1960.

The problem of chordwise divergence is treated with primary emphasis on slender delta wings which have cantilever support at the trailing edge. Experimental and analytical results are presented for four wing models having apex half angles of 5°, 10°, 15°, and 20°. The analytical results include calculations based on low-aspect-ratio theory, lifting-surface theory, and strip theory. A Mach number range from 0.8 to 7.3 is covered.

From authors' summary

6417. **Nelson, R. L., and Welsh, C. J., Some examples of the applications of the transonic and supersonic area rules to the prediction of wave drag,** NASA TN D-446, 46 pp., Sept. 1960.

The experimental wave drags of bodies and wing-body combinations over a wide range of Mach numbers are compared with computed drags of the supersonic area rule and with the results of equivalent-body tests. The wave drags computed by using the supersonic area rule are shown to be in best agreement with the experimental results for configurations employing the thinnest wings. A rapid method of computing wing area distributions and area-distribution slopes is given in an appendix.

From authors' summary

6418. **Talbot, L., Theory of the stagnation-point Langmuir probe,** *Physics of Fluids* **3**, 2, 289–298, Mar./Apr. 1960.

A theory of a Langmuir-type probe situated at the stagnation point of a blunt body in a supersonic partially ionized stream is presented. The classical Langmuir-probe theory is used in which the gas is ideally divided into two distinct regions, a sheath adjacent to the probe surface within which charges of one sign only are present, and the undisturbed plasma region with neutral charge.

The boundary-layer flow in the stagnation region is considered to be frozen in composition, the composition being a binary mixture of neutral molecules and ion-electron pairs. The conservation equations for the stagnation region boundary layer are solved subject to the assumptions of constant Prandtl number and a Chapman-Rubesin factor of unity.

By solving simultaneously the sheath and boundary-layer expressions, equations are obtained which relate probe potential and current to free-stream ion and electron densities ahead of the bow shock wave. An analysis of the stagnation point heat transfer shows that the heat transfer varies with probe potential. A numerical example is presented as well as a detailed examination of the validity of the assumptions made in developing the theory.

W. T. Snyder, USA

6419. **Craven, A. H., Base pressure at subsonic speeds in the presence of a supersonic jet,** Coll. Aero., Cranfield, Rep. 129, 26 pp., Mar. 1960.

This paper presents the results of an experimental investigation into the effect of supersonic jets upon the base pressure of a bluff cylinder in a uniform subsonic flow. The ratio of jet diameter to base diameter was 0.1875.

Jet stagnation pressures giving slight under-expansion of the jet cause an increase in the base pressure but for larger jet stagnation pressures the base pressure is again reduced.

A simple theory, based on a momentum integral, shows the dependence of the base drag upon the jet and free-stream speeds and upon the dimensions of the jet and the base.

From author's summary

6420. **Werner, W., A numerical approach to the question of stability of supersonic enclosures in an otherwise subsonic flow field,** AFOSR TN 60-256 (Mathematical Institute Technische Hochschule, München TN-12), 36 pp., Dec. 1959.

Time-dependent perturbations are considered of an exceptional solution of the plane transonic flow equations. This solution corresponds to steady continuous transonic flow past a profile with local supersonic regions. The perturbed flow fields are calculated by a method of three-dimensional characteristics and show regions, near the sonic line, where velocity gradients are large. Author concludes from this result that steady solutions exist for plane transonic flow with shocks. M. Holt, USA

6421. **Ionescu, M., and Darie, G., On the wave drag and rolling moment of the wings with supersonic leading edges** (in Roumanian), *Studii Si Cercetari Mecan. Apl.* **10**, 3, 683–693, 1959.

Authors consider the thin triangular wings with supersonic leading edges, whose incidence has a forced antisymmetrical variation. By a calculation artifice, the problem is reduced to that of a wing with subsonic leading edges for which the axial disturbance velocity is calculated with the aid of formula established by E. Carafoli and M. Ionescu [AMR **12**(1959), Rev. 3030]. The coefficients of the series development of this velocity are calculated for homogeneous motions of the order $n = 1, 2, 3, 4$. The results are used for determining the wave drag and rolling moment coefficients for the above-indicated bases.

Finally, the expression of these coefficients is given for an homogeneous flow of any n order.

T. Oroveanu, Roumania

6422. Maikapar, G. I., Calculation of the influence of centrifugal forces on surface pressure for a body of arbitrary shape in hypersonic flow, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 83-91, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

For the limiting case of a layer of vanishing thickness ($M_\infty \rightarrow 1$) behind the bow shock, a method is presented for calculating surface pressure on a body of arbitrary shape. The streamlines in the layer behind the shock are determined as unaccelerated particle paths at the body surface as a function of the initial velocities. The streamline distribution through the layer is then found from the equation of continuity, and the pressure at the body surface is computed when the streamlines are known.

From author's summary by F. L. Schuyler, USA

6423. Luniev, V. V., On the similarity of hypersonic viscous flows around slender bodies, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 273-280, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author derives equations of motion of a heat-conducting viscous fluid in curvilinear coordinates. He investigates the $M \gg 1$ flow over a slender, pointed, two-dimensional or axisymmetric body. Author then formulates similarity rules concerning the state properties, shock shape, and body shape.

F. L. Schuyler, USA

6424. Erickson, W. D., Real-gas correction factors for hypersonic flow parameters in helium, NASA TN D-462, 13 pp., Sept. 1960.

The hypersonic flow parameters for helium have been calculated from the thermodynamic properties of helium for stagnation temperatures from 0°F to 600°F and stagnation pressures up to 6,000 pounds per square inch absolute. The results are presented as simple correction factors which must be applied to the ideal-gas parameters. It has been shown that the error incurred by using the uncorrected ideal-gas parameters may be significant for high stagnation pressures and moderate temperatures. For stagnation conditions of 80°F and 4,000 pounds per square inch absolute, the free-stream static-pressure correction factor is approximately 1.13.

From author's summary

6425. Cresci, R. J., Hypersonic axisymmetric nozzles, *ARS J.* **30,** 7, p. 708 (Tech. Comments), July 1960.

6426. Hill, J. A. F., Initial calibration of the hypercore installation at M = 7.6, Mass. Inst. Technol., Naval Supersonic Lab. TR 437, 28 pp., Mar. 1960.

The Mach number and stagnation temperature distributions obtained in the calibration of a 12-inch diameter, $M = 7.6$ nozzle are presented. Model blockage limits are given for axisymmetric models at zero yaw and for low-aspect-ratio wings at large angles of attack. Brief descriptions are also given of the (unconventional) tunnel circuit and the nozzle design and fabrication processes.

From author's summary

6427. Cohen, I. M., Free molecule flow over nonconvex bodies, *ARS J.* **30,** 8, 770-772 (Tech. Notes), Aug. 1960.

6428. Chester, W., The propagation of shock waves along ducts of varying cross section, Advances in Applied Mechanics, Vol. 6, New York, Academic Press, Inc., 1960, 119-152.

The present paper considers the modification of the shock wave when the shock travels along a duct whose cross section changes continuously. The article consists of five chapters:

I. General introduction;

II. The steady-state theory. Here author developes his own point of view on the subject and gives the results of some numerical calculations and graphical representations.

III. Chisnell's theory. The approach used in this section is quite different from the steady-state theory. The problem is first considered for small change in cross-section area. Chisnell's extension to arbitrary area changes is then explained and criticized. There are given solutions for a small area change, for an arbitrary area change, and weak and strong shocks are considered.

IV. Comparison of two theories. (Steady-state and Chisnell theories) Here author makes conclusion: "The error in Chisnell's theory, compared with the steady-state theory, is 25% for a weak shock and 5.7% in the limiting case of a strong shock. Chisnell's result is least accurate in the region of $M = 1.5$, where the error is 48%."

V. Steady flow regime ahead of the shock. Here author considers the propagation a shock wave along a duct in which a steady flow has been established. Author says: "As before there is agreement between the two theories only when the area change is small."

T. Riabokin, USA

Boundary Layer

(See also Revs. 6372, 6418, 6423, 6449, 6518, 6522, 6523, 6524, 6526, 6527, 6528, 6529, 6530)

6429. Hayasi, N., On similar solutions of the steady quasi-two-dimensional incompressible laminar boundary-layer equations, *J. Phys. Soc. Japan* **15,** 3, 522-527, Mar. 1960.

Author first obtains three-dimensional boundary-layer equations from general equations of motion for orthogonal curvilinear coordinates. These are reduced to two-dimensions if one velocity component and the pressure gradient in the same direction are neglected. For this case a general similarity condition is stated and applied to the equations. Solutions are shown to exist only if velocity at outer edge of boundary layer satisfies specified power or exponential laws. New similar solutions are found using planar coordinates based on the body shape. One example based on flat plates is described in detail. Relationship to Blasius solution is clearly demonstrated. W. D. Baines, Canada

6430. Dutton, R. A., The velocity distribution in a turbulent boundary layer on a flat plate, Aero. Res. Counc. Lond. Curr. Pap. 453, 24 pp., 1959.

Author investigated five boundary layers along a smooth flat plate obtained with five different transition-promoting devices at a constant high Reynolds number. Transition devices were of two types, three with wires and two with glasspaper.

Author measured velocities using a flattened Pitot tube; skin friction, using round Pitot tubes on the surface (according to Preston technique) at five stations. The experimental velocity profiles in the inner boundary layer follow Nikuradse logarithmic law. Author suggests correction for effect of turbulent fluctuating velocities on Pitot measurements. Profiles of such corrected velocities follow Coles law in the inner layer.

In stations far from the leading edge, velocity profiles in outer boundary layer follow the same defect law for the same transition

device. But the defect law followed differs with the type of transition device. Author is in doubt whether discrepancy is to be attributed to a persistent effect of transition devices or to incomplete development of boundary layer. In region where logarithmic law and defect law overlap, form parameter follows Landweber, Coles law. M. Viparelli, Italy

6431. Deich, M. E., and Zaryankin, A. E., An approximate method for the calculation of a turbulent boundary layer at high velocities (in Russian), *Trudî Mosk. Energ. In-ta* no. 30, 218–231, 1958; *Ref. Zb. Mekh.* no. 6, 1959, Rev. 6419

The customary integral correlation derived by Karman for a compressible liquid is solved

$$\frac{d\delta^{**}}{dx} + \frac{u_1'\delta^{**}}{u_1}[2 + H - M^2] = \frac{\tau_w}{\rho_1 u_1^2} \qquad [1]$$

on the assumption that

$$H = H_0(1 + \alpha_1 M^2) = H_0\left[1 + \frac{2\alpha_1\lambda_1^2}{(k+1) - (k-1)\lambda_1^2}\right] \qquad [2]$$

and that the coefficient of friction c_f is

$$c_f = \frac{\tau_w}{\rho_1 u_1^2} = \frac{\rho_w}{\rho_1} R_w^{**-m}(\zeta + \alpha\Gamma) \qquad [3]$$

where $\zeta R_w^{**-m} = cf_0$ is the coefficient of friction with the gradient of pressure equalling zero, α is a constant coefficient, Γ is the parameter, equal to

$$\Gamma = \frac{u_1'\delta^{**}}{u_1} R_w^{**m}.$$

A general expression was obtained for the parameter Γ which may serve for the calculation of the boundary layer at any gradient of pressure (not taking into account heat exchange) for regions at a distance from the break-away. L. I. Arkhipova

Courtesy Referativnyi Zhurnal, USSR

6432. Lofquist, K., Flow and stress near an interface between stratified liquids, *Physics of Fluids* **3,** 2, 158–175, Mar./Apr. 1960.

Salt water was flowed turbulently under a pool of fresh water, as in estuaries. Interfacial agitation was varied by varying the density and flow rate of the salt water. The interface slope, velocity and density profiles and rate of mixing were measured and tabulated for 43 runs. Stress and effective velocity profiles were derived from the data and the equation of motion. During stable stratification the effective viscosity minimum equals (with laminar interface) or exceeds (agitated interface) the effective viscosity. The data are plotted dimensionlessly in terms of a Reynolds and a Froude number. The interfacial stress coefficient depends on both. The velocity profile is related to the interfacial stress similarly to turbulent flow near a rigid boundary.

C. F. Bonilla, USA

6433. Schowalter, W. R., The application of boundary-layer theory to power-law pseudoplastic fluids: similar solutions, *AIChE J.* **6,** 1, 24–28, Mar. 1960.

Two- and three-dimensional boundary-layer equations have been developed for fluids having the viscous stress-strain-rate relation $-T_{ij}/\rho = \mu_{eff}\, e_{ij}$ where ν_{eff} is a function of the invariants of $-e_{ij}(= u_{ij} + u_{j,i})$. The function ν_{eff} is chosen in such a way that for one-dimensional flow $T_{xy} = -\rho K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}$. The condition for the existence of similar solutions is investigated.

S. D. Nigam, India

6434. Evans, N. A., Heat transfer in the unsteady compressible laminar boundary layer on a flat plate, Proc. Heat Transf. Fluid Mech. Inst., Stanford, Calif., June 15–17, 1960; Stanford Univ., 1960, 77–91.

A semi-infinite flat plate ($x > 0$) moves in its own plane from rest (but not impulsively started) with variable velocity $-u_e$ in the x direction in a compressible fluid at rest at infinity. By various transformations the equation for the stream function is reduced to one already given for incompressible flow by Cheng and Elliott who solved it by expansion in powers of $\xi^{1/4}$ where $\xi = x/\int u_e dt$ [AMR **9**(1956), Rev. 4007]. In compressible flow there is an additional enthalpy equation which is solved by an expansion in powers of $\xi^{1/4}$ as far as the third power, for Prandtl numbers of 0.75 and 1.00 for prescribed surface enthalpy or heat transfer. Solution is given as a set of curves plotted on too small a scale to be of practical use, and by a table which will give the surface heat transfer or surface enthalpy expanded in powers of $\xi^{1/4}$. Author believes his solution to be valid up to $\xi = 0.5$, above which there are convergence difficulties. J. C. Cooke, England

6435. Pappas, C. E., and Okuno, A. F., Measurements of skin friction of the compressible turbulent boundary layer on a cone with foreign gas injection, *J. Aero/Space Sci.* **27,** 5, 321–333, May 1960.

Average skin friction was measured at both sub- and supersonic speeds ($0.3 \leqslant M \leqslant 4.7$) over a Reynolds number range of $0.9–5.9 \times 10^6$ and helium, air, and freon-12 as injectants. Theoretical evidence that the injection process reduces friction and heavy gases are less favorable was verified. Increasing Mach number appreciably decreases the benefits. Despite presence of large surface and base pressure allowances ($10–10^3$ times friction drag contributions) excellent fundamental data resulted including the boundary-layer trip influence. Authors omit comment on the specific axial permeability distribution of the fiberglas model.

J. R. Baron, USA

6436. Danberg, J. E., Measurement of the characteristics of the compressible turbulent boundary layer with air injection., U. S. Nav. Ord. Lab. Rep. 6683, Aero. Res. Rep. 67, White Oak, Md., 59 pp., June 1960.

The compressible turbulent boundary layer on a porous flat plate with air injection has been studied experimentally in the Naval Ordnance Laboratory hypersonic wind tunnel. Boundary-layer velocity and temperature profiles were obtained at a Mach number of 5.1, a Reynolds number of 1.5×10^7 per meter, wall to free-stream temperature ratios of 4.2 and 5.0, and rates of air injection from zero to 0.08 per cent of free-stream mass flow. Experimental values of skin friction were obtained from the velocity profiles and the results were compared with the theories of Rubesin and Persh. Neither theory predicts the experimental data on an absolute basis but Rubesin does describe the trend within ±25 per cent. The measured total temperature and velocity profiles compared favorably with the profiles assumed in Rubesin's theory. However, the Rubesin expression for the u^+ velocity at the interface between the laminar sublayer and the outer turbulent layer disagrees with experiment. The measurements indicate u^+ values of 14 ±3, independent of both wall temperature and injection rate.

From author's summary

6437. Bertram, M. H., Boundary-layer displacement effects in air at Mach numbers of 6.8 and 9.6, NASA TR R-22, 32 pp., 1959.

Measurements are presented for pressure gradients induced by a laminar boundary layer on a flat plate in air at a Mach number of 9.6 and for the drag of thin wings at a Mach number of about 6.8 and zero angle of attack. The pressure measurements at a Mach number of 9.6 were made in the presence of substantial heat transfer from the boundary layer to the plate surface. The measured pressure distribution on the surface of the plate was predicted with good accuracy by a modification to insulated-plate displacement theory which allows for the effect of the heat transfer and

temperature gradient along the surface on the boundary-layer displacement thickness. The total drag of thin wings with square and delta planforms was measured at a nominal Mach number of 6.8 over a reasonably wide range of Reynolds numbers. The total drag was found to be greater than can be explained by adding a classical value of laminar skin friction to the estimated pressure drag. The difference is, in general, explained by the increase in skin friction (20 to 40%) caused by the boundary-layer-induced pressures.

From author's summary by A. Q. Eschenroeder, USA

6438. Hopkins, E. J., Keating, S. J., Jr., and Bandettini, A., Photographic evidence of streamwise arrays of vortices in boundary-layer flow, NASA TN D-328, 20 pp., Sept. 1960.

Photographs are presented of various models coated with fluorescent oil to show evidence of surface vortices at a Mach number of 3.03. Vortex formation was evidently present on models with forward-facing steps, rearward-facing steps, wires, discrete surface particles, or unswept flat surfaces with sharp leading edges. Some photographs are also presented for the models coated with a sublimation material which clearly indicates the location of boundary-layer transition; however, it does not show the vortices as clearly as the fluorescent oil.

From authors' summary

6439. Takano, A., Wall temperature and heat transfer of a flat plate in the slip flow, *Trans. Japan Soc. Aero. Space Sci.* **2,** 2, 11-21, 1959.

Author continues efforts of earlier papers [AMR **12**(1959), Rev. 5103, and Proc. 8th Jap. Nat. Congr. Appl. Mech., 1958, pp. 285-291]. He attempts to find solution valid in the entire range from continuum to free-molecule flow. The derivation of boundary conditions is carried out by means of some questionable approximations. These are claimed to be valid for all flow regimes. The differential equations are linearized with respect to the free stream and the resulting Rayleigh-type equations are solved.

S. H. Maslen, USA

6440. Hama, F. R., Boundary-layer transition induced by a vibrating ribbon on a flat plate, AFOSR TN 60-290 (Inst. Fluid Dynam. Appl. Math., Univ. Maryland, TN BN-195), 32 pp., Feb. 1960.

To get a better over-all picture than is possible with a hot-wire, transition has been studied by observing the flow of dye in a water tank. The ribbon was about in the critical layer. When its vibration was sufficiently strong the vortex sheets it produced were rolled up to discrete straight vortices. The formation of these vortices can be taken as an indication that the limit of neutral stability is reached, as is shown by comparing with the experimental curve by Schubauer and Skramstad. Further on the straight vortices deformed into three-dimensional vortex loops which are stretched in the inhomogenous velocity field of the boundary layer. Eventually on the legs of the loop near its forward top a turbulence spot was created with random motion. A chain reaction of successive production of smaller loops on the legs of the initial loops is suggested as the mechanism of the final breakdown into turbulence. The development of discrete vortices, loops and turbulence spots appears to be the essential feature of transition here as in other cases previously investigated.

K. Wieghardt, Germany

6441. Sewell, K. G., A new analytical model for boundary layer transition, Proc. Heat Transf. Fluid Mech. Inst., Stanford, Calif., June 15-17, 1960; Stanford Univ., 1960, 106-119.

Paper presents a model of flat plate transition which supposes that Tollmien-Schlichting waves become unstable to three-dimensional disturbances, resulting in formation of horseshoe vortices which grow and break up into turbulent spots.

Author proposes a criterion for onset of three-dimensional instability, based on an argument which is far from convincing. He believes that oscillograms of Schubauer and Skramstad [NACA Rept. 909, 1948] give evidence of a modulating wave of lower frequency, which controls frequency of spot formation. Hence author calculates distance to the end of transition, using values for spot velocities from Schubauer and Klebanoff [NACA Rept. 1289, 1956; see AMR **9**(1956), Rev. 507].

Reviewer considers the evidence for this modulating wave is dubious in view of the successful treatment of spot formation as a random process by Emmons [*J. Aero. Sci.* **18,** 490-498, 1951; AMR **5**(1952), Rev. 486]. Author's calculation also seems to assume that spots form simultaneously on lines perpendicular to the flow.

Reviewer also disputes Fig. 9, which purports to show variation of transition Reynolds number with kinematic viscosity. This ignores the fundamental dimensional analysis of the problem, and the correlation cannot be correct.

Reviewer concludes that nothing in this paper should be accepted without very careful scrutiny.

L. H. Tanner, England

6442. Sandborn, V. A., and Wisniewski, R. J., Hot wire exploration of transition on cones in supersonic flow, Proc. Heat Transf. Fluid Mech. Inst., Stanford, Calif., June 15-17, 1960; Stanford Univ., 1960, 120-134.

Paper presents study of fluctuations in supersonic boundary layers on a cone, using a hot-wire anemometer. The data presented consists largely of measurement of the flatness factor of the signal, used as a measure of intermittency. There is an intermittent region at the outer edge of the layer which author considers is equivalent to that in subsonic layers, associated with turbulent spots. Another intermittent region appears near the wall, and author considers signal here suggests weak shock waves rather than turbulence. Interpretation of the flatness factor measurements is, unfortunately, not very clear, as oscillograms are described but not presented. Author uses flatness factor in outer region to deduce rate of spot formation and concludes that the rate increases in downstream direction.

Some measurements on a cone warming up from $-320°$ to $+40°F$ show that cooking below a certain point causes transition to occur earlier. There is no evidence of a different type of transition, however.

L. H. Tanner, England

6443. Gibbings, J. C., On boundary-layer transition wires, Aero. Res. Counc. Lond. Curr. Pap. 462, 32 pp., 1959.

The existing literature on boundary-layer transition due to a single spanwise wire is reviewed. The main parameters which influence the position of the transition point are the free-stream turbulence, the Mach number, and the pressure gradient. For incompressible flow over a flat plate with low turbulence transition occurs at the wire, when the wire Reynolds number based on the free-stream velocity is $Uk/\nu \approx 850$, with k as diameter of the wire. The effect of free-stream turbulence is to make this Reynolds number a function of both the degree of turbulence and of wire position. Compressibility makes it a function of the Mach number and the pressure gradient makes it a function of the Pohlhausen parameter $\lambda = \frac{\delta^{*2}}{\nu}\frac{dU}{dx}$ with δ^* as displacement thickness of the boundary layer and $U(x)$ as free-stream velocity. Curves are given for all these functions.

When transition occurs downstream of the wire, the reciprocal of the wire Reynolds number varies linearly with the position of the transition point over most of the range of the position of transition point.

H. T. Schlichting, Germany

6444. Curle, N., The estimation of laminar skin friction, including the effects of distributed suction, *Aero. Quart.* **11,** 1, 1-21, Feb. 1960.

This paper extends Stratford's analysis [Aero. Res. Counc. Lond. Rep. Mem. no. 3002, 1954] of a laminar boundary layer near separation to yield the full distribution of skin friction along a wall, including the effects of suction. The central idea is that the boundary layer may be divided into two parts. In the outer part, where viscous effects are small, the total head is constant along streamlines, and in the inner part near the wall pressure and viscous forces are in equilibrium. The joining of the solutions in these two parts, with continuity of a suitable collection of flow variables, leads to yet another one-parameter method of obtaining approximate solutions of the boundary-layer equations. The method suffers from some of the defects of such methods, but, from the examples considered, it appears that it nevertheless has a very useful range of validity. I. Proudman, England

6445. Chung, P. M., and Anderson, A. D., Dissociative relaxation of oxygen over an adiabatic flat plate at hypersonic Mach numbers, NASA TN D-140, 28 pp., Apr. 1960.

Assumptions made are laminar flow, Prandtl number and Lewis number unity and $\rho\mu$ = constant; also a formula for the recombination rate is used, based on experiments due to Camac and others, in which this rate varies as $1/T^2$. An extension of the Karman-Pohlhausen method is used and the resulting integral equations were solved on a digital computer for the adiabatic wall case for flight at altitudes from 50,000 to 200,000 ft and free-stream Mach numbers from 5 to 20. The results show that the rate of approach to equilibrium conditions with distance downstream from the leading edge is slow, and it is inferred that if the wall were cooled to a value of 700 °K the heat-transfer rate would be sensibly the same as for a chemically inert gas for the range of altitudes and speeds considered.

Reviewer believes the results to be of considerable qualitative interest and would welcome the application of the method to the case of the boundary layer behind a strong shock.

A. D. Young, England

Turbulence

(See also Revs. 6142, 6412, 6430, 6431, 6432, 6441, 6442, 6522, 6523, 6524, 6601, 6709, 6710)

6446. Edmonds, F. N., Jr., Spectral relations in homogeneous turbulence of an incompressible fluid, Proc. Sixth Midwest. Conf. Fluid Mech., Austin, Texas, Sept. 1959; Austin, Tex., Univ. Press, 1959, 140-148.

A method is discussed for obtaining "kinematic" relations between different spectra of a homogeneous turbulent field in an incompressible fluid. The relations are obtained within the framework of a linear theory, i.e. a theory in which nonlinear terms in the equations of the problem are neglected. It is then shown that these relations are valid, subject to certain restrictions, when nonlinear effects are considered. The method is discussed in connection with turbulence caused by thermal instability, but its application to other types of turbulence is emphasized.

From author's summary by R. P. Pearce, Scotland

6447. Eskinazi, S., Mixing of wakes in a turbulent shear flow, NASA TN D-83, 53 pp., Sept. 1959.

This interesting, well-documented, experimental investigation concerns the wakes of circular cylinders (Reynolds number R = 2240) in fully developed channel flow (R = 86,000). The wake decays about three times as rapidly as when in a uniform flow, although the velocity deficit displays similarities with wakes in a free stream except near the edge of the wake. The decay is even more rapid when close to one wall, but slows up when the wake spreads to the wall. The enhanced mixing rates are attributed to high turbulence levels at the start of the wake, its diffusion into a momentum-transferring turbulent field, and the axial pressure gradient, which influences are discussed on the basis of semi-empirical hypotheses. A. Powell, USA

6448. Laurence, J. C., Turbulence studies of a rectangular slotted noise-suppressor nozzle, NASA TN D-294, 85 pp., Sept. 1960.

Turbulence studies, made in a model noise-suppressor-type nozzle, were aimed at the statistical properties of the fluctuating flow which may cause the attenuation of the sound power in some frequency bands and at the effect of the nozzle spacing on these properties. Mean and turbulent velocity profiles, intensity, scale, spectra, and probability densities of the fluctuating velocities and convection velocities of the pressure and velocity eddies are reported. The turbulence intensity, scale and spectral distribution, and the mean velocity profiles are changed by the spacing-to-width ratio of the rectangular slots.

From author's summary

Aerodynamics

(See also Revs. 6159, 6163, 6371, 6397, 6413, 6418, 6420, 6428, 6437, 6444, 6486, 6602, 6605, 6606, 6608, 6618, 6621, 6643)

6449. Weissinger, J., Theoretical investigations on ring airfoils, AFOSR TN 60-343 (Institut für Angewandte Mathematik der Technischen Hochschule Karlsruhe Germany), 113 pp., Jan. 1960.

Theory of incompressible flow over a ring airfoil with a central body is studied, and yields formulas for the lift and pitching moment. Laminar boundary layers on ring airfoils are also studied. Results of numerical calculations for the case of ellipsoidal central bodies and for boundary layers are given in graphical form. Report also contains extensive tables of velocities induced by a ring vortex with cosine distribution of circulation and its associated trailing vortex cylinder. G. N. Ward, England

6450. Chaudhuri, S. N., A brief survey of the non-linear wing theories, *J. Sci. Engng. Res., India* **3**, 2, 363-376, July 1959.

This brief survey on nonlinear wing theories covers most of the literature available on the subject right from the mid-twenties when Betz pointed out that the nonlinearity in $C_L \sim \alpha$ could be expected on the basis of viscous effects for very low aspect-ratio wing. The theory proposed by Kuchemann has been described in some detail because it is an interesting and useful approach for aerodynamicists engaged in the design of supersonic aircraft and missiles. From author's summary

6451. Belokonov, V. M., The linear theory of a thin wing of small span in a transonic flow (in Russian), *Trudi Kuibyshevsk. Aviats. In-ta* no. 5, 9-33, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6130.

The problem is solved in a linear setting on the basis of the theory of a lifting surface. The density of the double layer at every point of the surface of the wing is represented by the linear function of the angle of attack and the local angle of declination of the median surface of the wing from the plane of the chords and by the unit functions of the distribution of the compactness of the double layer of sources and flows $\Gamma_{\varepsilon}(x, z)$ and $\Gamma_{\alpha}(x, z)$. Each one of these functions is presented in the form of a double series

$$\Gamma(x, z) = \nu_0 l \sum_m \sum_n a_{mn} \mu_m(\varphi) \nu_r(\theta)$$

$$\mu(\varphi) = \frac{\sin (m-1)\varphi}{m-1} - \frac{\sin (m+1)\varphi}{m+1} \text{ with } m \geqslant 2$$

$$\mu_0 = \varphi + \sin\varphi,\ \mu_1 = \varphi - \frac{1}{2}\sin 2\varphi$$

$$\nu_n(\theta) = \sin n\theta$$

where φ and θ are the angular coordinates along the chord and span. Two systems of linear algebraical equations are obtained which satisfy the boundary conditions. Tables are furnished for the integral of influence of the distribution of compactness of the double layer over the section on the inductive velocity and for the integrals enabling a rapid determination to be made for the normal component of the inductive velocity. Results are given for the calculations of a round wing and of a comparison of experiment and calculation on N. E. Kochin's theory ["Theory of the round wing," Collection of papers, **2**, 1949]. S. I. Kuznetsov

Courtesy Referativnyi Zhurnal, USSR

6452. Lockwood, V. E., Lift generation on a circular cylinder by tangential blowing from surface slots, NASA TN D-244, 38 pp., May 1960.

The results of an investigation on the generation of lift on a circular cylinder which had a fineness ratio 8 with a 2.5-diameter end plate are presented. The investigation was made to determine to what extent the lift and drag characteristics were influenced by slot number, slot combination, slot position, and blowing momentum coefficient. The results could be correlated with those of a jet-flap wing on the basis of equal drag.

From author's summary

6453. Jones, J. G., The pressure distribution on a wing with a subsonic leading edge in the presence of antisymmetric body shaping, *Aero. Quart.* **11**, part 1, 51–70, Feb. 1960.

The configuration considered is a lifting wing-body combination comprising a wing centrally mounted on a quasi-cylindrical body. The body shape is obtained by adding small distortions, antisymmetric with respect to the wing plane, to a basic circular cylinder.

The analysis is based on the linearized theory of compressible flow, and general expressions for the pressure distribution on the wing surface are obtained by application of Evvard's method. This paper is concerned with two particular cases in which explicit expressions involving tabulated functions for the pressure distribution due to the upwash field can be obtained. They are (1) body distortions in which the cross sections are circular and (2) body distortions of arbitrary cross section, but with the restriction that the leading edge is nearly sonic. Numerical results are given for several representative examples. J. R. Spreiter, USA

6454. Carafoli, E., and Nastase, A., Determination of the surface of a wing fitted with a pressure distribution separation ridge (in Roumanian), *Studii Si Cercetari Mecan. Apl.* **10**, 4, 995–1012, 1959.

Authors consider a thin wing in supersonic flow, whose projection on a horizontal reference plane, parallel to the undisturbed stream velocity, is triangular. The problem of determining the equation of the wing surface is equivalent to that of finding the disturbance velocity vertical component. Calculations are performed for a high order conical motion and assuming that the wing is fitted with a ridge which separates it into two regions with different variations of the axial velocity. As these velocities are given under the form of homogeneous polynomials, the incidence variation is found and then the wing surface is determined by integration. Particular attention is given to the case in which the ridge is centrally placed, as in such a case a new singularity of the form $\frac{1}{x}$ appears in the modified expression of the incidence. It is shown that wings with centrally placed ridges, whose surfaces tend to infinity at the origin can exist only for conical motions of the order $n \geqslant 3$. A continuity condition of the velocity horizontal components must be also satisfied at the origin.

Examples of calculation are given for several positions of the ridge including the centrally placed ridge which leads to the above-mentioned particular case.

T. Oroveanu, Roumania

6455. Lam, S. H., Interaction of a two-dimensional inviscid incompressible jet facing a hypersonic stream, AFOSR TN 59-274 (Princeton Univ., Dept. Aero. Engng. Rep. 447; ASTIA AD 212 708), 31 pp., Mar. 1959.

Idealized theoretical investigation of problem encountered when cooling on a blunt-nosed slender body is sought by jet injection in direction opposite to free stream. Assumptions that jet flow is inviscid, irrotational and incompressible can hardly be justified. Analysis, carried out in the hodograph, is limited to model problems of a simple source, and of a particular configuration representing a semi-infinite two-dimensional channel with constant cross section bound by flat plates of infinitesimal thickness; for the latter case alternatives are discussed according to whether flow separates from, or remains attached to, leading edge of channel. Results include shape of interface streamline and drag of configuration. Within the limits of analysis the essential parameter is the ratio between dynamic heads of jet and free stream, while mass flow rate is not important.

R. Vaglio-Laurin, USA

6456. Mirels, H., and Thornton, P. R., Effect of body perturbations on hypersonic flow over slender power law bodies, NASA TR R-45, 30 pp., 1959.

Inviscid hypersonic slender-body theory, in the limit as the free-stream Mach number becomes infinite, is used to find the effect of slightly perturbing the surface of slender two-dimensional and axisymmetric power law bodies. The body perturbations are assumed to have a power law variation (with streamwise distance downstream of the nose of the body). The results of numerical integrations of the governing ordinary linear differential equations presented for (1) the effect of boundary-layer development on the flow over two-dimensional and axisymmetric slender power law bodies, (2) the effect of very small angles of attack (on two-dimensional power law bodies), and (3) the effect of blunting the nose of very slender wedges and cones. Differential equations for finding the effect of a power law lateral perturbation of the centerline of a slender power law body are also formulated.

From authors' summary by F. A. Williams, USA

6457. Heyson, H. H., Jet-boundary corrections for lifting rotors centered in rectangular wind tunnels, NASA TR R-71, 91 pp., 1960.

A theoretical analysis provides values of the correction factor. The results indicate that at high speeds the corrections are the same as those for a wing, but that at low speeds, for the cases considered, there is a large tunnel-induced upwash at the rotor. Increasing the rotor size decreases the correction factors for the wide wind tunnels but has little effect upon the correction factors in deep narrow wind tunnels. The corrections are equivalent to a wind-tunnel-induced rate of climb, and considerable care will be required in the application to very low speed flight conditions.

R. C. Binder, USA

6458. McLemore, H. C., and Peterson, J. B., Jr., Aerodynamic characteristics of a large-scale unswept wing-body-tail configuration with blowing applied over the flap and wing leading edge, NASA TN D-407, 213 pp., Sept. 1960.

Tests were conducted in the Langley full-scale tunnel to determine the effects of blowing boundary-layer control on the aerodynamic characteristics of a large-scale, wing-body-tail configuration having an unswept wing with an aspect ratio of 2.86 and a thickness ratio of 0.04. The tests were conducted for a range of angles of attack from approximately $-4°$ to $23°$ for a Reynolds

number of approximately 5.2×10^6 corresponding to a Mach number of 0.08. Longitudinal stability and control characteristics were obtained for three tail heights, and lateral-control characteristics were obtained for two aileron and several spoiler configurations.

From authors' summary

6459. Crabtree, L. F., The rotating flap as a high-lift device, Aero. Res. Counc. Lond. Curr. Pap. 480, 28 pp., 1960.

This novel device consists of a small autorotating or driven auxiliary wing mounted below trailing edge of main wing. Experimental results obtained by other investigators sometime ago for lift, drag, and moment coefficients, as well as power required to rotate auxiliary wing, are presented, discussed, and analyzed. Data are for very low Reynolds numbers. For a 2.67 aspect ratio wing of NACA 23015 airfoil section with a full span flap of chord equal to 1/4 chord of wing, both wing and flap with end plates, a maximum lift coefficient as large as 3.8 together with a drag coefficient of about 1.38 is obtained at a Reynolds number of 140,000 based on the wing chord. Moment coefficients are large, for this case equal to about −1.8.

A theory for airfoil with rotating flap is developed and presented. One result is locus of flap position for maximum lift for a given angle of attack of main wing. The experimental data confirm this theoretical result. Authors state that because of very limited information and because of very low test Reynolds numbers much more experimental work is needed before application to reduction of aircraft approach speed can be seriously considered.

N. Tetervin, USA

6460. Turner, T. R., Low-speed investigation of a full-span internal-flow jet-augmented flap on a high-wing model with a 35° swept wing of aspect ratio 7.0, NASA TN D-434, 42 pp., Aug. 1960.

An investigation of a full-span internal-flow jet-augmented flap on a semispan jet-transport model has been made in the Langley 300-mph 7- by 10-foot tunnel. The large diving moments were trimmed by blowing over the deflected elevator or by downward blowing from a fuselage nose jet. The jet momentum coefficient range was from 0 to 6.33, the flap deflection range was from 0° to 70°, and the angle-of-attack range extended from as low as −25° to the stall angle.

From author's summary

6461. Stanisic, M. M., On the unsteady motion of a delta wing in supersonic flight, *J. Aero/Space* i. **27**, 5, 399–400 (Readers' Forum), May 1960.

Volterra's method of solution of the wave equation is used to obtain the acceleration potential, generated by a thin wing oscillating in a supersonic flow, by means of an integral superposition. This is an extension of the work of Heaslet-Lomax-Jones [NACA TN 1412, 1947; AMR 1(1948), Rev. 499].

J. P. Guiraud, France

6462. Richardson, D. W., Graphic presentation of optimum flight profiles, *Aero/Space Engng.* **19**, 9, 16–17, 46, Sept. 1960.

The technique of dynamic programing has been used to determine optimum flight profiles for interceptor aircraft. A further development of this method has resulted in a ground-based computer program which outputs a printed graphical presentation of the families of optimum flight profiles. This visual display performs a variety of functions, particularly those involving the problems of selecting general flight control laws and logic to be designed into airborne flight control computers. Typical tactical situations are presented and analyzed.

From author's summary

6463. Melik-Sarkisyan, Z. A., Motion of an aeroplane before a landing and its energy reserve when taking into account the wing's deflection (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Aviats. Tekhn.* no. 1, 78–86, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2466.

The equations of the motion of an airplane are investigated for the last part of its aerial flight before landing, to cover two cases: landing with the use of parachute braking, and high-speed landing (the parachute brake not being used). The reserve of mechanical energy of the airplane at the moment of touching down is determined as the sum of kinetic energy of the motion of the center of gravity, of the potential energy equal to the product of the weight of the airplane by the possible transposition of the center of gravity due to the cushioning effect of all-round compression, and of the internal energy of the wing determined by its deflection deformation at the moment of the airplane's touch-down. For some special cases comparisons are made of the results of calculations for the vertical velocity of drop in height at the moment of touching down, for the magnitude of the "coefficient of weighability" (the weight of the airplane, not equilibrated by the lifting power) and for the reserve of energy of the airplane with the corresponding magnitudes, recommended in the textbooks for calculating the chassis buffer action. The vertical velocity determined by the standard recommendations is considerably in excess of the velocity when using a parachute brake, but a high-speed landing with determined angles for loss of height may prove to be close to the real velocity. In actual conditions the "coefficient of weighability" at the moment of touching down is found to be smaller than the 0.25 recommended in the textbooks, more especially so in high-speed landings. The divergence in the magnitudes for the reserve of energy of an airplane determined by the author and in conformity with textbook recommendations is, in the main, due to differences in the magnitudes of the vertical landing speeds, already commented on in the paper, with a few corrections due to consideration of the energy of deformation of the wing. The author does not indicate how to use the full reserve of energy and the "coefficient of weighability" of the airplane for calculating the chassis buffer action, which is obtainable by computation.

G. S. Aronin

Courtesy Referativnyi Zhurnal, USSR

Book—6464. Etkin, B., Dynamics of flight—stability and control (Wiley Books in Space Technology Series), New York, John Wiley & Sons, Inc., 1959, xiv + 519 pp.

Author is to be congratulated for writing this text. The up-to-date treatment of the subject makes it a valuable contribution to the field. It is well suited as an undergraduate and first-year graduate text for students in aeronautical engineering and as a reference book for the industry. The book is organized into 14 chapters and 4 appendices. Good bibliographies and a list of symbols are included with each chapter. The following is an abbreviated description of the most important parts of the book: In the first chapter (Introduction) the general notions are defined and explained from the physical point of view. In chapters 2 and 3 (Static stability, and control), the concepts of balance, stability, moments, maneuverability are treated in detail. The influence of the various components (tab, elevator, thrust, etc.) are analyzed in detail. The concept of structural flexibility is introduced at this early point.

Chapter 4 (General equations of unsteady motion) derives the fundamental equations of motion, first for a rigid body, and then additional concepts such as spinning rotors, gyroscopic effects, structural deformations are introduced and their contributions are evaluated and discussed. The reader is given a build up of complexity from the simple model to the system concept. Chapter 5 is an extensive, detailed treatment of the stability derivatives. The aerodynamic transfer functions as well as aeroelastic derivatives are introduced.

Chapters 6 and 7 deal, as their title indicates, with the longitudinal and lateral stability of uncontrolled motion. The fundamental concepts of stability in linear systems are well covered and illustrated with examples. Chapter 8 (Some mathematical aids)

derives and applies the necessary mathematical tools such as Laplace transforms, transfer functions, random variables, power spectrum and response to random input. In chapter 9 (Response of the airplane to actuation of controls) author develops the theory and equations of longitudinal and lateral response on a system basis by considering the contributions of the elevator, aileron and rudder. Large disturbance maneuvers are also covered.

Chapter 10 (Flight in turbulent air) introduces turbulence and treats the response of an aircraft to atmospheric turbulence on the basis of power spectrum analysis. Chapter 11 (Inverse problems) is a short treatment of problems in control analysis and design where the motion is specified and guidance procedure is to be found. Chapter 12 (Automatic stability and control) treats feedback controls, system stability, Nyquist diagrams, gain and phase margins, the root-locus method and a brief discussion of automatic pilots.

In Chapter 13 (Specialization to missiles) the previously developed work for piloted aircraft is applied to nonrolling and rolling missiles. Chapter 14 (Machine computation and simulation) treats the use of digital and analog computers to obtain solutions to some of the previously derived equations. Appendix A is a short treatment of vector analysis. Appendices B, C and D contain 80 pages of extensive aerodynamic data presented in the form of graphs. H. H. Hilton, USA

6465. Shanks, R. E., and Smith, C. C., Jr., Low-speed measurements of static and oscillatory lateral stability derivatives of a 1/5-scale model of a jet-powered vertical-attitude VTOL research airplane, NASA TN D-433, 19 pp., Sept. 1960.

Force tests of the static and dynamic lateral stability characteristics of a VTOL airplane having a triangular wing mounted high on the fuselage with a triangular vertical tail on top of the wing and no horizontal tail have been made in the Langley free-flight tunnel. The static lateral stability parameters and the rolling, yawing, and sideslipping dynamic stability derivatives are presented without analysis. From authors' summary

6466. Jernell, L. S., and Wong, N., Investigation of the static longitudinal stability characteristics of a 0.067-scale model of a four-stage configuration of the Scout research vehicle at Mach numbers of 2.29, 2.96, 3.96, and 4.65, NASA TN D-554, 25 pp., Sept. 1960.

An investigation was made to determine the effect of fin area and second-stage flare angle on the static longitudinal stability of a 0.067-scale model of a four-stage Scout vehicle. The tests were performed at Mach numbers of 2.29, 2.96, 3.96, and 4.65 at Reynolds numbers of about 4.6×10^6 per foot. The effect of Reynolds number was investigated by conducting additional tests at Mach numbers of 2.29 and 3.96 at Reynolds numbers of 5.64×10^6 and 6.49×10^6 per foot, respectively. From authors' summary

6467. James, C. S., Aerodynamic performance and static stability at Mach number 3.3 of an aircraft configuration employing three triangular wing panels and a body of equal length, NASA TN D-330, 34 pp., Aug. 1960.

Wing-tunnel measurements of performance and static stability at combined angles of attack and sideslip were made on a supersonic aircraft configuration at a Mach number of 3.3 and a Reynolds number of 5.46 million. A maximum lift-drag ratio of 6.65 (excluding base drag) was measured at a lift coefficient of 0.100 and an angle of attack of 3.6°. The lift-drag ratio remained greater than 3 to a lift coefficient of 0.35. Estimated performance was in reasonable agreement with experiment. Static stability characteristics were favorable within the test range. Longitudinal and directional centers of pressure were close to the respective centroids of projected planform and side area. From author's summary

6468. Llewellyn, C. P., and Wolhart, W. D., Effect of Reynolds number on the lateral-stability derivatives at low speed of sweptback- and delta-wing-fuselage combinations oscillating in yaw, NASA TN D-398, 45 pp., Aug. 1960.

Results were obtained from the models oscillating in yaw over an angle-of-attack range from 0° to 32° for the delta-wing models and from 0° to 28° for the sweptback-wing model. The Reynolds number range was from 0.7×10^6 to 7×10^6 for the sweptback-wing model and from 0.9×10^6 to 9×10^6 for the delta-wing models. The tests were run for amplitudes of oscillation from 2° to 10° and reduced-frequency parameters from 0.028 to 0.113. From authors' summary

6469. Henderson, W. P., The longitudinal aerodynamic characteristics of a sweptback wing-body combination with and without end plates at Mach numbers from 0.40 to 0.93, NASA TN D-389, 22 pp., May 1960.

The effect of end plates on the longitudinal aerodynamic characteristics of a sweptback wing-body combination is presented for high subsonic speeds corresponding to a Reynolds number (based on the mean aerodynamic chord) of approximately 2.3×10^6. The wing had 45° sweepback of the quarter-chord line, an aspect ratio of 4, a taper ratio of 0.3, and NACA 65A006 airfoil sections parallel to the plane of symmetry, and was mounted near the rear of a body of revolution having a fineness ratio of approximately 8. From author's summary

Vibration and Wave Motion in Fluids

(See also Revs. 6159, 6405, 6548, 6572, 6594, 6655, 6662, 6664, 6665, 6716)

6470. Ablow, C. M., Wave refraction at an interface, *Quart. Appl. Math.* **18**, 1, 15–29, Apr. 1960.

A plane wave in one of two perfect gases moves toward the parallel plane interface between the gases. The wave is either continuous or headed by a shock front weak enough that entropy changes may be neglected. Using Riemann's solution of the appropriate hyperbolic partial differential equation, four equations are derived giving the details of the reflected and refracted wave motions. The equations are of first-order integrodifferential or implicit functional form depending on the boundary conditions and must be solved simultaneously for four functions of a single independent variable. The equations are suitable for numerical step-by-step solution. R. C. Binder, USA

6471. Cumberbatch, E., The impact of a water wedge on a wall, *J. Fluid Mech.* **7**, 3, 353–374, Mar. 1960.

This paper deals theoretically with the impact of a water wave on a plane wall. The shape of the wave before impact is considered to be a two-dimensional wedge which is assumed to strike a wall at right angles to its path. The wedge is assumed to be infinite in extent and to have a uniform translational velocity V before impact. The choice of a wedge shape enables the problem to be formulated in terms of a velocity potential for an irrotational flow $\Phi = \Phi(\lambda, \mu, t)$, where λ and μ are similarity variables defined by x/Vt and y/Vt, respectively. A solution is obtained by suitably joining solutions valid at large and small distances from the wall.

In the general problem of wave impact on a wall the total exchange of momentum with the wall up to a given time might be crudely estimated by supposing that all the fluid which would have crossed the plane of the wall had the wall not been there has lost all its momentum and the rest has lost none. The force on the wall could then be obtained by differentiation with re-

spect to time. The analysis of this paper indicates that in two cases (wedges of semi-angles 45° and 22.2°) the force is greater than the value so obtained by factors 2.4 and 1.6, respectively.

T. Hayashi, Japan

6472. Wiegel, R. L., Transmission of waves past a rigid vertical thin barrier, *Proc. Amer. Soc. Civ. Engrs.* **86,** WW 1 (*J. Water Harbors Div.*), 1-12, Mar. 1960.

Wave-tank data which show the transmission of waves past a barrier extending from above the surface to some distance below are compared with estimates of transmission coefficients based on ratio of total power to power transmitted below the level of the barrier. Approximation consists of neglecting the influence of the barrier on the flow pattern and power transmitted below the barrier.

C. Cox, USA

6473. Jarlan, G. E., Observations of surface waves at Ripple Rock underwater demolition, Nat. Res. Counc. Canada, Div. Mech. Engng. and Nat. Aero. Establishment, Quart. Bull. no. 1, 43-64, Jan./Mar. 1960.

The salient features of the wave system following an underwater explosion are briefly described and the mathematical treatment of such a system reviewed. The records of the surface fluctuations following an underwater explosion are given. The two first waves observed were of a complex nature but the celerity of the leading wave was similar to that for a solitary wave. The complicated boundary geometry and diffusion effect prevented the application of the mathematical treatment to the wave records although the period of a seiche induced by the explosion agreed roughly with that calculated using a simple geometrical approximation to the shape.

G. H. Lean, England

6474. Voitsenia, V. S., Plane problem on oscillation of a body under two surface-separating liquids, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **22,** 6, 1121-1140, 1958. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Waves generated by oscillations of a two-dimensional body under surface of separation of two liquids, with lower, heavier liquid of infinite depth, upper liquid of finite thickness, are investigated following method of Kochin. Solutions are first obtained in form of Fourier integrals for a pulsating vortex and source. Case of an oscillating body is then formulated as a Fredholm integral equation of the second kind with the density of a surface distribution of pulsating sources as the unknown function. Solution by successive approximations is suggested. It is found that on the free boundary and on the boundary of separation at a distance from the body the wave profiles are due to the superposition of two wave shapes, one of which is dominant on the free boundary, the other on the boundary of separation.

L. Landweber, USA

6475. Budiansky, B., Sloshing of liquids in circular canals and spherical tanks, *J. Aero/Space Sci.* **27,** 3, 161-173, Mar. 1960.

Theoretical calculations are made of the natural modes and frequencies of small-amplitude sloshing of liquids in partially filled circular canals (tubes) and spherical tanks, induced by lateral accelerations of the containers. The differential equations are those given by Lamb ["Hydrodynamics," page 363]. Boundary conditions and nondimensional values are introduced leading to an integral equation. To find the kernel function, conformal mapping is used. For circular canals the solutions are the odd-order Legendre polynomials. The first eigenvalue shows the fundamental frequency to be

$$w_1 = (g/R)^{1/2} \quad (R = \text{radius of canal})$$

A similar approach for the spherical tank provides modes and frequencies for the nearly full and half full cases. The nearly empty case has been solved by Lamb (p. 291). These results are being used as a basis for estimating frequencies for arbitrary depth of liquid. Discussion and numerical results are given at end of paper.

C. Jaeger, England

6476. Kochina, N. N., and Mel'nikova, N. S., Expansion of a piston in water, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23,** 1, 123-133, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The problem of a shock wave propagating in water caused either by a one-dimensional, by a cylindrical or by a spherical piston which changes its speed suddenly from one constant speed to another constant speed is analyzed. The partial differential equations (momentum, continuity and condition for isentropic change of state on either side of the shock) are reduced to three total differential equations by means of the assumption of similarity solutions. Using the boundary conditions and rather general equations of states and general equations for the internal energy, these equations are reduced to those describing the same problem in a gas. This is possible by defining a new variable in such a manner that the difference between the equation of state and the equation for the internal energy in the water and the gas disappears.

As a result, the shock propagation velocity, the pressure and density ratio before and immediately after the shock, as a function of the piston velocity is graphically given. For the one-dimensional case, the states do not vary, of course, except across the shock. For the cylindrical and spherical case, two graphs show the change of the velocity, the density and the pressure as a function of the distance behind the shock with a suitable defined shock strength function as a parameter.

Reviewer misses the value of the constants in the equations of state used for calculating the curves presented.

H. P. Eichenberger, USA

6477. Sretenskii, L. N., Calculations for the tangential forces of wave resistance on a sphere in motion in a circular orbit (in Russian), *Trudi Morsk. Gidrofiz. In-ta Akad. Nauk SSSR* **11,** 3-17, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2597.

The sphere, which is situated under the free surface of a heavy incompressible liquid makes a steady circular motion in a horizontal plane. The action of the sphere is replaced by the action of a dipole because, due to both the progressive motion of the sphere in an undelimited liquid and to the basis for the linear theory of waves, the potential of velocities of the disturbed motion of the liquid is determinable. A calculation is carried out for the horizontal component of the hydrodynamic force with the aid of this potential and the linearized expression for the pressure in the liquid; a form of analysis is furnished for the magnitude of the wave resistance presented in the shape of a series with Bessel's functions.

M. D. Khaskind

Courtesy Referativnyi Zhurnal, USSR

Fluid Machinery

(See also Revs. 6289, 6337, 6570, 6602, 6618, 6680)

6478. Iversen, H. W., Rolling, R. E., and Carlson, J. J., Volute pressure distribution, radial force on the impeller, and volute mixing losses of a radial flow centrifugal pump, *ASME Trans.* **82** A (*J. Engng. Power*), 2, 136-144, Apr. 1960.

The radial force in the test centrifugal pump as obtained from the measured pressure distribution in the volute is a reasonable design approximation to the measured radial force. The pressure measurement need not be elaborate since results for the test pump were obtained with simple volute side-wall pressure taps.

The analysis of the mixing process in the volute, while subject to many simplifications, resulted in volute pressure distributions and radial forces which were on the order of the measured radial forces. The analysis also yielded an explanation of the mechanical-energy losses due to mixing in the volute and a consequent

head-capacity modification to the impeller head that shows the influence of the volute in predicting the pump performance.

From authors' summary by S. I. Wiselius, Holland

6479. Coester, R., Theoretical and experimental investigations on cross-flow turbo-blowers (in German), ETH, Mitt. Inst. Aero., Zurich, no. 28, 57 pp., 1959.

A cross-flow blower has a rotor consisting of a cage-type blade cascade with blades parallel to the axial direction. Gas flows across the rotor in a plane perpendicular to the rotating axis, entering along one part of the periphery and leaving along another part.

Author suggests several designs with experimental verifications to improve the efficiency at low flow coefficient. A maximum efficiency of about 60% was arrived at for a region of volume flow lying somewhere between that of conventional turbo-blowers and piston-type or rotational-type compressors.

L. S. Dzung, Switzerland

6480. Clark, E. L., Jr., and Ordway, D. E., An experimental study of jet-flap compressor blades, *J. Aero/Space Sci.* **26,** 11, 698–702, 738, Nov. 1959.

Experimental results are given for increasing lift on airfoils in cascade by employing jet-flap principle. Each airfoil of modified NACA 65-12 (10) section had 4-in. chord with 0.013-in. trailing edge blowing slot directed down at 15.9° to chord line. When tested in low-speed cascade tunnel, increase in blowing quantity gave increases in pressure rise and turning angle through cascade.

While authors are primarily interested in using jet-flap technique to overcome "rotating stall," reviewer believes that it is of more interest for increasing pressure rise through stage of large fan or compressor. Therefore results from proposed extension of experiments to single-stage compressor are welcomed, as these tests would permit more accurate assessment of practical merits of such systems.

F. G. Blight, Australia

6481. Krivoshein, V. F., The effect of profile change in the elementary stage blading of axial compressor upon its characteristics (in Russian), *Teploenergetika* no. 7, 32–36, July 1959.

Using simplifying assumptions of small variations, the author derives approximate formulas for the calculation of changes in basic parameters of an elementary stage of axial-flow compressor when varying the stagger angle of the stage cascades. Several diagrams indicate relatively good agreement between calculated and measured values.

J. Polasek, Czechoslovakia

6482. Acton, O., Vane angle design of an axial compressor blade cascade (in Italian), *Aerotecnica* **39,** 2, 66–69, Apr. 1959.

6483. Arsen'ev, L. V., Calculation for the portion of a turbine between the inlet and the outlet valves on partial loads (in Russian), *Nauchno-Tekhn. Inform. Byul. Leningrad Politekhn. In-ta* no. 3, 60–66, 1958; Ref. Zh. Mekh. no. 6, 1959, Rev. 6209.

The proposed method of calculation for the characteristics of turbines is based, in common with other known methods, on the application of the equation of continuity for each pair of adjacent rims. This equation enables a linkage to be made of the degree of decrease of pressure in a pair of adjacent rims in the partial and calculation regimes through the peripheral velocity in these regimes. The fundamental equation for the calculation has the form

$$x_{pi}{}^{*} = \frac{\beta_{pi}{}^{*}}{\beta_{pi_0}{}^{*}} = x_{Hi}{}^{*} \; \frac{e_{Hi}{}^{*} E_{Hi}}{e_{Hi_0}{}^{*} E_{Hi_0}}$$

Here x_{pi} is the relation of the coefficients of delivery in the partial and calculation regimes in the ith working rim, $x_{Hi}{}^{*}$ also in the ith guiding rim, $e_{Hi}{}^{*}$ and $e_{Hi_0}{}^{*}$ are degrees of lowering the pressure in the guiding rim in the partial and the calculation regimes respectively, E_{Hi} and E_{Hi_0} are certain complexes of the parameters in the guiding rim in the partial and the calculation regimes respectively, when

$$E_{Hi} = f\left(e_{Hi}{}^{*}, \frac{u}{c_1}, \varphi\right)$$

If the necessary parameters in the calculation regime are known, then, having assigned the magnitude $e_{H_1}{}^{*}$ in the first guide for the rim and the relation u/c_1, in the first stage, it is possible to determine the degree of lowering of the pressure in the first working rim. Then having recorded the equation analogous to the previous one, for the next pair of rims it is possible, knowing the degree of lowering in the first working rim, to determine the magnitude of $e_{H_1}{}^{*}$ in the second guiding rim and so forth, up to the last rim. If the distribution of the falls of pressure over the whole of the rims is known then the subsequent calculations for the parameters of the turbine characterizing the investigated regime of work offer no difficulties. Nomograms are furnished, which simplify the application of the given method for the determination of falls in pressure in the rims to unrated regimes and also the sequence of the calculations in question. Gas dynamic functions are not used in the proposed method though their application would ordinarily simplify the calculations to an appreciable extent. The method was worked out for transonic velocities of the flow in the portion of the turbine between the inlet and outlet valves through which the steam passes. The selection of velocity coefficients φ and ψ in the jet apparatus and the working wheel in variable regimes is not investigated.

V. Kh. Abiats

Courtesy Referativnyi Zhurnal, USSR

6484. Dechev, V. I., Investigation of turbine and pump cascade profiles in a flow of viscous incompressible liquid (in Russian), *Trudi Khar'kovsk. Politekhn. In-ta* **16,** 145–157; 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2572.

The author derives formulas for the forces acting on the profile of a cascade in a plane flow of a viscous liquid, incorrectly applying the theory of impulses in the operation. The example given shows the divergence between the experimental and the calculated magnitudes of the forces, though when computing the latter the author used the empirical formulas for the outlet angle of the flow and for the thickness of the boundary layer. For the present-day position on this problem the following literature may be referred to: the works of L. G. Loitsianskii [The mechanics of liquid and gases, Moscow, Gostekhteorizdat, 1957] and of M. E. Deich [Technical gas dynamics, Moscow-Leningrad, Energoizdat, 1953].

G. Yu. Stepanov

Courtesy Referativnyi Zhurnal, USSR

6485. Wrench, J. W., Jr., Calculation of Goldstein factors (AML problem 42.1-54), David W. Taylor Mod. Basin Rep. 1111, 25 pp., Sept. 1955.

Purpose of the report is the calculation of the original expression of the Goldstein factor for screw propellers with various number of blades and for a greater range of the factor $\mu_0 = \omega R/v$, ω being the angular velocity, R the radius of the propeller and v the velocity of advance of the propeller.

The values of the approximate expression of the coefficients $a_m{}^{*}$ given by Goldstein are calculated on the DTMB UNIVAC for a range from 3 to 6 of the number of blades and μ_0 ranging from 0.25 to 6. Desk-machine calculations have revealed that this approximation leads to unacceptable errors when μ_0 is less than 2.

Author presents a method for solving the infinite system of equations for the coefficients a_m by successive approximation when μ_0 exceeds unity. On a particular numerical case he gives the derived successive approximation of the a_m's. The values of the numerical differences between the coefficients $a_m{}^{*}$ calculated according to Goldstein and the coefficients a_m by the author are indicated, because the terms of the Goldstein factor are also com-

puted on the UNIVAC for the above-mentioned ranges using the coefficient a_m*. Tables of these terms are appended.

Some numerical values of the Goldstein factor are given for comparison calculated both with a_m* and a_m.

T. Gerey, Hungary

Book—6486. Marinescu, A., Theory of helicopter [Teoria elicopterului], Bucharest, Editura Academiei Republicii Populare Romine, 1960, 267 pp., 12.60 Lei. (Paperbound)

This book is a brief but complete presentation of the main aspects of the modern theory of helicopter. The subject is divided into the following chapters: Chap. I, Preliminary notions; Chap. II, Aerodynamics of helicopter rotors; Chap. III, Considerations on the aerodynamic reactions on the rigid blade rotors, with applications to antitorque rotors, convertoplane propellers and VTOL aircrafts; Chap. IV, Calculation of helicopter performances; Chap. V, Helicopter stability; Chap. VI, Strength calculation of helicopter rotors; Chap. VII, Vibrations of helicopter rotors; Chap. VIII, On the vibrations of other helicopter parts.

Main characteristic of the book lies in the practical value of the solutions obtained and in the high level theoretical treatment of the problems. Author's own contribution is to be noted particularly in the chapters concerned with the stability of helicopters with cantilever blade rotors, the blade being considered as elastically deflected from the rotation plane, as well as in those dealing with rotor and fuselage vibrations.

Reviewer believes that owing both to the manner of approaching the problems and to the simple solutions obtained, book may be of help to those working in the field.

S. N. Savulescu, Roumania

6487. Gessow, A., and Gustafson, F. B., Effect of blade cutout on power required by helicopters operating at high tip-speed ratios, NASA TN D-382, 18 pp., Sept. 1960.

A numerical study was made of the effects of blade cutout (varying from 0 to 0.5 of the rotor radius) on the power required by a sample helicopter rotor traveling at tip-speed ratios of 0.3, 0.4, and 0.5. Substantial reductions in profile-drag power (as much as 55% at a tip-speed ratio of 0.5) were indicated when the cutout was varied from 0 to 0.3 of the rotor radius. Cutout also effected significant increases in ability of the rotor to overcome parasite drag and reduced rotor stall. From authors' summary

Flow and Flight Test Techniques and Measurements

(See Revs. 6145, 6457, 6459, 6506, 6529, 6571, 6576)

Thermodynamics

(See also Revs. 6148, 6165, 6387, 6415, 6418, 6424, 6504, 6506, 6519, 6530, 6554, 6566, 6570, 6650, 6715, 6724)

Book—6488. Gilmont, R., Thermodynamic principles for chemical engineers, Englewood Cliffs, N. J., Prentice-Hall, Inc. 1959, xxv + 339 pp. $11.

Author emphasizes the fundamental approach to thermodynamics, and for this reason the book should be of value to those seeking a firm grasp of the basic principles.

Since particular emphasis is placed on the development of chemical thermodynamics, third-year chemistry and chemical engineering students should find the book a very valuable reference for their physical chemistry course.

A serious shortcoming of the book is the lack of sample problems. Usually, considerable aid to the learning process of engineering students is given through the solution of illustrative problems. However, this shortcoming can be easily remedied by the instructor. Not much emphasis is placed on chemical reaction equilibria, a subject of considerable interest and importance to chemical engineers.

Chapter 10, "Generalized charts of thermodynamic functions," should be of interest to the engineering student. Author should be commended for including a chapter on irreversible thermodynamics. Reviewer agrees with author's belief that this subject will be of considerable interest and importance in the future to the chemical engineer. K. Nobe, USA

6489. Brown, G. M., Thermodynamics, *Indust. Engng. Chem.* **52**, 5, 451-455 (Chemical engineering fundmentals review), May 1960.

Book—6490. Golubev, I. F., Viscosity of gases and gaseous mixtures: A reference manual [Vyazkost' gazov i gazovykh smesei: Spravochnoe rukovodstvo], Moscow, Gosudarstvennoe Izdatel'stvo Fiziko-Matematichoskoi Literatury, 1959, 375 pp. 12.40 r.

The rapid progress in many branches of technology is being seriously hampered by inadequate information about properties of materials. Among those, the viscosity of gases and gaseous mixtures plays an important part. In spite of many and extensive efforts reflected in a staggering number of papers scattered throughout hundreds of scientific journals, the user who needs reliable values does not know where to turn. The author of this small booklet has tried to facilitate this for him, and one would like to think that volumes similar in scope, and covering all the remaining transport properties of gases, will soon appear in this country. One would, however, be hard put to it to think of a publisher who could profitably undertake this essential task, and one cannot but hope that official support for such a venture would be forthcoming.

The booklet is well conceived. It takes the reader from the basic concept through a review of theories of viscosity and, very importantly, through a review of experimental methods. It discusses experimental results in great detail, culminating in 115 + XI tables in the text and the appendix, and in 161 references to original papers. These contain many results and references to Soviet Russian work, most of which is largely unknown, and somewhat inaccessible in the West.

Although one would admire the scope and sweep of the organization of the subject matter in this booklet, one cannot extend one's admiration to the details of execution and presentation. The manual bears all the marks of hasty writing and gives a distinct impression of mustiness when it discusses theories of viscosity, or even theories of experimental viscometry. The author, a well-known experimenter, loses himself, rather uncritically, in various empirical correlation schemes without attempting to discover which of them should be retained, and which should be discarded. He is content with reproducing a good deal of subject matter from older publications, without attempting to improve them in the light of later developments. For example, when discussing the theory of capillary measurements, he quotes S. Erk [Forschungsheft no. 288, 1927] almost verbatim, and ignores the later contributions to the theory of the inlet-length correction due to L. Schiller, S. Goldstein, and G. S. Atkinson, and H. L. Langhaar.

The references are also inadequate and it is clear that the author did not seek them out systematically, relying rather on what came across his desk in a random manner. For example, in quoting the reviewer's work, the author discusses his earliest work which was improved and re-evaluated in subsequent publications, not mentioned. Apart from being incomplete, and unselective, the references contain numerous typographical errors.

Summing up, the reviewer wishes highly to commend the author in attempting an important, and needed task. The result is admittedly imperfect and does not accurately reflect the present state of knowledge on the subject. J. Kestin, USA

6491. Johnson, E. F., Molecular transport properties of fluids, *Indust. Engng. Chem.* **52,** 5, 447–450 (Chemical engineering fundamentals review), May 1960.

6492. Heims, S. P., Effects of chemical dissociation and molecular vibrations on steady one-dimensional flow, NASA TN D-87, 66 pp., Aug. 1959.

As the temperature of a gas increases its thermodynamic properties depart significantly from those of an ideal gas. Present paper considers the effects of harmonic molecular vibration and dissociation on the steady, one-dimensional, adiabatic flow of a homonuclear, diatomic gas such as O_2, N_2, etc. Comparison with exact thermodynamic data for O_2 indicates that neglect of electronic excitation is unimportant below about 5000°K. Vibrational energy levels are assumed to follow a Boltzmann distribution characterized by a vibrational temperature generally different from the translational temperature.

A general expression for entropy in nonequilibrium flows is developed in terms of density, translational temperature, degree of dissociation, and vibrational energy referenced to given initial conditions. Explicit relations for one-dimensional flow properties are derived for the four possible isentropic flows: frozen reaction and vibration, equilibrium reaction and vibration, frozen reaction and equilibrium vibration, and equilibrium reaction and constant vibrational temperature.

Where chemical and vibrational relaxation occur, a condition other than the initial state must be specified for a solution. For no dissociation, equations are presented for constant pressure, density, temperature, and streamtube area. Finally simultaneous chemical and vibrational relaxation behind a normal shock in a constant area duct is discussed. Relations are developed from which the vibrational energy and degree of dissociation can be determined from simultaneous pressure and temperature measurements behind the shock. The converse problem is also briefly discussed, based on a degree of dissociation-distance relation that accounts for dissociation-vibration coupling.

As author indicates, his approach lies between those using tabulated thermodynamic properties which include electronic energy changes, higher-order corrections, etc., and those employing Lighthill's ideal dissociating gas model.

H. E. Brandmaier, USA

6493. Lutz, O., On the thermo-gas dynamics of flow in cylindrical tubes (in German), *Ing.-Arch.* **28,** 178–183, Mar. 1959.

Solution curves of a differential equation are constructed which relate the dynamic head to the stagnation temperature in a gas flowing along a constant temperature tube of constant cross section. The differential equation is formulated from a rather empirical relationship between heat and pressure losses in turbulent flow which was suggested by Reynolds. The phenomenon of thermal choking is discussed in terms of these curves.

A. F. Pillow, Canada

6494. Ishigai, S., and Sekoguchi, K., On the flashing flow of saturated water through horizontal pipes, *Bull. JSME* **2,** 8, 563–568, Nov. 1959.

The energy equation for a single-phase flow is usually applied to predict the pressure drop of the flashing flow of steam-water mixtures. This energy equation must be solved in such a way that the solution will not only give the relations among the pressure drop, the mass flow rate and other factors, but also be applicable without discrepancy to the critical flow.

In this paper, a new correction factor for friction loss is introduced to satisfy the aforementioned requirements and is determined by experiment. This factor is the ratio of the true coefficient of friction for the flashing flow to the calculated coefficient of friction for a homogeneous flow. An empirical formula is determined for it from the experiments over a pressure range from about 3.5 to 8 atm, a range of the length of horizontal pipe line from 1 to 10 m, and a range of the internal diameter of pipe 10.0, 6.2 and 4.1 mm.

From authors' summary by T. Hayashi, Japan

6495. Kohn, A., and Philibert, J., Contribution to the study of solidification of alloys (in French), *C. R. Acad. Sci., Paris* **249,** 20, 2073–2075, Nov. 1959.

6496. Stil'bans, L. S., and Fedorovich, N. A., The operation of refrigerating thermoelectric elements in nonstationary conditions, *Soviet Phys.-Tech. Phys.* **3,** 3, 460–463, Nov. 1958. [Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **28,** 3, 489–492, Mar. 1958 by Amer. Inst. Phys., Inc., New York, N. Y.]

The operation of a refrigerating thermoelectric element under nonstationary conditions is studied theoretically and experimentally. It is shown that the inertia of the thermoelectric element is proportional to the square of its linear dimensions. The inertia also depends on the strength of the operating current, and can be decreased by a large factor by using a pulsed current with amplitude exceeding the optimum value of the current for stationary operation. With the pulsed type of operation the thermoelement can give for a short time interval a cooling effect considerably larger than the maximum effect obtainable under stationary conditions.

From authors' summary

6497. Somers, E. V., and Kelly, J. C. R., Thermoelectric power, *Mech. Engng.* **82,** 7, 40–42, July 1960.

6498. Pereleshina, A. P., The results of an experimental investigation of thermoelectromotive force in thermistors (in Russian), *Inzhenerno-Fizicheskii Zh.* **3,** 4, 119–122, Apr. 1960.

6499. Thring, M. W., Potential heat in fuel: direct conversion to electricity, *J. Inst. Fuel* **33,** 233, 294–300, June 1960.

6500. Danishevskii, S. K., Selection and calibration of tungsten and molybdenum wire for thermocouples, *Measurement Techniques* no. 5, 333–338, June 1960. (Translation of *Izmeritel'naya Tekhnika, USSR* no. 5, 25–29, May 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Heat and Mass Transfer

(See also Revs. 6144, 6160, 6214, 6228, 6235, 6309, 6370, 6371, 6372, 6373, 6374, 6394, 6415, 6418, 6434, 6435, 6439, 6445, 6446, 6509, 6528, 6648, 6675, 6695, 6711, 6713, 6720, 6722, 6724)

6501. Eckert, E. R. G., Hartnett, J. P., Irvine, T. F., Jr., and Sparrow, E. M., Heat transfer, *Indust. Engng. Chem.* **52,** 4, 327–339, Apr. 1960.

6502. Stolz, G., Jr., Numerical solutions to an inverse problem of heat conduction for simple shapes, *ASME Trans.* **82** C (*J. Heat Transfer*), 1, 20–26, Feb. 1960.

Problem deals specifically with a spherical solid having constant physical properties and no internal heat generation. The object of the inverse problem is to determine the time variation in heat flux at the surface of the material if an interior temperature is known as a function of time. A numerical method is proposed which involves inverting the usual direct problem where the surface heat flux is specified. A numerical example is carried out for quenching a sphere in oil.

R. Siegel, USA

6503. Swann, R. T., Calculated effective thermal conductivities of honeycomb sandwich panels, NASA TN D-171, 24 pp., Dec. 1959.

Heat transfer through a honeycomb sandwich panel is calculated taking into account modification of conduction in the cell walls due to radiation. The interaction of conduction with radiation creates a nonlinear temperature gradient in the walls and it is shown that the effect on the total heat transfer may be a major one.

Conduction through the air in the cells is estimated separately. Convection within the cells is neglected; where the temperature gradient is large and downward, e.g. on the lower surfaces of wings, etc., this effect will have to be added.

Comparison with experiment shows reasonable agreement. Nondimensional curves for quick estimates are given.

E. Pribram, England

6504. Yang, K.-T., and Szewczyk, A., An approximate treatment of unsteady heat conduction in semi-infinite solids with variable thermal properties, *ASME Trans.* **81** C (*J. Heat Transfer*), 3, 251-252 (Tech. Briefs), Aug. 1959.

This very short paper presents an approximate procedure for the calculation of unsteady heat conduction in semi-infinite solids with variable thermal properties. It is claimed to be an improvement over previous efforts in this area since it yields physically sensible results for cases where thermal properties have a large dependence on temperature. Instead of using polynomials to represent an unsteady temperature profile an exponential form is used. Good agreement is shown for several cases where the method of the paper is compared with exact solutions.

W. S. Aiken, Jr., USA

6505. Enig, J. W., A method for the rapid numerical solution of the heat conduction equation for composite slabs, U. S. Nav. Ord. Lab. Rep. 6666, 19 pp., Aug. 1959.

Problem is temperature in a slab of two materials, with one face insulated and the other face heated, with heat flux being a nonlinear function of surface temperature. Author's method starts with the analytic solution (in series form) for problem when temperature of heated face is a known function of time. Using the relation between heat input and surface temperature, an equation for the actually unknown surface temperature is derived in the form of a nonlinear integral equation. This is solved by a numerical method and the resulting value for surface temperature is used in the analytic solution to calculate the temperature at other points.

The numerical solution for surface temperature is based on replacing the first derivative by the difference ratio and then using Newton's method of solution. There is a discussion of the error due to using only a finite number of terms of the infinite series, but no discussion of the error introduced by replacing the derivative by the difference. Comparison with exact solution for a simple case (constant heat flux) gives good agreement, and for a more complicated case agreement was obtained with a numerical solution of the usual kind for heat conduction equations using an implicit finite difference scheme. Present method is said to be much faster than the usual implicit or explicit difference methods, especially when temperature at only a few points is required.

H. G. Landau, USA

6506. Hansen, C. F., Early, R. A., Alzofon, F. E., and Witteborn, F. C., Theoretical and experimental investigation of heat conduction in air, including effects of oxygen dissociation, NASA TR R-27, 43 pp., 1959.

One-dimensional, nonsteady heat transfer in a gas at constant pressure is analyzed for the case in which the coefficient of thermal conductivity and the thermal diffusivity are functions of temperature, the medium is semi-infinite, the initial temperature is uniform, and the boundary temperature is constant. To facilitate obtaining numerical solutions, the thermal diffusivity is taken to be directly proportional to the integral of thermal conductivity for temperatures less than temperatures at which oxygen dissociation occurs, and is taken to be constant for temperatures at which oxygen dissociation occurs. Obtained solutions are combined with measured heat fluxes in a shock tube in order to obtain values of the integral of the thermal conductivity for realized air temperatures. Percentage of deviations of experimental results from theoretical predictions are of same magnitude (20%) as uncertainties in measurements and in theory.

E. L. Knuth, USA

6507. Goodman, T. R., Nonsteady incompressible heat transfer for arbitrary bodies and all Prandtl numbers, AFOSR TN 60-843 (Allied Research Associates, Inc., Boston, Mass.), 31 pp., Sept. 1960.

The title problem is solved using an integral method, and ignoring viscous dissipation. A partial differential equation is derived which yields as special cases Lighthill's nonuniform heat-transfer formula and the nonsteady heat conduction in a slab. The differential equation is then specialized to the nonsteady but uniform heat transfer on a flat plate. Comparisons with other solutions are made when available, and it is shown that the integral method produces accuracy of a few per cent in these limiting cases. Finally, the heat-transfer formula which has been derived is used to calculate the temperatures in a thin skin subject to a jump in external velocity.

From author's summary

6508. Guinn, G. R., Aerodynamic heating charts for solid propellant rocket motors, *ARS J.* **30,** 8, 776-778 (Tech. Notes), Aug. 1960.

The solutions are presented for transient temperatures in a semi-infinite composite solid when the heat input is approximated by a fifth-degree polynomial of time. These solutions have been numerically evaluated and are presented in charts from which the temperature history at several locations in a typical liner-propellant combination can be computed for a given aerodynamic heating input.

From author's summary

6509. Hellwege, K.-H., Knappe, W., and Semjonow, V., Quasi-stationary measurement of specific heat and heat conductivity in plastics (in German), *Z. Angew. Phys.* **11**, 8, 285-290, Aug. 1959.

Method used consists of supplying heat at an intermediate radius of a circular cylindrical specimen through a resistance coil, and measuring the rise of temperature at the axis and also the temperature difference between two thermocouples at different radii outside the coil. Heat flow at the specimen surface is prevented by a "guard-ring" jacket and by testing in a high vacuum. Paper describes experimental techniques both of testing and of producing the specimens, which can be either cast or extruded. Author's error analysis concludes that accuracies of determination of specific heat and thermal conductivity are plus or minus three per cent and plus or minus twenty per cent, respectively. Results from extruded specimens show greater deviations through the many additional variables introduced into the production process. Results are given for some commercially available plastics over the range -150°C to $+100^\circ$C. Disagreement with work of previous investigators is shown in the transformation region of silicon compounds.

D. G. Wilson, England

6510. Sokolova, I. N., Calculation of the heating of shells at high speeds, *ARS J.* **30**, 4, 375-379, Apr. 1960. (Translation of *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Energetika i Avtomatica* (*Bull. Acad. Sci. USSR, Div. Tech. Sci., Power and Automation*), no. 3, 90-94, 1959.

This paper describes a method using polynomial approximations for obtaining approximate solutions of the one-dimensional heat-conduction equation subject to general boundary conditions. Com-

parison of the approximate and exact solutions for some simple cases is presented.

While the method is applicable to the title problem under the simplifying assumptions stated in the text, no numerical results for such a problem are given.

Similar methods have been reported in several papers [AMR **11** (1958), Rev. 651; AMR **10**(1957), Rev. 2291; and AMR **3**(1950), Rev. 1544] to which no reference is made. A novel feature of the present treatment is the use of separate parabolas, each covering a part of the range, instead of a single higher-order polynomial for increased accuracy. W. Squire, USA

6511. Eriksson, B. E., An experimental study of heat transmission in the surface layers of the Skagastöl glacier (in English), *Kyltekn. Tidskr., Stockholm* **18**, 2, 21–30, Apr. 1959.

Paper is third in series by author on this subject. A heat balance is formulated for the Skagastöl Glacier. Individual terms in the heat balance are formulated and discussed. Observed meteorological data were used in the calculations. Temperatures to a depth of 20 m were determined for two periods of several months duration. Problem of finding temperature distributions was complicated by the irregular nature of the surrounding temperature. Author uses a hydraulic analogy to solve the transient heat balance and describes the procedure in detail. A comparison between calculated and measured discharge rates shows good agreement. Reviewer feels that paper suffers from lack of experimental data to substantiate methods. Although paper is of primary interest to glaciologists, engineers will be interested in the use of the hydraulic analogy. R. W. Van Aken, USA

6512. Seban, R. A., The influence of free stream turbulence on the local heat transfer from cylinders, *ASME Trans.* **82** C (*J. Heat Transfer*), 2, 101–107, May 1960.

The local heat-transfer coefficients, the local pressure coefficient and the local recovery factor were each measured, for air flowing transversely, at 150 to 350 ft/sec, on three cylinders, ... one of elliptic cross section (2 in. × 6 in.), one of circular cross section (1.25-in. diam), and one of circular cross section (1.87-in. diam) fitted with a downstream splitter plate. The turbulence was produced by screens of several mesh dimensions, with most tests made with ¼-inch mesh. Results for tests without screens are also reported, but a low-frequency oscillation in the wind tunnel then existed.

In spite of the low-frequency oscillation of the flow without the screens, such flow produced results qualitatively different in important respects from the results with the screens. The addition of screens produced effects on heat transfer qualitatively similar to those reported by Giedt, namely greater heat transfer at and near the stagnation point, and, for angles of $90°$ to $110°$ from the stagnation point, a greater and more rapidly rising heat transfer with increasing angle, then a drop in heat transfer with angle for angles above $110°$.

Lack of knowledge of the intensity and scale of turbulence seriously limits the general value of the results, as the author admits. An earlier reference, not mentioned by the author, gives average heat-transfer coefficients as a function of turbulence intensity, ... namely E. W. Comings, J. T. Clapp, and J. F. Taylor, "Air turbulence and transfer processes—Flow normal to cylinders," in *Indust. Engng. Chem.* **40**, no. 6, June 1948, p. 1076.
R. H. Norris, USA

6513. Wong, T. J., Goodwin, G., and Slye, R. E., Motion and heating during atmosphere reentry of space vehicles, NASA TN D-334, 18 pp., Sept. 1960.

Flight-corridor depths are shown to decrease as reentry speeds increase but may be widened three- to fivefold by use of aerodynamic lift. Nonequilibrium boundary-layer flows will be encountered during reentry from space voyages but the effect upon the convective heating appears to lessen the severity of the heating problem. Radiative heating may be as severe as convective and unfortunately occurs at the same time in the flight path.
From authors' summary

6514. Griffith, J. D., and Sabersky, R. H., Convection in a fluid at supercritical pressures, *ARS J.* **30**, 3, 289–291 (Tech. Notes), Mar. 1960.

Earlier heat-transfer studies near the critical point of several fluids have reported heat-transfer coefficients which are too high to be explained by the strong temperature dependence of the fluid properties. It has also been suggested that these large increases are due to the formation and collapse of low density bubble-like aggregates near the heating surface, very much like the vapor bubbles in nucleate boiling. This work was undertaken to determine visually if the bubble-like motion existed in the heating region, and if so what the effect was on the heat-transfer rate.

A horizontal electrically heated wire was mounted inside a high pressure cylinder containing Freon 114A (P_{crit} = 478 psia, T_{crit} = 294°F). A schlieren system was used to view and photograph the heated wire through quartz windows in the ends of the cylinder. The heat flux was measured as a function of the wire temperature.

The results presented in the article are (1) a graph of heat flux versus wire temperature at various P and T bulk conditions above and below the critical point and (2) two schlieren photos at a fluid bulk temperature of 80°F for a supercritical and a subcritical pressure. The subcritical photo was of nucleate boiling. Both photos were similar in appearance thus showing the presence of bubble-like activity at supercritical pressures. There were no observable increases in the heat transfer when the bubble-like aggregates were present at the supercritical pressures and/or temperatures. The authors feel that these results may not necessarily be true of other test conditions or other fluids. They are planning to extend the work to other fluids. J. M. Savino, USA

6515. Zakkay, V., Heat transfer at a corner, *J. Aero/Space Sci.* **27**, 2, 157–158 (Readers' Forum), Feb. 1960.

Experimental results are presented on the heat transfer from an axially symmetric blunt body with a $50°$ cone half-angle followed by a cylindrical section. The tests are performed at Mach number 8 and at two Reynolds numbers, 0.16×10^6/in. and 0.12×10^6/in., with stagnation-temperature-to-wall-temperature ratio of 2.3.
S. Eskinazi, USA

6516. Covert, E. E., On an approximate solution of the energy equation and its application to the McClimans' data, Mass. Inst. Technol., Naval Supersonic Lab. TR 403, 27 pp., Nov. 1959.

The energy equation for low-speed pipe flow is transformed by means of the Kirchhoff transformation. An approximate solution is found in terms of the variable which indicates that the usual parameters in forced convection can be generalized to allow for high rates of heat transfer and variable fluid properties.
From author's summary

6517. Roderick, D. J. I., Murray, M. V., and Wall, A. G., Heat transfer and draught loss in the tube banks of shell boilers, *J. Inst. Fuel* **32**, 225, 450–463, Oct. 1959.

Experiments were made to determine value of "a" in relationship $U_c = ak\, Re^{0.8}/D$ where U_c is convection coefficient between gas and water, k is gas conductivity, D the smoke tube diameter and Re the Reynolds number.

Measurements were taken of heat loss and pressure drop of flue gases passing through smoke tubes of an economic boiler. Heat loss was determined by metering gas flow and by suction pyrometer temperature readings at tube inlet and outlet. Pressure losses were determined by pressure tappings in combustion chamber and in tube wall near exit.

Heat losses corrected by subtraction of calculated radiation loss

expected from gas combustion were used to estimate "a" in 14 ft 6 in. long tubes of 3 in., $2\frac{1}{2}$-in. and $1\frac{3}{4}$-in. O.D. The 3-in. tube sometimes was tested with "retarders."

Results showed "a" to have value of 0.022 and measured pressure drops compare well with those estimated from graphs given in Appendix. "Retarders" increase "a", a value as high as 0.038 being measured in one case.

Design curves of the effect of tube length on gas temperature drop based on "a" = 0.022 are given in Appendix for tubes of 0.5-3.5-in. diameter, but reviewer wonders if a range of 7 : 1 in D is justified when based on experiments covering less than 2 : 1, especially when no temperature traverse was made along tube to determine effect of entry. D. B. Leason, England

6518. Hausen, H., and Duwel, L., Application of the equivalent diameter in equations for the heat transfer to unsymmetrically heated plain channels (in German), *Kältetechnik* **11**, 8, 242-249, Aug. 1959.

For channel flow, is equivalent diameter the same if one or two sides are heated? To answer, authors restudy turbulent boundary layer with heat transfer when viscosity varies. An expression is derived relating velocity distribution, viscosity coefficient, and a turbulent viscosity coefficient; also an expression for wall heat transfer in terms of velocity profile.

Authors carry out calculation for constant μ and turbulent $u \sim y^{1/7}$. They find that one- and two-sided heating results coalesce as Re and/or Pr become very large. Then equivalent diameter may be used in computing Re for use in calculating Nu.

Reviewer notes that much rests on claim that knowing $u(y)$, e.g., from experiments, the ratio of turbulent to molecular momentum exchange can be found. In fact, authors' Eq. [13] contains three quantities, not just the two mentioned, so all that can be found is a relation between the ratio and the unknown (in general) distribution of μ. In worked example μ = constant, so difficulty is avoided. M. Finston, USA

6519. Maikapar, G. I., On laminar flow in pipes of liquids with different viscosity (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 7, 108-114, July 1958.

Laminar equations of motion for channel and pipe flow are solved for various boundary conditions, assuming central core flow separated from walls by symmetrical or annular region filled with different fluid. Density and transport properties for each fluid are assumed constant, but dissipation terms are included in energy equation. Pressure (and temperature for case of insulated walls) is assumed to vary linearly in flow direction. Author finds pressure drop and heat transfer to depend strongly on viscosity, but only weakly on heat conductivity, of fluid adjacent to wall.

D. Coles, USA

6520. Bosworth, R. C. L., and Groden, C. M., Thermal transients associated with natural convection, *Austral. J. Phys.* **13**, 1, 73-83, Mar. 1960.

Authors propose, without any consideration of precise physical significance of model, that thermal transients in natural convection should be investigated using an analogous electric circuit consisting of a separate convection element (resistance and inductance in series) and conduction element (resistance and capacitance in parallel) arranged in parallel. Known results for this circuit are re-expressed in such form that equivalent components can be found from direct measurements of the transient temperature.

Reviewer considers that model is interesting but that its application to real physical systems may be more complicated than authors imply, and he questions advisability of reporting such analogies without any discussion of their mathematical or physical significance. B. R. Morton, England

6521. Bosworth, R. C. L., Attempts to measure the inductive element associated with the natural convection of heat, *Austral. J. Phys.* **13**, 1, 84-94, Mar. 1960.

This is apparently an interim report on experiments with thermal transients about a heated wire immersed in liquid and analysis of results in terms of model described in associated paper [See preceding review]. Measured values of equivalent thermal resistances, capacitance and inductance are all found to vary with temperature difference. Although instrumental and calculation errors are discussed in detail, again little attempt is made to discuss physical significance or adequacy of experiments. Results are quoted in dimensional form thus hindering interpretation, and no attempt is made to relate them to relevant Rayleigh number. Reviewer looks forward to more carefully argued treatment and analysis of results. B. R. Morton, England

6522. Fujii, T., On the development of a vortex street in a free convection boundary layer, *Bull. JSME* **2**, 8, 551-555, Nov. 1959.

The experimental apparatus used by the author [see following review] in determining the free-convection heat-transfer coefficients from a vertical cylinder to ethyleneglycol or water is modified to permit the observation of the development of the boundary layer by schlieren method or by the behavior of small aluminum particles suspended in the fluid. Qualitatively, the process of transition from laminar to turbulent boundary layers could be detected. The region of transition corresponds to a range of Grashof number times Prandtl number between 4×10^9 and 10^{10}. An attempt is also made to analyze the experimental data theoretically.

N. Z. Azer, Egypt

6523. Fujii, T., Experimental studies of free convection heat transfer, *Bull. JSME* **2**, 8, 555-558, Nov. 1959.

Results of experimental measurements of local free-convection heat-transfer coefficients from a vertical cylinder, about 360 mm in height and 76 mm in diameter, to ethyleneglycol or water are presented. From ethyleneglycol data, Nusselt number was found to be proportional to Grashof number times Prandtl number raised to the power $\frac{1}{4}$, for both laminar and turbulent boundary layers and the constant of proportionality was different in both cases. On the basis of available experimental data on the subject, reviewer believes that the exponent $\frac{1}{4}$ for turbulent boundary layer is questionable. Temperature distributions in laminar, transient and turbulent boundary layers are also measured. N. Z. Azer, Egypt

6524. Fujii, T., An analysis of turbulent free convection heat transfer from a vertical surface, *Bull. JSME* **2**, 8, 559-563, Nov. 1959.

With the use of integral momentum and energy equations, two approximate analytic solutions for the flow and heat transfer in free convection boundary layer on a vertical plate are obtained. The first solution corresponds to what the author called transient turbulent boundary layer, in which the boundary layer is divided into laminar sublayer and turbulent layer with various assumptions made for the flow field within the two layers (temperature and velocity distributions are similar along the plate and are linear in the laminar sublayer and follow $\frac{1}{7}$ power law near the edge of the layer, η_s the ratio of the thickness of the laminar sublayer y_s to the thickness of the boundary layer δ is constant along the plate and arbitrarily taken 0.0203 and the eddy diffusivity for heat ε_k and eddy diffusivity for momentum ε_ν are also arbitrarily taken 7 times the thermal diffusivity K and kinematic viscosity ν, respectively).

The second solution corresponds to the turbulent boundary layer. The flow field is also divided into laminar sublayer and turbulent layer with the same velocity and temperature distributions as in the first solution. Relations for heat flow and shearing stress on the wall developed for forced convection turbulent flow are used. It is also assumed that the laminar sublayer Reynolds number R_{es} assumes a constant value 160 ($R_{es} = u_s y_s/\nu$, u_s velocity at the

edge of laminar sublayer). A relation between Nusselt number, Grashof number and Prandtl number is derived from each solution. It is claimed that the theoretical expression derived in the first solution agrees with experimental measurements carried out by the author for a range of $GrPr$ between 10^{10} and 10^{11}.

Reviewer believes that assumptions of η_s being a constant having a value 0.0203, $\varepsilon_K = 7K$ and $\varepsilon_\nu = 7\nu$ need to be justified experimentally before any conclusive remarks can be made. It is also worth pointing out that the approach of the second solution is similar to that of Eckert and Jackson [NACA Rep. 1015, 1951] except that in the latter the division of the boundary layer into two layers was not followed and the assumption of $R_{es} = 160$ was avoided.

N. Z. Azer, Egypt

6525. Morton, B. R., Laminar convection in uniformly heated horizontal pipes at low Rayleigh numbers, *Quart. J. Mech. Appl. Math.* **12,** 4, 410–420, Nov. 1959.

Author considers combined forced and free convection in a horizontal pipe. Solution assumes uniform peripheral wall temperature and is applicable to fully developed region, that is, to region where secondary as well as main flow is independent of axial position. The three momentum equations, the energy equation, and the continuity equation are reduced to three equations involving a dimensionless axial velocity, temperature difference, and stream function. Solution valid for small Rayleigh numbers is obtained by expanding these three quantities in power series in Rayleigh number. It was necessary to go to second-order corrections in order to obtain an effect of free convection on Nusselt number and resistance coefficient, although first-order correction gave a distortion of velocity and temperature profiles. It was shown that the forced convection solution is in error by about 10% when the product of Reynolds and Rayleigh number equals 3000. Reynolds number is based on axial pressure gradient and Rayleigh number on axial temperature gradient.

Reviewer believes paper is a worthwhile contribution to literature on forced and free convection.

R. G. Deissler, USA

6526. Sparrow, E. M., Eichhorn, R., and Gregg, J. L., Combined forced and free convection in a boundary layer flow, *Physics of Fluids* **2,** 3, 319–328, May–June 1959.

Similarity solutions of the temperature and velocity boundary-layer equations are sought when a free convection term is present. If the free-stream velocity has a power law dependence upon x, the distance along the boundary layer, then the ordinary differential equations governing the velocity and temperature profiles contain only two nondimensional parameters, Pr and α^2. Here Pr is the Prandtl number and α^2 is Gr/Re^2, where Gr and Re are local Grashof and Reynolds numbers based on x.

Detailed numerical studies are made (for a Prandtl number of 0.7) of the two cases: (1) the wall is at constant temperature; (2) there is a constant heat flux at the wall. The dependence of Nusselt number and wall skin friction upon α is studied, both when buoyancy aids and when it opposes convection. The asymptotes to the curves for small and large α give the limiting cases of forced and free convection. Ranges of α in which these approximations suffice are delineated. Details of the profiles for various α are given. For opposing buoyancy, separation velocity profiles arise when α exceeds a certain value.

A. F. Pillow, Canada

6527. Hausen, H., New equations for free and forced convection (in German), *Allgemeine Wärmetechnik* **9,** 4/5, 75–79, 1959.

For title problems author finds following results: for free convection, $Nu = 0.11\,(GrPr)^{1/3} + (GrPr)^{0.1}$ correlates vertical plate data for $10^{-4} < GrPr < 10^{12}$; for laminar flow in tube, $Nu = 3.65 +$
$$\frac{0.19\left(Pe\,\dfrac{d}{L}\right)^{0.8}}{1+0.117\left(Pe\,\dfrac{d}{L}\right)^{0.467}}$$
when $10^{-4} < \dfrac{L}{Pe\,d} < 1$; for turbulent flow, $Nu =$
$$0.037\,(Re^{0.75} - 180)\,Pr^{0.42}\left[1+\left(\frac{d}{L}\right)^{2/3}\right]\left(\frac{\mu_w}{\mu_F}\right)^{0.14}.$$
Note change from usual $Pr^{1/3}$ dependency in last result. Symbols have usual meanings.

M. Finston, USA

6528. Mori, Y., On a laminar free-convection flow and heat transfer of electrically conducting fluid on a vertical flat plate in the presence of a transverse magnetic field, *Trans. Japan Soc. Aero. Space Sci.* **2,** 2, 22–26, 1959.

The free convection flow and heat transfer of an electrically conducting fluid on a vertical plate in the presence of a transverse magnetic field is analyzed for the case of a steady flow and steady magnetic field with zero excess charge density in the fluid.

For $Pr = 1$ and the magnetic field intensity varying as the 1/4 power of the distance from the leading edge of the plate, similarity transformations of the magnetohydrodynamic boundary-layer equations to ordinary differential equations are found. The latter are integrated numerically. It is found that the velocity profiles are flattened and the heat-transfer rate reduced by the presence of the magnetic field.

L. M. Grossman, USA

6529. Baldwin, L. V., Sandborn, V. A., and Laurence, J. C., Heat transfer from transverse and yawed cylinders in continuum, slip, and free molecule air flows, *ASME Trans.* **82**C (*J. Heat Transfer*), 2, 77–86, May 1960.

Adopting Nusselt and Reynolds numbers based on gas properties evaluated at total temperature, new and existing data for heat transfer from cylinders is correlated in the experimental range Mach number 0.001 to 6.0, Reynolds number 0.02 to 3×10^5, Knudsen number 4×10^{-6} to 37. In subsonic flow the air velocity normal to the cylinder axis should be used in the unyawed Nusselt number correlation in order to correlate yawed cylinder results.

Experimental recovery temperature data indicate the existence of free molecule flow for cylinders above a Knudsen number of 5.

J. F. Clarke, England

6530. Kihara, T., Taylor, M. H., and Hirschfelder, J. O., Transport properties for gases assuming inverse power intermolecular potentials, Univ. Wisconsin, Theor. Chem. Lab. WIS-OOR-27, 20 pp., Mar. 1960.

A research article on the evaluation of a number of integrals (related to molecular cross sections) which are needed in the calculation of the coefficients of viscosity, heat conductivity, diffusion and thermal diffusion of dilute gases. Gas molecules are assumed to either attract or repel each other with a force varying inversely as some power of the separation distance. This force law is too simplified for most real gases at ordinary temperatures, but there are a number of situations in which it is a reasonable approximation. Perhaps the most important of these are gases at very high temperatures, in which molecular collisions are so energetic that the ordinary long-range attractive van der Waals forces are unimportant. Reviewer would have expected to see this article published in a regular research journal, such as the *Journal of Chemical Physics.*

E. A. Mason, USA

6531. Philip, J. R., General method of exact solution of the concentration-dependent diffusion equation, *Austral. J. Phys.* **13,** 1, 1–12, Mar. 1960.

Author considers diffusion (heat-conduction) problem
$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\,\frac{\partial\theta}{\partial x}\right] \qquad [1]$$

with conditions

$$\theta = 0 \text{ for } t = 0,\ x > 0, \qquad \theta = 1 \text{ for } x = 0,\ t > 0 \qquad [2]$$

and a similar problem with rather different boundary conditions, formulated in the paper. Exact solutions of these problems are investigated, i.e. solutions expressible in explicit formulas.

According to a previous paper of the author, substituting $\varphi = xt^{-1/2}$, the problem is reduced to finding the function $\theta(\varphi)$ or, more exactly, the inverse function $\varphi(\theta)$, fulfilling the equation

$$D(\theta) = -\frac{1}{2}\frac{d\varphi}{d\theta}\int_0^\theta \varphi(\theta)\,d\theta. \qquad [3]$$

To any function $\varphi(\theta)$ (which satisfies certain conditions), there corresponds a function $D(\theta)$, according to [3]. Author presents tables with typical φ and corresponding D. For all chosen functions is $\lim D = 0$ or $\lim D = \infty$ for $\theta \to 0$. This seems to be restrictive for author's method. But he shows that functions φ exist such that D has a finite non-zero limit for $\theta \to 0$.

In practical problems, D is prescribed and φ is to be found. D is then to be approximated with functions corresponding to simple functions φ. This would require the construction of a "library" of such correspondences. Reviewer believes that, from the mathematical point of view, the question of convergence of approximative solutions would then be of interest.

K. Rektorys, Czechoslovakia

6532. Pattle, R. E., Diffusion from an instantaneous point source with a concentration-dependent coefficient, *Quart. J. Mech. Appl. Math.* **12,** 4, 407–409, Nov. 1959.

Author reports concentration distribution solutions for s dimensions ($s = 1, 2,$ or 3), when the diffusion or seepage coefficient varies according to $D = D_0(C/C_0)^n$ with $n > 0$. Diffusant occupies bounded region whose diameter increases as the $1/(sn + 2)$ power of time, in the manner of the ink in a blot.

L. E. Scriven, USA

6533. Bayley, F. J., Air-cooling methods for gas-turbine combustion systems, Aero. Res. Counc. Lond. Rep. Mem. 3110, 34 pp., 1959.

Report was originally published in 1951, and is a summary of work done at the National Gas Turbine Establishment. Porous wall, louvered, and localized injection systems are treated analytically, as well as external convection. Experimental results for some systems are reported. Analytic methods are based on convection heat transfer only.

Author states: "It is shown that 'sweat', or effusion-cooling, is by far the most effective and efficient method, while the use of 'louvered' surfaces represents the nearest practical approach to this ideal which is possible while suitable porous materials remain unavailable."

G. M. Ketchum, USA

6534. Papell, S. S., Effect on gaseous film cooling of coolant injection through angled slots and normal holes, NASA TN D-299, 27 pp., Sept. 1960.

A study was made to determine the effect of coolant injection angularity on gaseous film-cooling effectiveness. A corrective term containing coolant injection angularity was developed to qualify a correlating equation presented in NASA TN D-130 that was limited to tangential injection of the coolant. Data were also obtained for coolant injection through rows of holes normal to the test plate. The slot-correlating equation was adapted to fit these data by the definition of an effective slot height. An additional corrective term was then determined to correlate these data.

From author's summary

6535. Hahnemann, H. W., Determining the diffusion resistance and the capillary fluidity coefficient deriving from stationary and intermittent processes (in German), *ZVDI* **102,** 5, 157–158, Feb. 1960.

Authors have considered molecular diffusion and gross motion associated with the transport of water in air-filled porous materials. The effect of the fraction of the pores filled with an aqueous phase was taken into account on the basis of experimentally determined coefficients. The treatment is limited to the one-dimensional case, and a partial differential equation describing the transport is presented. Experimental measurements of the capillary and viscous resistance to the gross motion of the liquid phase, expressed as an apparent kinematic viscosity, have been determined as a function of the fraction of the pore volume filled with an aqueous phase. Such data were established for nine different materials varying from clay to beechwood. The resistance to the gross motion was determined from transient and steady-state measurements. The correlation of the effective viscosity with the water content of the pores was far from single valued.

B. H. Sage, USA

6536. Liu, V. C., Note on diffusive separation of gas mixtures in flow fields, *J. Appl. Phys.* **29,** 8, 1188–1189, Aug. 1958.

6537. Neu, R. F., Comparison of localized heat-transfer rates in a liquid-oxygen-heptane rocket engine employing several injection methods and oxidant-fuel ratios, NASA TN D-286, 39 pp., June 1960.

The effects of injection process and oxidant-fuel ratio upon local values of heat transfer were studied in an 1800-pound-thrust oxygen-heptane rocket engine. Circumferential variations in local heat flux as high as 2 : 1 were observed. These heat-flux variations were influenced by the injector type and the oxidant-fuel ratio. Within the chamber, the longitudinal variations of heat flux were smaller, generally, than the circumferential variations. Each experimental case is explained individually in terms of such factors as injector spray pattern, spatial heat release, and patterns of hot gas flow. No general model is proffered that explains these distributions of local heat flux for the different injectors and operating conditions.

From author's summary

6538. Weatherston, R. C., and Smith, W. E., A method for heat rejection from space powerplants, *ARS J.* **30,** 3, 268–269 (Tech. Notes), Mar. 1960.

Paper describes a preliminary analysis of capacity-type heat exchanger for dissipating heat from space power plants. The proposed system consists of a rotating drum driving a belt in contact with it. Waste heat from the power plant was first transported to the drum, thus raising its temperature. The belt conducts heat from the hot drum and dissipates energy by radiation as it leaves the drum and travels through space. Upon returning to the drum, it has been sufficiently cooled and the cycle repeats.

Author demonstrate that the use of such capacity heat exchanger of optimum design makes possible a significant reduction in weight of the cooling system as well as the entire nuclear power plant.

B. T. Chao, USA

6539. Wing, L. D., Evaluation of thermal problems at relatively low orbital altitudes, *Aero/Space Engng.* **19,** 3, 58–60, 64, Mar. 1960.

This paper contains a rapid and simple method of estimating temperature levels due to radiation on the surface of vehicle orbiting the earth at altitudes small by comparison with earth radius. Also a method of cooling is proposed which counteracts strong temperature fluctuations of the skin, using good insulators for sandwiched skin design and recirculation of air for cooling purposes.

No comparisons are made of this method with other proposed methods of equalizing the skin temperature. In particular, increased expenditure of energy due to pumping and corresponding increase of weight due to pumping equipment are not mentioned. Similarly stress problems connected with the use of heat insulators for structural elements are not discussed.

This reviewer feels that the proposed method of skin temperature estimate would have been much more useful if, by simple trigonometric considerations relating to the orbit of the vehicle, the extreme values of the heat-flux oscillations could be figured explicitly. This refers primarily to the contribution of direct solar radiation. B. Zarwyn, USA

6540. Lefebvre, A. H., and Herbert, M. V., Heat-transfer processes in gas-turbine combustion chambers, Instn. Mech. Engrs., Prepr., 1959, 3-13.

A rationalized semiempirical procedure for estimating heat transfer in gas-turbine combustors is given. Reasonable consistency with tests is maintained when this method is applied to a few configurations of combustors. The comparison of three designs given in Table 1, which shows that the tubo-annular combustor has the highest flame-tube temperature, is somewhat misleading. For a constant engine diameter and a number of other similar conditions, possible variations in designs are many. The reason for a higher flame-tube temperature in the tubo-annular design than in other designs is probably because the former design is too large in some dimension, as well as radiation between flame tubes in the former design. The qualitative conclusions may be considered as known facts justified from this study. The authors say nothing about a paper by D. G. Stewart on scaling of combustors ["Selected Combustion Problems-II," AGARD, Butterworths, London, 1956, pp. 384-413]. It would be interesting to check the general validity of Stewart's heat-transfer and chemical correlations in the light of the present results. S. L. Soo, USA

6541. Chirkov, A. A., and Stefanovskii, B. S., The dominating method of heat transfer in the cylinders of internal combustion motors (in Russian), *Trudi Rostovsk. In-ta Inzh. Zh.-d. Transp.* no. 21, 96-111, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6441.

Basing their arguments on the analysis of numerous investigations, experimental data and calculations of their own, authors advance points showing the unsuitability of the known Briling formula and of its modification for the evaluation of heat transfer in a cylinder. The gross coefficient of heat emission α_g calculated by this formula is over valued, in the view of the authors, while the distribution of heat transmitted to the wall of the cylinder, as presented by the formula, to account for convection and the radiation of gases is considered as irregular by them. On the assumption, with some reservations, of the similarity of the heat emission from the working bodies to the walls of the cylinders of compressors and internal combustion motors the authors find a formula for the coefficient of heat emission at the expense of convection α_k. Subsequently a calculation is made on the basis of the general theory of heat transfer for the heat passed to the wall of the cylinder at the expense of radiation from the gas space and radiation from the sooty particles of the flame. As a result, graphs are given to show the changes in the gross coefficient of heat emission of gas α_g in the cylinder from which, in contradiction to the calculations by Briling's formula, it is obvious that the dominant means for the transfer of heat in the motor appears to be radiation and not convection. A formula is proposed for the calculation of the coefficient of heat emission

$$\alpha_g = 44\frac{\lambda 10^4}{3\sqrt{D}}\sqrt{\frac{p\,c_m}{\mu\,10^4\,T}} + n\,\varepsilon_v\,\varepsilon_\omega\,\sigma\frac{\left(\frac{T}{100}\right)^4-\left(\frac{T_\omega}{100}\right)^4}{T-T_\omega}$$

where p and T are the running values for the pressure and temperature in the cylinder, T_ω is the temperature of the cylinder, c_m is the mean speed of the piston, D is the diameter of the cylinder, ε_ω and ε_v are the effective degrees of blackness of the wall and the gas space, μ is the dynamic viscosity of the gas, λ is the coefficient of heat conductivity of the gas, n is a numerical coefficient. The paper concludes with a proposal to prevent overheating of cast iron pistons of two-stroke motors by aluminizing or by diffusional chromeplating of the bottom of the piston to induce a sharp reduction of the coefficient of radiation of the piston's surface.

R. Ya. Panfilov
Courtesy Referativnyi Zhurnal, USSR

6542. Panchev, St., On the evaporation of freely falling large drops in the atmosphere (in Russian), *Dokladi Bolg. Akad. Nauk* **10**, 5, 355-358, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2613.

The solution of the title problem led the author to Bessel's equation, the solution of which for the velocity of the drops approximates satisfactorily with the expression for the falling of a drop of constant mass. The author continues by finding the linkage between the final and the initial dimensions of the drop and of the path it follows. This relation enables the spectrum to be ascertained for the magnitude of raindrops at the earth's surface as a function of the height of the cloud, of the initial dimensions of the drop, and of the relative humidity.

G. A. Varshavskii
Courtesy Referativnyi Zhurnal, USSR

6543. Mayatskii, G. A., A theoretical and experimental investigation of heat exchange in media in droplet condition with large Reynolds numbers (in Russian), Avtoref. Diss. Kand. Tekhn. Nauk, Kuibyshevsk. Industr. In-ta, Kuibyshev, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2737.

6544. Otte, W., Spraying phenomena in the cooling tower (in German), *Brennstoff-Wärme-Kraft* **10**, 8, 371-373, Aug. 1958.

Purpose is to distribute the entering warm water evenly over the surface area of the cooler. Furthermore, there is a heat exchange between the water drops in the spray and the air leaving the cooling tower, the amount of which depends on the number, size, and distribution of the droplets. Author discusses the trajectory of the spray, effect of updraft of air, design measures for producing small droplets, and the arrangement of spraying outlets in the tower. The nozzle is a smooth opening to which the water is fed at a low head, of about 1.5 to 2 ft; the water issues from the nozzle in a smooth jet, which impinges on a plate with serrated edges, which break up the jet into droplets. A large number of such outlets are arranged in such a configuration that the droplets are evenly distributed within the upward current of the cooling air.

K. J. DeJuhasz, USA

6545. Otte, W., Experiments with spraying nozzles (in German), *Brennstoff-Wärme-Kraft* **12**, 5, 217-218, May 1960.

Continuation of Otte, W. [preceding review]. Using a nozzle of 26-mm diam and a water discharge rate of 10.1 cu m (2700 gal/hr) several types of distribution plates were used, with different diameters, depths, and numbers of serration. The results were compared as regards evenness of coverage of the sprayed area.

K. J. DeJuhasz, USA

6546. Seleznev, A. A., Making use of the roughness of surfaces of heat exchange to reduce the weight of the heat exchanger (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Aviats. Tekhn.* no. 1, 106-112, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2723.

An investigation was carried out on the basis of results previously obtained by the author [*Teploenergetika* no. 7, 45-47, 1955] to see whether it is possible to reduce the weight of a heat exchanger by using rough tubes. However in rough tubes hydraulic resistance is higher than in smooth tubes, and this results in an increase in energy output. By using the formula obtained in the present paper it is possible to determine the relative decreases in the dimensions of the heat exchanger and the increase of energy output for different parameters of roughness and for the various regimes of work of the heat exchanger. It is shown by reference to concrete examples of calculation that the utilization of roughness

of surfaces as a means of intensifying heat exchange is economically more profitable than intensification of heat exchange by increase of Reynolds number. V. N. Kalashnik

Courtesy Referativnyi Zhurnal, USSR

6547. Overcashier, R. H., Todd, D. B., and Olney, R. B., Some effects of baffles on a fluidized system, *AIChE J.* **5,** 1, 54–60, Mar. 1959.

Some characteristics are reported for the fluidization of an air-microspheroidal catalyst system in a 16-in.-diameter bed equipped with baffles. The back-mixing characteristics and retention-time distributions of gas and solids, allowable gas and solids velocities, entrainment rate, and bed density are studied as functions of baffle design.

It is shown that the use of baffles narrows the retention-time spectrum and permits either concurrent or countercurrent flow while not seriously reducing gas or solids through-put or solids holdup. From authors' summary

Combustion

(See also Revs. 6209, 6533, 6540)

6548. Schmidt, P., Periodically repeated ignition through shock waves (in German), Arbeitsgemeinschaft für Forschung des Landes Nordrhein-Westfalen no. 82, 57 pp., Oct. 1958.

In first part of paper, experimental results on pulse jets and on newly developed "resonant shock-wave tube" are presented. Author regards these results as evidence for periodic reignition due to shock waves. In second part, temperature reached in shock wave of observed strength is first computed by standard gasdynamic method and is found to be much below required ignition temperature. This analysis is followed by "molecular-kinetic" treatment, which, in author's opinion, supports his view of shock-wave ignition. Last part of paper suggests new applications of pulsating-combustion devices, e.g. for VTOL airplanes, chemical processing, and "high-energy treatment of matter."

Reviewer regards presented experimental and theoretical evidence for shock-wave ignition as unconvincing, and considers rapid mixing of residual burnt gas with fresh combustible gas under influence of shock wave as more likely explanation of observed effects. G. H. Markstein, USA

6549. Buckley, D. H., Swikert, M. A., and Johnson, R. L., Ignition of a combustible atmosphere by incandescent carbon wear particles, NASA TN D-289, 16 pp., Sept. 1960.

An investigation was conducted to determine whether carbon wear particles abraded from rotating carbon elements in contact with a metal surface could ignite a combustible gas atmosphere. Experiments were run with a carbon vane in sliding contact with a rotating 440-C stainless-steel flat disk in a chamber filled with a combustible atmosphere of air and propane. When an electric potential was applied across the vane-disk interface, incandescent carbon wear particles were obtained from the vane specimen and fires were sometimes produced. Ignition of the gas mixture occurred only when incandescent carbon wear particles were present. From authors' summary

6550. Newman, M. M., and Robb, J. D., Investigation of minimum corona type currents for ignition of aircraft fuel vapors, NASA TN D-440, 12 pp., June 1960.

The present report is essentially a limited supplementary study continuing an earlier investigation (NACA TN 4326) on lightning hazards in relation to aircraft fuel tanks. These studies on minimum corona-type currents that might cause fuel vapor ignition in fuel vent areas establish a much lower range of ignition currents that could present an indirect lightning hazard. Laboratory studies of minimum sparking currents versus time durations were made in relation to possible fuel vapor ignition from potentials induced from lightning discharge currents along the aircraft skin at discontinuities. Such pulses might be brought inside the aircraft along poorly bonded conductors into areas possibly having explosive fuel leak vapors present. Individual pulses of less than 10 microseconds duration required currents of over 1 ampere for ignition of fuel vapor. From authors' summary

6551. Fletcher, E. A., Dorsch, R. G., and Allen, H., Jr., Combustion of highly reactive fuels in supersonic airstreams, *ARS J.* **30,** 4, 337–344, Apr. 1960.

The feasibility of adding heat to supersonic airstreams by combustion was studied in a small wind tunnel. Aluminum borohydride, pentaborane, hydrocarbon-aluminum borohydride mixtures, trimethylaluminum, diethylaluminum hydride, alkylboranes, alkylboron hydrides and vinylsilane were tested. The first three ignited easily and burned well. The others either failed to ignite or burned only in the diffuser. Trimethylaluminum and diethylaluminum hydride produced light and other evidence of heat evolution when water was simultaneously injected into the tunnel. Gross effects on flow were studied by observation of shock patterns and water sprays injected from the tunnel walls. Examples are given which illustrate the use of these techniques in aerodynamic studies in larger wind tunnels. From authors' summary

6552. Shchelkin, K. I., Measurement of the speed of propagation of turbulent combustion, *ARS J.* **30,** 1, 76–77 (Russian Supplement), Jan. 1960. (Translation of *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Energetika i Avtomatika* no. 2, 137–138, 1959.)

Bolz and Burlage of the U.S. computed flame speed by taking successive photographs of a growing spherical flame. Their computation method according to this paper is not valid where the width of the combustion zone is large compared to the spherical flame radius. An approximate estimate of the influence of the width of the zone is presented but lacks in authenticity due to assumptions required. Reviewer believes that views presented are worth considering in future investigations of this nature.

J. T. Hamrick, USA

6553. Scholte, T., The influence of hydrogen or deuterium on the flame-propagation velocity of carbon monoxide and air mixtures (in German), *ZVDI* **102,** 17, 673–677, June 1960.

The flame-propagation velocity of mixtures consisting of air and pure carbon monoxide does not depend on the concentration of the latter. By adding small quantities of hydrogen or deuterium the square of the flame-propagation velocity increases proportionally to the amount of agents added. Additions of hydrogen compounds also produce similar acceleration of the flame-propagation velocity. The results obtained provide an insight into the reaction mechanism and confirms that the diffusion of the radicals from the zone of flame into the unburned mixture plays a more important role than heat conductance. From author's summary

6554. Van Tiggelen, A., Kinetical parameters in premixed laminar flames, AFOSR TN 59-1149 (Univ. of Louvain, Lab. for Inorganic Chem. TN-2), 43 pp., Feb. 1960.

The burning velocities, obtained with methane-oxygen flames at different mixture ratios and different initial temperatures, are almost perfectly represented by a formula where the activation energy is 38 kcal and the reaction order with respect to oxygen is 1.4 and to methane -0.4. The formula corresponds exactly to the theory with a chain mechanism including a peroxide radical.

From a kindred investigation performed on acetylene-oxygen mixtures a similar empirical formula is obtained. A chain reaction with the formation of a peroxide must also occur in these flames.

The last chapter relates some experimental results with stoichiometric ternary methane-hydrogen-oxygen mixtures at different dilutions with nitrogen.

From author's summary by H. Behrens, Germany

6555. Zucrow, M. J., Osborn, J. R., and Pinchak, A. C., Luminosity and pressure oscillations observed with longitudinal and transverse modes of combustion instability, *ARS J.* **30**, 8, 758–761 (Tech. Notes), Aug. 1960.

Combustion pressure oscillations were observed in two experimental gaseous bipropellant rocket motors. The geometries of the two motors were different so that one rocket motor tended to exhibit only the longitudinal mode while the other displayed only transverse modes of oscillations. The local pressure and the luminosity of the combustion gases were simultaneously recorded while the rocket motors were operating with combustion pressure oscillations. The results indicate that the reaction mechanism which sustains the longitudinal mode is similar to the aerothermodynamic interaction which supports the transverse modes of combustion pressure oscillation. In addition, the results are in agreement with and support the "shock or pressure wave" mechanism of combustion pressure oscillations as postulated by Zucrow and Osborn.

From authors' summary

6556. Crump, J. E., and Price, E. W., "Catastrophic" changes in burning rate of solid propellants during combustion instability, *ARS J.* **30**, 7, 705–707 (Tech. Notes), July 1960.

Experimental results of a research program on combustion instability in solid propellants have revealed a "catastrophic" change in burning rate which is correlated with the occurrence of a harmonic relationship between a longitudinal acoustic mode and a tangential acoustic mode of the propellant chamber.

From authors' summary

6557. Salamandra, G. D., and Tsukhanova, O. A., Formation of a shock discontinuity ahead of a flame front, *ARS J.* **30**, 1, 66–72 (Russian Supplement), Jan. 1960. (Translation of *Physical Gasdynamics, USSR Acad. Sci.* 151–162.)

Results are presented in the form of photographs taken with high-speed photographic equipment and curves which show the results of theoretical calculations. Photographic equipment and experimental procedure are described in detail. Distance from the point of ignition to point at which the shock discontinuity is formed is determined for mixtures of hydrogen and oxygen varying from 33.3 to 70% hydrogen. It is shown that this distance is dependent upon the chamber shape as well as the gaseous medium and that theoretical calculations agree with experimental results. Formation of shock discontinuities in short chambers and complication introduced by reflected disturbances is demonstrated and commented upon.

Authors do not reference extensive work in the field by U. S. investigators. These are given in a review accompanying the paper. Paper is believed to be a useful addition to the literature.

J. T. Hamrick, USA

6558. Salamandra, G. D., Interaction between a flame and a shock discontinuity, *ARS J.* **30**, 1, 73–76 (Russian Supplement), Jan. 1960. (Translation of *Physical Gasdynamics, USSR Acad. Sci.* 163–166.)

High-speed photographs of hydrogen-oxygen flame front shock wave interaction are presented with interpretation by the author. Reviewer was unable to follow interpretation at several points due to loss of photographic detail in reproduction. The author concludes that a shock discontinuity while being formed will not as a rule pass through a flame front traveling in opposition but will do so after the shock is formed. In the latter case deformation of the flame front results in increased combustion surface and higher flame propagation speed.

J. T. Hamrick, USA

6559. Chekalin, E. K., and Sobolev, G. K., Some features of the gas flow in a laminar Bunsen flame (in Russian), *Inzbenero-Fizicbeskii Zb.* **1**, 4, 72–75, 1958; *Ref. Zb. Mekb.* no. 3, 1959, Rev. 2507.

An investigation is conducted on the field of velocities in a free isothermal current and in a Bunsen laminar flame forming above a burner of round section. The method is described for following the process by introducing solid particles into the gaseous flow. A diagram was obtained of the velocities in a free isothermal current flowing from the nozzle of the burner; the hydrodynamic structure of the Bunsen flame was also examined.

A. S. Ginevskii
Courtesy Referativnyi Zburnal, USSR

6560. Vulis, L. A., Flow problems met with in applied gas dynamics (in Russian), Investigation of the physical bases of the working processes in furnaces and ovens, Alma-Ata, Akad. Nauk KazSSR, 1957, 15–53; *Ref. Zb. Mekb.* no. 3, 1959, Rev. 2508.

A survey is furnished of the individual results of the investigations on the free turbulent currents in an incompressible liquid and gas. The first part of the article is devoted to an examination of some of the fundamental aspects of the semiempirical theory of currents in an incompressible gas. A class is separated out which characterizes the simplest self-modelling motions, among which will be found universality of the profile of velocity and other characteristics derived in relative coordinates for different transverse sections of the current. In addition, methods are examined for the solution of the problem of the twisted current, for the thermal problem of a two-dimensional current and for the current which is semidelimited, for the problem of a current flowing about a cone, and also for problems dealing with complex turbulent motions (regarding the wake, the current in forward flow, the current in return flow or oriented at an angle to the axis of the current's flow).

In the second part of the article some general properties of turbulent gas currents are investigated; in particular, an approximate method developed by the author for the investigation of gas flows is described; this is based on the assumption of the determining role played in the processes of turbulent exchange in current flows of compressible gas by the density of the impulse flow, the surplus heat content, substances, etc. In its application to self-modelling motions this assumption merges with the hypothesis on the universality of the profiles of the density of the impulse flow, etc., in jet flows of a compressible gas and the conservation of the same relationships with the coordinates as in the corresponding jets of incompressible gas. It is stated that the assumption on the universality of the profile of the density of the impulse flow in jet flows is confirmed by the results of experiments in a number of special cases.

In the third part an investigation is carried out on some questions relating to the jet theory of a flame.

A. S. Ginevskii
Courtesy Referativnyi Zburnal, USSR

6561. Gol'denberg, S. A., Combustion in a turbulent flow (in Russian), *Inzbenero-Fizicbeskii Zb.* no. 1, 53–64, 1958; *Ref. Zb. Mekb.* no. 3, 1959, Rev. 2367.

Author investigates the process of combustion in a jet of a homogeneous mixture in a free space. He finds the formula given below for the length of the zone of combustion. This resulted from the solution of the differential equation for the transfer of heat and mass in a turbulent flow consisting of hydrocarbon-air mixtures and mixtures of CO with air (reaction of the first order); here, consideration had to be given to the interaction of the mean quadratic pulsation velocity and the turbulent velocity for the propagation of the flame U (generation of the turbulence of the flame):

$$L = \frac{Wd^2}{2\sigma U}\left\{\left[-\frac{4\sigma U}{d^2 W}\log_e \frac{\bar{c}}{\bar{c}_0} + \left(\frac{K}{\overline{W}}\right)^2\right]^{1/2} - \frac{\overline{K}}{\overline{W}}\right.$$

where W is the velocity of the flow; d is the diameter of the burner; c, c_0 are the averaged relative concentrations of the fuel mixture at the beginning and at the end of the zone of combustion respectively; $\bar{K}$ is the effective constant of the velocity of the chemical reaction, σ is an experimental coefficient depending on c and T. This approximate relation indicates that pure volumetric and pure surface combustion appear to be two boundary cases of the same process of frontal and volumetric combustion. It is shown that the experimentally established relation of L to the pressure P

$$L \sim P^{-0.2}\ (W = \text{const}), \qquad L \sim P^{-0.75}\ (R = \text{const})$$

can be derived from the formula for L. Experiments also establish the theoretical curves for the relation of the completeness of the combustion to L/d for various values of the coefficient for the excess air α at atmospheric pressure. L. S. Dmitriev

Courtesy Referativnyi Zhurnal, USSR

6562. Ingebo, R. D., Photomicrographic tracking of ethanol drops in a rocket chamber burning ethanol and liquid oxygen, NASA TN D-290, 18 pp., June 1960.

An ultra-high-speed tracking camera developed at NASA Lewis Research Center was used to photograph ethanol jets breaking up into drops and burning in a rocket combustor 4 inches from the injector face. The Nukiyama-Tanasawa and log-probability expressions for drop-size distribution gave good results in analyzing the experimental data. Drop-velocity data were also obtained with the tracking camera. From author's summary

6563. Segal, H. M., Design method for spherical grains, *ARS J.* **30,** 4, 370–371, Apr. 1960.

Through the application of descriptive geometry techniques a practical design method for obtaining the burning area of a spherically shaped grain is obtained, thus providing information for the plotting of a thrust-time curve. The application of the method is shown for a spherical grain with a diameter of 20 inches. Although the method lacks the precision of the mathematical approach it does offer the advantages of simplicity, speed and sufficient accuracy for normal design requirements.

J. F. Lee, USA

6564. Kantorovich, B. V., and Finyagin, A. P., Influence of excess air on the process of combustion of a pulverized form of fuel (in Russian), *Inzhenero-Fizicheskii Zh.* no. 1, 24–33, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2369.

The process is investigated of the reaction of burning pulverized fuel which is being continuously entrained by a flow of air. A kinetic equation is brought in to describe the velocity of the complete burning out of the pulverized fuel in a unit of volume of the combustion zone. This equation is solved by the authors for two extreme cases: a diffusional isothermal regime of combustion and a kinetic anisothermal regime. It was shown that in both cases the length of the zone of combustion depends on the coefficient of excess air α, and there is a value of α at which this length has a minimum magnitude. A description is also given of laboratory apparatus used, and experimental results are put forward confirming the theoretical deductions. V. V. Smirnov

Courtesy Referativnyi Zhurnal, USSR

Prime Movers and Propulsion Devices

(See also Revs. 6194, 6214, 6414, 6415, 6537, 6540, 6541, 6556, 6562, 6563, 6649, 6650, 6727)

6565. Schmidt, E., The rotary engine; genesis of a new type of internal-combustion engine with surprising characteristics (in German), *ZVDI* **102,** 8, 293–297, Mar. 1960.

A new form of internal-combustion engine is described in which gas spaces varying in volume are formed between a continuously rotating shaped runner and a cylinder of epitrochoid cross section. Such engines may be built with either stationary or rotary cylinders. The preferred form has a three-corned runner rotating in a cylinder which approximates a figure 8. The configuration is not necessarily limited to this form, but can be made with runners of 2, 3, 4 or 5 corners. With the higher numbers of corners, the epitrochoid becomes more sharply cusped. No analysis is given of the forms of these figures, but leading results such as the relative speeds of eccentric and runner and the maximum pressure ratios obtainable are stated. M. A. Mayers, USA

6566. Huber, E. W., Thermodynamic testing of the rotary engine (in German), *ZVDI* **102,** 8, 298–314, Mar. 1960.

The rotary engine described in the previous abstract is a four-cycle engine. Indicated diagrams are given for a test engine, and performance figures are given at various speeds. The first machine tested had rather serious leakage losses, but this was improved in a later machine leading to the belief that the development will proceed rapidly. M. A. Mayers, USA

6567. Froede, W., Development work on rotary combustion engines (in German), *ZVDI* **102,** 8, 314–322, Mar. 1960.

Details of design and design development of the rotary engine described in the previous abstracts.

M. A. Mayers, USA

6568. Nagao, F., and Hirako, Y., Estimation of the part-load performance of turbocharged two-cycled diesel engine, *Bull. JSME* **2,** 7, 390–397, Aug. 1959.

A turbocharged diesel engine is designed referring to one set of values for the operating variables at the design point, i.e., to the set of the design point values for the variables, such as the scavenging pressure and temperature, the turbine inlet pressure and temperature, the engine speed, the excess air ratio, the engine output, the rotational speed of turbocharger, and the component efficiencies. While previous papers were concerned with determining the design point performance of the turbocharged two-cycle diesel engine, the present paper is concerned with the determination of the performance characteristics of the same engine unit operating under off-design conditions or at part-load, where the variables presented above have a set of values different from those at the design point. From authors' summary

6569. Nagao, F., and Hirako, Y., Estimation of the operating characteristics of a turbocharged four-cycle diesel engine, *Bull. JSME* **2,** 7, 398–404, Aug. 1959.

The operating characteristics of a turbocharged four-cycle diesel engine are estimated by means of the equilibrium running line drawn by linking together the performance characteristics of the components of a given engine unit.

In estimating the engine performance corresponding to the equilibrium running line, two methods are employed: One is the theoretical method as shown on a two-cycle engine in the preceding paper, and the other is an experimental one with an actual engine which is mechanically supercharged.

From authors' summary

6570. Meijer, R. J., The Philips hot-gas engine with rhombic drive mechanism, *Philips Tech. Rev.* **20,** 9, 245–262, May 1959.

After a brief recapitulation of the hot-gas cycle as applied to an engine provided with a displacer piston, certain new design features of the Philips hot-gas engine are described. An account is given of a new kind of drive mechanism in which the required power and displacer piston movements are obtained by means of twin crank and conrod mechanisms offset from the engine axis; the crankshafts are coupled by two gear-wheels, and rotate in

opposite senses.... The new drive mechanism also permits complete dynamic balancing of the inertial forces of moving parts, even in a single-cylinder engine. Other innovations concern the design of the regenerator compartment, the cooler and the heater,... whose weight can be kept quite low despite the high gas pressure. The pre-heater has also been redesigned, with the result that burner efficiencies of up to 90% are now reached. In addition, a fast-acting system has been developed for regulating the engine output, this being controlled by altering the mean working pressure of the gas in the cylinder.

These various features have been applied in the design and construction of an experimental single-cylinder engine of about 40 hp containing hydrogen (helium could also be used) as the working fluid and burning light fuel oil.... The nominal speed of the engine is 1500 rpm.... Tests on this engine... show that... at 700°C it attains an over-all efficiency of up to about 38% (engine alone, without auxiliaries). The maximum....specific power was...120 hp per liter swept volume...

The specific weight of the engine is 5 kg (11 lb) per hp. (From author's summary)

No estimates of costs, or the potential of the engine, are given in the article so no conclusion can be drawn as to its economic value; but it is certainly a well thought out and interesting engine.

A. O. White, USA

6571. Danilov, Yu. I., Choice of an indicator for the polytropic expansion curve for gases being discharged from an internal combustion engine (in Russian), *Trudi Mosk. Aviats. In-ta* no. 92, 60–70, 1957; *Ref. Zh. Mekh.* no. 2, 1959, Rev. 1323.

A method is described for the selection of an indicator (varying with time) for expansion when the amount of work done by the working body in the cylinder is not constant, account being taken of the complete combustion of the working mixture and the cooling of the gas by the walls of the cylinder taking place during the exhaust process. The assumption is made that the heat input from complete combustion in the exhaust process is proportional to the heat output from the cylinder's walls. The flow value of the polytropic index is determined as the function of two variables: the velocities of heat liberation $dq/d\tau$ and the changes of the magnitude of the work done by the gas in a time period $d(Al)/d\tau$, where dq and $d(Al)$ are linked together in the thermodynamic process. The author departs from using the magnitude $d(Al)d$ and uses the more convenient parameter $dq/d\tau$, the derivative of pressure by time. The relation, determined in accordance with the method given, is introduced for the polytropic index as a function of the angle of rotation of a crankshaft.

A. M. Rozhdestvenskii

Courtesy Referativnyi Zhurnal, USSR

6572. Chang, C.-C., and Hsu, C.-T., Aerodynamic instability of supersonic inlet diffusers, *ARS J.* **30**, 5, 468–475, May, 1960.

Paper serves useful purpose of bringing together results of other workers. Main objective of paper was to show instability of cold supersonic inlet diffuser is essentially dependent on viscous dissipation. Analytical approach is based on assumption of quasi-steady flow. Inlet acoustic impedance is found by solving frictional one-dimensional unsteady compressible flow equations for external diffusion zone. Small perturbation method is used with perturbation values given by harmonic functions. In external diffusion zone the average axial velocity and the entropy perturbation are considered to vary linearly with distance. Capture area perturbation is neglected. These conditions give same flow equations for both external and internal diffusion zones. Results are compared with experimental data of Stoolman [Ph.D. Thesis, Cal. Inst. Tech., 1953]. Instability theory of Trimpi [AMR **9**(1956), Rev. 3677] is confirmed and the theory of Sterbentz and Evvard [AMR **9**(1956), Rev. 812] shown valid for the "steady" range of subcritical operation.

R. A. A. Bryant, Australia

6573. Senn, W. L., Jr., and Repetti, R. V., Specific impulse nomograph, *ARS J.* **30**, 5, 489–491 (Tech. Notes), May 1960.

A graphical method is presented which makes possible the rapid determination of specific impulse once the various parameters of a rocket propellant system are known.

From authors' summary

6574. Cockshutt, E. P., Dow, D. F., Levy, G. G., Rueter, F., and Sharp, C. R., Turbojet thrust augmentation: flight test results for a reheat system employing fuel injection upstream of a two-stage turbine, Nat. Res. Counc., Canada, LR-272, 45 pp., Jan. 1960.

Flight experiments on a reheat system, in which part of the reheat fuel was injected upstream of a two-stage turbine, are described. The system was fitted to an Orenda 14 engine installed in a Sabre 6 aircraft.

The reheat system was stable over the range of conditions explored, and operated satisfactorily at the ceiling of the aircraft with a tailpipe static pressure of 3.85 psia.

A net thrust boost of 32% was obtained at a flight Mach number of 0.82. Combustion efficiency decreased slowly from 86% at sea level static to 79% at 50,000 feet.

With the fixed oversize reheat nozzle, consistent relights were obtained up to 45,000 feet (corresponding to an altitude of 52,000 feet with a variable area nozzle in the closed position). Above 45,000 feet, results were erratic.

No serious mechanical or performance troubles attributable to preturbine injection of the reheat fuel were experienced.

From authors' summary

6575. van Manen, J. D., and Kamps, J., The effect of shape of afterbody on propulsion, Soc. Nav. Arch. Mar. Engrs., Ann. Meet., New York, Nov. 12–13, 1959; Adv. Pap. 2, 23 pp.

The boom in high-powered tanker-building after the last war led to many cases of vibration troubles and cavitation-erosion damage on the propeller blades. For this reason an extensive systematical investigation into the effect of shape of afterbody on propulsion has been carried out at the NSMB. The basic ship type was a 39,000-ton-deadweight tanker, the maximum size that can pass the Suez Canal in loaded condition. The variations in afterbody which were studied follow: I–Optimum shape from an efficiency point of view, based on statistical data of the NSMB (moderately U-shaped sections). II–Extremely V-shaped sections. III–Extremely U-shaped sections. IV–Cigar-shaped stern with Mariner rudder arrangement (Hogner afterbody). V–Extremely U-shaped sections combined with a screw in a nozzle. VI–Extremely U-shaped sections combined with two screws, each fitted in a nozzle, one above the other. VII–A twin-screw arrangement. Resistance and self-propulsion tests were carried out with four- and five-bladed propellers in the loaded and light condition. Wake measurements with pitot tubes and flow observations with tufts have been made. Cavitation tests in the tunnel with adjustable flow were carried out. Finally, the variations in the propeller thrust and torque during one revolution were recorded and analyzed.

From authors' summary

6576. Fellous, J. R., Theoretical and experimental contribution to the study of the wake in guided flows, Parts I and 2 (in French), *Rev. Inst. Fr. Petrole et Ann. Comb. Liquid* **14**, 10, 1307–1337, Oct. 1959; **14**, 11, 1546–1570, Nov. 1959.

Paper deals specially with the aerodynamic properties of wakes behind V-shaped flame holders (V.F.H.) used in some post-combustion ducts for turbojets, and gives information of fundamental and practical interest.

In the first part author examines (1) the wake configuration, the velocity along the wall of V.F.H., and the drag, by means of a two-dimmensional Helmholz-Kirchhoff scheme; (2) the development of the laminar boundary layer which determines the begin-

ning of every phenomena occuring in the wake; (3) the reasons why it is impossible to elaborate an analytical theory of turbulent wake.

The second part describes experiments on some V.F.H. (height: $b = 2$ to 5 cm, half top angle: $\beta = 15$ to 30°) in air flow (rectangular duct 16×18 cm) of 20-31 m/s velocity. Measurements of the quantity Q of air penetrating in the wake, its stay-time τ and the drag of the V.F.H. were performed by means, respectively, of an infrared absorption (by CO_2 injected in the flow) method and balance. Some results used to elaborate flame-holding devices now under study at S.N.E.C.M.A. are given.

N. Manson, France

6577. Arens, M. Application of air-breathing jet engines to high Mach number vehicles, *Aero/Space Engng.* **19,** 8, 26-31, 46, Aug. 1960.

6578. Rao, G. V. R., Approximation of optimum thrust nozzle contour, *ARS J.* **30,** 6, p. 561 (Tech. Notes), June 1960.

6579. Moult, E. S., Current problems in aero engine design, *J. Roy. Aero. Soc.* **64,** 593, 259-272, May 1960.

6580. Gordon, L. J., Ballistic effect of pyrolyzed liner in solid propellant motor firings, *ARS J.* **30,** 5, 502-503 (Tech. Notes), May 1960.

Any combustion chamber liner material lost during burning of a solid propellant motor contributes to the momentum of the exhaust even though it adds no energy to the stream. A relationship yielding the true propellant specific impulse from the measured thrust-time integral, the propellant weight and the weight of liner lost in a static test firing is derived. A relationship between vehicle burnout velocity and these parameters is also given.

From author's summary

6581. Wang, C. J., Anthony, G. W., and Lawrence, H. R., Thrust optimization of a nuclear rocket of variable specific impulse, *ARS J.* **29,** 5, 341-344, May 1959.

A simple relation is given relating specific impulse and flow rate of a propellant gas under dissociating conditions. From this an optimum thrust program is found for case of constant gravity and zero aerodynamic drag. A large advantage over the usual constant-thrust program is demonstrated.

From authors' summary by S. H. Maslen, USA

6582. Raether, M. J., and Seitz, R. N., Pierce gun design for an accelerate-decelerate ionic thrust device, *ARS J.* **30,** 7, 640-642 (Tech. Notes), July 1960.

6583. Moeckel, W. E., Baldwin, L. V., and English, R. E., Lubarsky, B., and Maslen, S. H., Satellite and space propulsion systems, NASA TN D-285, 47 pp., June 1960.

This report summarizes results of a study of low-thrust systems for space and satellite propulsion undertaken in 1957 by members of the staff of the Lewis Research Center. Particular emphasis is placed on electric propulsion. After an introduction to the general characteristics of electric propulsion systems, some of the most promising electric power generators and thrust generators are reviewed. Electric propulsion systems are compared to chemical and nuclear rockets for satellite sustaining and orbit control, and for manned and unmanned missions to Mars.

From authors' summary

6584. Moeckel, W. E., Propulsion methods in astronautics, Advances in Aeronautical Sciences, Vol. 2 (Proc. of the First International Congress in the Aeronautical Sciences, Madrid, Sept. 8-13, 1958); Pergamon Press, 1959, 1078-1097.

Magneto-fluid-dynamics

(See also Revs. 6159, 6418, 6528, 6582, 6681)

6585. Gambirasio, G., On the electrical behavior of an ideal plasma, *Physics of Fluids* **3,** 2, 299-302, Mar./Apr. 1960.

Problem studied is that of an electric field, perpendicular to a constant magnetic field in an ideal plasma, abruptly increased from zero to a constant value. The resulting time-dependent current flow is studied by means of Laplace transforms. Exact and approximate solutions for some simple cases are obtained and discussed. Reviewer wonders how the magnetic field remains constant in the presence of time-dependent currents.

R. A. Gross, USA

6586. Barger, R. L., Brooks, J. D., and Beasley, W. D., An experimental study of the ionization of low-density gas flows by induced discharges, NASA TN D-431, 21 pp., Sept. 1960.

Induced discharges are advantageous for ionizing low-density flows in that they introduce no electrode contamination into the flow and they provide a relatively high degree of ionization with good coupling of power into the gas. In this investigation a 40-megacycle oscillator was used to produce and maintain induced discharges in argon and mercury-vapor flows. Methods for preventing blowout of the discharge were determined, and power measurements were made with an in-line wattmeter. Some results with damped oscillations pulsed at 1,000 pulses per second are also presented.

From authors' summary

6587. Steginsky, B., Magnetohydrodynamic cavities, *ARS J.* **30,** 7, 642-643 (Tech. Notes), July 1960.

Some features of magnetohydrodynamic cavity formation are discussed in qualitative terms. Examination of the dependence of the final steady-state configuration on the initial conditions in two cases reveals incompatibilities which may arise in the physical interpretation of the problem if a proper "model" is not specified.

From author's summary

6588. Talbot, C. F., Pressure data obtained from a continuously flowing plasma, Mass. Inst. Technol., Naval Supersonic Lab. TR 319, 20 pp., Jan. 1960.

This report presents data on the static pressure profile associated with the continuous flow of partially ionized helium through a tubular test section. The data yielded were insufficient to fix the state of the flow; several hypotheses are advanced to explain the behavior.

From author's summary

6589. Chang, C. C., and Lundgren, T. S., Flow of an incompressible fluid in a hydromagnetic capacitor, *Physics of Fluids* **2,** 6, 627-632, Nov./Dec. 1959.

Hydromagnetic capacitor consists of a torus of rectangular cross section, filled with an electrically conducting, incompressible fluid. Inner and outer cylindrical walls are perfectly conducting electrodes, top and bottom plane walls are nonconducting. If an electromotive force is maintained across the electrodes, and a magnetic field applied in axial direction, fluid will be set in rotation about torus axis at a rate such that viscous forces will balance Lorentz forces in steady operation.

Motion equations are derived from Cowling's equation set, i.e. with free charge and displacement currents neglected. Although problem is axisymmetric with simple boundary conditions, it is difficult to solve analytically unless simplifying assumptions are made of a small torus thickness as compared to its radial extension and no viscosity effect at the circumferential walls. Steady flow problem with constant potential difference between cylindrical wall is first solved and formulas derived for velocity and induced field. Calculation of total viscous and Joule dissipation

rates yields result in agreement with result obtained in considering whole device as a circuit element with corresponding resistance. Ratio of viscous and Joule dissipation effects is also calculated, showing essentially equal effects for Hartman numbers $M > 3$. Total kinetic energy stored is finally computed, showing a maximum value at $M = 1.5$ approximate, with M^2-proportional decrease for $M > 2$-values.

As a second step, simplest unsteady problem is analyzed, i.e. charging the device from rest by a constant electromotive force source. Further simplification of assuming small value of parameter $\lambda = (\mu_0 \sigma \nu)^{1/2}$ (μ_0 magnetic permeability, σ electrical conductivity, ν kinematic viscosity) must be made for solving for velocity and induced field by means of trigonometric series. Finally, behavior of hydromagnetic capacitor under charginc conditions is shown to be equivalent to that of an electric parallel multicapacitor circuit combined with a parallel resistance. For large M-values, equivalent circuit simply consists of one resistance and one capacitor in parallel connection. Validity of assumption of small λ-value is discussed.

P. Schwaar, USA

6590. Brinkman, H. C., The vortex equations of magnetohydrodynamics (in English), *Physica* **25**, 11, 1063–1066, Nov. 1959.

The vortex equations are written in terms of a material vorticity, an electric vorticity and a magnetic vorticity (or electron cyclotron frequency). The electric vorticity is defined as the cell of the current density divided by the product of the unit charge and one-half the particle density. Kelvin's theorem for the circulation is derived and the electric and magnetic coupling are clearly shown. The analogy to Kelvin's theorem for the electic circulation is derived [Eq. 16.2].

In the opinion of the reviewer the form of the theorem can be simplified by use of Faraday's law and using the fact that the time derivative in Faraday's law is not transformed into a convection operation.

E. E. Covert, USA

6591. McClimans, T., A preliminary experimental study of skin friction in magnetogasdynamics, Mass. Inst. Technol., Naval Supersonic Lab. TR 396, 74 pp., Oct. 1959.

This report presents the theory of hydromagnetics and its application to skin friction forces. With a continuous flow plasma jet, experiments were conducted for the flow of high temperature air in a circular tube. The friction forces were determined by pressure drops along the tube.

Fluid temperatures in the experiments ranged from 6000°R to 8000°R, which gave low Hartmann numbers.

Feasibility of further experiments in the ionized gas region is discussed.

From author's summary

6592. Zabusky, N. J., Hydromagnetic stability of a streaming cylindrical incompressible plasma, *Physics of Fluids* **3**, 2, 278–288, Mar./Apr. 1960.

A dispersion relation is derived and analyzed for the case where the equilibrium velocity of an incompressible, nonresistive, cylindrical plasma has a spiral motion along magnetic field lines. The symmetric hydromagnetic equations are used to derive the plasma hydromagnetic pressure.... The presence of flow introduces overstable modes. For $m = 0$ the time divergences are removed by flow. For $m = 1$ the divergences are enhanced by flow such that the growth rates and oscillation frequencies increase linearly with the flow velocity. The smaller the wavelength of the disturbance, the larger are the overstable eigenvalues.

From author's summary by R. A. Gross, USA

6593. Montgomery, D., Stability of large amplitude waves in the one-dimensional plasma, *Physics of Fluids* **3**, 2, 274–277, Mar./Apr. 1960.

Problem of the stability of the nonlinear plasma oscillations of Bernstein, Greene, and Kruskal is discussed. The eigenvalue equation for the perturbed distribution function possesses an expansion in powers of a parameter proportional to the maximum value of the equilibrium electrostatic potential. The stability of any given distribution can be inferred from consideration of the zeroth order alone.

From author's summary by R. A. Gross, USA

6594. Holter, O., Jensen, E., Kildal, A., Knoff, J. B., and Voyenli, K.-F., Theoretical researches in magneto-hydrodynamics, AFOSR TR 59-92 (Inst. Theor. Astrophys.; ASTIA AD 219 370), 19 pp., June 1959.

Researches have been made in the general field of magnetohydrodynamics by a special research team of the Institute of Theoretical Astrophysics, Oslo. The subjects covered are plasma oscillations in the classical sense; general theory of wave propagation in a plasma; the form and importance of viscous forces in plasma phenomena when magnetic fields are active; plasma shock waves; and forces in a plasma conditioned by certain deviations from thermal equilibrium, in particular deviations from equipartition of kinetic energy.

Each team member has produced a paper summarizing the results obtained during the calendar year 1959. The papers are printed separately, and the report gives only a brief summary of the contents of the printed papers, which again, in nearly all cases are shortened English versions of more comprehensive papers originally written in Norwegian.

From authors' summary by D. C. Leigh, USA

6595. Zhigulev, V. N., Analysis of weak discontinuities in magnetohydromechanics, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 1, 107–113, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Using the method of characteristics, the weak discontinuity in a magnetohydrodynamic flow is discussed. The treatment bears close parallelism to an American paper published about the same time. ["Note on magnetohydrodynamics, VIII. Nonlinear wave motion" by K. O. Friedrichs and H. Kranzer, Air Research and Development Report, N.Y.O.-6486, July 31, 1958].

C. T. Chang, Sweden

6596. Voyenli, K., On plane stationary shock waves in a plasma, AFOSR TN 60-424 (Univ. Oslo, Inst. Theor. Astrophysics Scient. Rep. no. 3), 88 pp., 1959.

A review and some extensions of the theory of plane hydromagnetic shocks with arbitrary magnetic field orientation are given using both classical and relativistic treatments. Primary effort here is in the derivation and the solution of the jump conditions across the shock.

H. Yoshihara, USA

6597. Olsen, H. N., Thermal and electrical properties of an argon plasma, *Physics of Fluids* **2**, 6, 614–623, Nov./Dec. 1959.

Temperatures ranging from 10,000 to 25,000°K have been measured spectroscopically in thermal plasmas of atmospheric pressure argon arcs at currents in the range of 200 to 800 amp. Electrical properties of the plasmas have been derived from measured radial temperature distributions using Spitzer's theory for the temperature dependence of electrical conductivity of a completely ionized gas. Existence of local thermal equilibrium has been demonstrated by the agreement between excitation temperatures determined from both atomic and ionic spectral line intensities. Agreement between values of electrical quantities obtained by direct measurement and those derived from measured temperatures based on the assumption of thermal equilibrium demonstrates the internal consistency of the experimental and analytical methods.

From author's summary by F. Tamaki, Japan

6598. Regirer, S. A., Concerning the thermal effect in the flow of an electrically conducting fluid between parallel walls, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 5, 1346–1350, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

With thermal and electrical conductivity and magnetic permeability assumed constant, the viscous flow between infinite parallel walls subjected to a uniform magnetic field, normal to the wall, is studied for the cases of constant viscosity and of viscosity varying as $(1 + \text{const} \cdot T)^{-1}$, where T is temperature. In the first case, a closed-form solution is obtained and the relative effects of Joule and viscous dissipations are compared. In the second case, series solutions are derived and shown to be absolutely convergent for sufficiently slow viscosity variation with T and sufficiently weak magnetic fields.

M. V. Morkovin, USA

6599. Levich, V. G., and Myamlin, V. A., Motion of mercury globules in the gravitational field and in a magnetic field (in Russian), *Zh. Fiz. Khimii* **31,** 11, 2453–2457, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2354.

The problem is solved of the fall of a mercury drop into an electrolytic solution in the presence of a constant magnetic field, directed perpendicularly to the field of gravity. It is assumed that the distribution of currents in the drop does not alter while the distribution of velocities differs but little from that occurring without a magnetic field [V. G. Levich, "Physico-chemical hydrodynamics," Gostekhizdat, 1952]. It is shown that under the action of a supplementary force from the direction of the magnetic field a motion of the drop appears on the current, perpendicular to the force of gravity and the magnetic field. A formula is obtained for the velocity of the motion. If a drop with a radius of 0.1 cm falls into a solution of the electrolyte in glycerine with a density for the surface charge of the double layer equal to 10^{-5} C/cm^2 then in the presence of a field of 10^8 gauss that velocity, in agreement with the evaluation brought in, will be of the order of 10^{-2} cm/sec.

G. E. Gershuni

Courtesy Referativnyi Zhurnal, USSR

6600. Steg, L., and Sutton, G. W., The prospects of MHD power generation, *Astronautics* **5,** 8, 22–25, 82–86, Aug. 1960.

Major applications, other than a short-duty cycle generator for space vehicles, await answers to major questions about magnetohydrodynamic phenomena, and the advent of fruitful inventions.

From authors' summary

6601. Gershman, B. N., Influence of the magnetic field on the turbulent instability in the ionosphere (in Russian), *Nauchn. Dokladí Vyssh. Shkoly. Fiz.-Matem. Nauk* no. 1, 114–118, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6045.

An estimate is made of the influence of the magnetic field on the criteria of appearance of turbulence because of the prevalence of ionospheric winds. To this end author utilizes the solution for the problem on the appearance of instability in an even flow between two parallel planes with a magnetic field H_0 situated perpendicularly to the boundary planes. A similar problem has been solved previously [L. H. Thomas, *Phys. Rev.*, **91,** 780–783, 1953; J. T. Stuart, *Proc. Roy. Soc. Lond.* (A) **221,** p. 189, 1954] without taking into account, however, the anisotropy of the conduction. This omission is taken up by the author. It is shown that in this case the basic parameter Q is equal to

$$Q_{\text{eff}} = \frac{a^2 H_0^2}{4\pi\rho\nu\delta_{\text{eff}}}$$

where $2a$ is the thickness of the flow, ρ is the density, ν is the viscosity, while

$$\delta_{\text{eff}} = \delta\left(1 + \frac{\delta_H^2}{\delta^2}\right) = \frac{c^2}{4\pi\sigma}\left(1 + \frac{\sigma^2}{\sigma_H^2}\right)$$

where the conduction $\sigma = e^2 N/m(\nu_e + \nu_m)$, $\sigma_H = e^2 N/m\omega_n$; ν_e, ν_m are the effective numbers of electrons, ions and molecules in collision; N is the concentration of electrons (charge e, mass m); ω_H is the gyro frequency of the electrons. For isotropic conduction in Q, instead of δ_{eff} we have $\delta_0 = c^2/4\pi\sigma_e$, where $\delta_e = e^2 N/m\nu_e$. δ_{eff} is always $> \delta_0$, and consequently $Q_{\text{eff}} < Q\delta_0$, that is, taking account of anisotropy always results in a diminution in the stabilizing influence of the magnetic field. The rest of the results remain unaltered and, in particular, the principle of the distribution of velocities is preserved. The author makes use of the works of Thomas and Stuart, replacing Q by Q_{eff} for investigating the stabilizing influence of the magnetic field and shows that at altitudes of 75–80 km it is small, while at a height of 120 km it might affect wind movements which might reach the order of tens of kilometers.

S. A. Kaplan

Courtesy Referativnyi Zhurnal, USSR

Aeroelasticity

6602. Craven, A. H., The aerodynamic derivatives of an aerofoil oscillating in an infinite staggered cascade, Coll. Aero. Cranfield, Rep. 125, 17 pp., Jan. 1960.

Results indicated in title are obtained in complex integral form by use of incompressible flow thin airfoil theory and solution of an integral equation given in "Integral equations," by S. G. Mikhlin, (Pergamon Press, 1957). Reviewer believes that failure to distinguish between imaginary indices for harmonic time dependence and for complex conjugate velocities negates results for phase differences between adjacent blades other than 0 or Π. Author gives partial check with previous theories for these two cases. No numerical results are presented.

F. Sisto, Jr., USA

6603. Gainer, P. A., and Aiken, W. S., Jr., Modified matrix method for calculating steady-state span loading on flexible wings in subsonic flight, NASA Memo 5-26-59L, 62 pp., June 1959.

A rapid iterative method for computing the steady-state span loading on flexible wings in subsonic flow is presented, using tables of downwash factors to find the required aerodynamic influence coefficients for application of lifting-line theory. Spanwise distributions of angle of attack, aerodynamic loading, shear, bending moment, and pitching moment are obtained as functions of the product of the dynamic pressure and the rigid-wing lift-curve slope. For the sample calculation presented, the time required for solution was of the order of one-tenth the time required by previous methods.

A. H. Sacks, USA

6604. Hooper, W. E., A theoretical approach to some rotor blade flutter and forced vibration problems, *J. Helicop. Assn.* **12,** 282–307, Dec. 1958.

The equations of motion in matrix form of a rotor blade are developed using normal modes. Several numerical examples are presented. Author concludes that, for flutter mode representation, the modes are, in order of importance: rigid pitch, first torsion, rigid flap, first flap bending, second torsion.

H. H. Hilton, USA

Proceedings of the National Specialists Meeting on Dynamics and Aeroelasticity, Fort Worth, Texas, Nov. 6–7, 1958; New York, Institute of the Aeronautical Sciences, 135 pp., $6. (Paperbound) (Revs. 6605–6617)

6605. Hsu, P.-T., Some recent developments in the flutter analysis of low-aspect-ratio wings, 7–26.

For calculating flutter, aerodynamic derivatives have to be available for arbitrary motion of the lifting surface. For modern aircraft, two-dimensional derivatives (strip method) are no longer

sufficient. General three-dimensional analytic solutions are, however, not available.

Author describes a method for numerical calculation of pressure distributions on wings of arbitrary planform using the kernel function method and the collocation method. The error in the pressure distribution is minimized on the average by proper choice of upwash collocation points and integration points respectively. The average error can be brought to zero, if the upwash function is represented by a polynomial of certain maximum order and the upwash collocation points are chosen from a quadrature equation for polynomial functions. In span-direction, one thus gets Multhopp's well-known downwash collocation stations. As general form of the pressure distribution function the steady-state incompressible solution is chosen.

Method is applied to several two-dimensional cases, where exact solutions are available. Comparison shows satisfactory agreement at least for translational motion of the wing. As an example three different small-aspect-ratio wings are computed and solutions compared with other approximate solutions and with experiments. The results seem to be encouraging, but for higher modes of wing motion a rather large number of collocation points will probably be necessary. Even then the method presented may compare favorably with other known methods. Method is readily adaptable to high-speed computer. H. Merbt, Sweden

6606. Cunningham, H. J., and Woolston, D. S., Developments in the flutter analysis of general plan form wings using unsteady air forces from the kernel function procedure, 27–36.

For arbitrary planform and arbitrary motion of the wing, aerodynamic forces have to be calculated numerically in most cases. Several computation methods have been proposed during the past 20 years. Author gives a short review of the so-called kernel function method and its application. This method is applicable throughout the Mach number range and has recently been used—both in the subsonic and supersonic ranges—to calculate aerodynamic loads and flutter boundaries with the help of high-speed computing machines.

Various such results are reviewed in this paper and critically discussed by comparing them with other theoretical results and with experiments. Comparison seems to indicate that the kernel function method as applied mainly by the NACA has given a very promising means for routine calculations in static and dynamic aeroelastic problems. H. Merbt, Sweden

6607. Abramson, H. N., Investigation of a method for the measurement of subsonic oscillatory aerodynamic influence coefficients, 37–51.

See AMR **12**(1959), Rev. 6345.

6608. Pines, S., An elementary explanation of the flutter mechanism, 52–58.

This paper presents a simple description of a large class of flutter phenomena. A frequency coalescence theory is presented which accounts for what is called "catastrophic flutter." Insight is demonstrated as to the most effective pertinent flutter parameters so that some guidance may be had in designing flutter fixes. The theory offers a simple explanation of many of the puzzling characteristics of classical flutter. No attempt is made to cover all flutter phenomena; in particular, single-degree-of-freedom flutter is not included within its scope. However, bending torsion flutter, panel flutter, and control surface flutter involving more than one degree of freedom are explained. The paper also presents a closed-form solution of the flutter equations for the simple two-degree-of-freedom case and indicates an approach for the solution of the more general multi-degree-of-freedom flutter problem.

From author's summary by R. L. Bisplinghoff, USA

6609. Head, A. L., Jr., A philosophy of design for flutter, 59–65.

6610. Mahaffey, P. T., The use of low speed flutter model tests and trend curves to establish flutter boundaries, 66–68.

It is convenient to plot results of flutter calculations in the form of so-called flutter envelopes together with the flight envelope in a plot of the stiffness-altitude parameter versus Mach number, revealing directly critical regions of flutter. The flutter envelope often consists of straight portions in the subsonic and supersonic regions respectively and a "compressibility hump" in the transonic region. Starting from this fact, author suggests the following test method: only one low-speed point of the flutter envelope is determined with the help of an accurate flutter model. All other flutter tests are made with very much simplified models to get the trend of the flutter envelope in both subsonic and supersonic regions and the relative height of the transonic hump. These measurements are then corrected so that the flutter envelope passes through the one correct low-speed point assuming that the observed ratio of the two measurements of the stiffness-altitude parameter at this point can be applied to the whole length of the boundary determined with the help of the simplified models.

One example is given. Method must be applied with great caution. It will especially be difficult to predict how simple the high-speed models can be made with respect to simulation of density ratios, mode shapes, and frequency ratio as to still give the trend of the envelope reasonably correct.

H. Merbt, Sweden

6611. Asher, G. W., A method of normal mode excitation utilizing admittance measurements, 69–76.

The application of admittance measurements to the determination of natural undamped vibration modes of scaled elastic models is discussed. Proposed methods are useful in separating two modes with nearly identical frequencies. Pertinent experimental techniques are discussed. B. A. Boley, USA

6612. Nelson, H. C., Zapotowski, B., and Bernstein, M., Vibration analysis of orthogonally stiffened circular fuselage and comparison with experiment, 77–87.

A Rayleigh-type analysis of the natural frequencies of an orthogonally stiffened, freely supported circular fuselage section is presented, in which the stiffened cylinder is replaced by an equivalent shell of uniform thickness. Numerical results are given, showing the effect on the frequency of the model patterns and of geometrical and structural dimensions. Good correlation with experiment is reported. B. A. Boley, USA

6613. Archer, J. S., A stiffness matrix method of natural mode analysis, 88–97.

A systematic method of free vibration analysis utilizing the IBM 704 is presented with particular attention to its application in thin, low-aspect-ratio wings of various shapes and various types of construction. Experimental results for a reinforced-skin delta wing, for a solid honeycomb core fin, and for a delta plate are presented, and are in good agreement with the calculated ones.

B. A. Boley, USA

6614. Zimmerman, N. H., Landing gear vibrations induced by skidding tires—theoretical and experimental study, 98–111.

Nonlinear single-degree-of-freedom analysis shows good experimental correlation. Influence of many factors on stability and limit cycle amplitude are included for design criteria. Author enlarges on factors considered by Wignot and Hoblit.

C. R. Freberg, USA

6615. Kistler, E. L., Jr., and Capalongan, F. F., Some studies of the dynamic motions of hypervelocity, high altitude vehicles, 112–117.

For development of autopilots for airplanes and missiles, study of the dynamics of the free aircraft is often essential for successful

design. Paper investigates frequencies and damping in a two-degree-of-freedom longitudinal short-period mode of a triangular wing-body combination at 8 < Ma < 30 and 100,000 feet < height < 200,000 feet using an analog computer. Level-flight and shallow-angle-entry conditions are investigated. Satisfactory solutions could only be obtained by very special set-up of the computer and very careful scaling because of the extremely low damping at relatively high frequencies. Reasonable accuracy was obtained for small perturbations using linear aerodynamic theory. Nonlinearities in the basic equations and in the aerodynamic terms could be omitted in most cases.

The shallow-angle-entry case can be solved approximately using the linear level-flight solutions for frequency and by approximating the damping envelope through the apparent damping due to air density. H. Merbt, Sweden

6616. Merrick, V. K., The influence of rate of change of altitude and acceleration on the longitudinal dynamic stability of aircraft and missiles, 118-127.

For modern aircraft and missiles the influence of acceleration and rate of change of altitude can, for dynamic stability investigations, no longer be neglected,—the result being a system of second-order differential equations with variable coefficients. Author presents an approximate solution for a system with one variable and shows that this solution is suitable for aircraft and missile small-perturbation analysis. It is possible—yet rather laborious—to extend that solution to systems of more than one variable.

For the case of longitudinal motion and oscillatory perturbation in direction normal to flight path limits of validity of conventional stability theory have been investigated for influence of acceleration and rate of change of altitude. Graphs of the results are given. For very stable configurations, influence is not large but increases rapidly with decreasing stability. In this range, conventional stability calculations cannot be applied. For the case of a short-range ballistic missile even time-histories of angle of incidence and velocity are calculated from the approximate solution.

H. Merbt, Sweden

6617. Deutschman, J. N., and Schlessinger, M., Dynamic model tests of an unconventional landing configuration, 128-135.

Bell $X-2$ airplane with nose gear and aft oleo-shock strut supported skid is analyzed by models. Landing problems, model design and test analysis are described. Dynamics and stability are successfully predicted. C. R. Freberg, USA

End of Symposium

Aeronautics

(See also Revs. 6179, 6187, 6366, 6368, 6465, 6563, 6608, 6614, 6616, 6617)

Book—6618. Cristescu, J., Coaxial double propellers; Single propellers (in French), Publ. Scient. Tech. Min. Air, France no. 356, 276 pp., 1959.

Methods for analysis of coaxial double propellers turning in opposite senses are studied along with those for analysis of single propellers. First chapter presents derivation of propeller aerodynamic equations following six different methods, according to various assumptions in regard to assumed number of blades, variation of circulation along blades and along circle swept by blades; and induced velocities. Chapter II presents computation procedures for doublet of maximum efficiency by the six methods developed in chapter I. Chapter III deals with model tests of double propellers and their comparison with the computed values. Chapter IV presents methods of computation of variable circulations along blades of coaxial double propeller during one rotation. Chapter V deals with single propellers; Chapter VI with single propellers with large loads; Chapter VII with single propellers having a hub; and Chapter VIII with a second solution with average velocities. Of the six procedures proposed by author, best agreement between doublet computations and experimental results is obtained assuming variable circulation, finite number of blades, different induced velocities for both propellers and circulation concentrated around the blades and not distributed along the circle swept by the propellers. For single propellers, variation of circulation along blade is assumed to be elliptic with respect to parameter formed by the square of the relative distance to the axis. Circulation is also assumed concentrated around the blade. An Appendix shows several integrations used in the book.

A. Balloffet, USA

6619. Miele, A., Equations of motion of a rocket-powered aircraft, Boeing Scientific Research Labs., Seattle, Wash., Flight Sciences Lab. Rep. 20, DI-82-0038, 24 pp., Jan. 1960.

Equations of motion of a rocket-powered aircraft are derived in a formal sense, and the necessary approximations rendering these equations into a useful form in an engineering sense are noted. No significant new results are obtained from such an analysis, although this may not be intended. C. C. Wan, USA

6620. Huston, R. J., and Winston, M. M., Data from a static-thrust investigation of a large-scale general research VTOL-STOL model in ground effect, NASA TN D-397, 66 pp., Aug. 1960.

Model force data and pressure distribution on wing and flaps at center of wing semispan, measured at two ground heights above the ground, are presented without analysis for various combinations of propeller disk loading, wing tilt, and flap deflection.

From authors' summary

6621. Newsom, W. A., Jr., Effect of ground proximity on aerodynamic characteristics of two horizontal-attitude jet vertical-take-off-and-landing airplane models, NASA TN D-419, 20 pp., Aug. 1960.

The first model had a tilting wing-engine assembly which was set at 90° incidence for take-off and landing. The second model had a cascade of retractable turning vanes to deflect the exhaust of the horizontally mounted jet engines downward for vertical take-off and landing while the entire model remained in a horizontal attitude. Tests were made for a range of heights above the ground and included force tests and tuft studies of the flow field caused by the jet exhaust. The results indicate that the deflected-jet model suffered losses in lift as high as 45% near the ground because of a low-pressure region under the model caused by the entrainment of air by the jet exhaust as it spread out along the ground. The tilt-wing model experienced a loss of lift of less than 3% near the ground. From author's summary

6622. Lazzarino, L., Considerations on shortened and vertical take-off and landing of aircraft (in Italian), *Aerotecnica* **39,** 5, 213-218, Nov. 1959.

The technical evolution of some widely used types of civil and military aircraft makes more and more unsatisfactory the conventional solutions of the take-off and landing problems.

A great deal of research work has been accomplished and is being carried out at present for finding of new solutions of the indicated problems....

The application to an aircraft of devices, with the aim of improving its take-off and landing maneuvres, implies generally some worsening of flight characteristics that may be very important in relation to the principal purpose of the aircraft. The possibility of setting up evaluation methods of the various flight characteristics is discussed, with the aim of obtaining a quantitative

comparison between different solutions and between various possible design outlines of each solution; some comparison criteria are proposed in relation to some particular cases of outstanding interest.

Successively the effects of gusts on the flight behavior and safety of aircraft, flying at low speeds during take-off and landing maneuvres, are discussed. From author's summary

6623. Rogallo, F. M., Lowry, J. G., Croom, D. R., and Taylor, R. T., Preliminary investigation of a paraglider, NASA TN D-443, 25 pp., Aug. 1960.

Preliminary tests of flexible wing gliders indicate stable, controllable vehicles at both subsonic and supersonic speeds. Such vehicles may be made extremely light with available materials. The results of this study indicate that this concept may provide a lightweight controllable paraglider for manned space vehicles.

From authors' summary

6624. Houbolt, J. C., and Batterson, S. A., Some landing studies pertinent to glider-reentry vehicles, NASA TN D-448, 22 pp., Aug. 1960.

The results presented may serve as guidelines for consideration of landing problems of glider-reentry configurations. The effect of the initial conditions of sinking velocity, angle of attack, and pitch rate on impact severity and the effect of locating the rear gear in various positions are discussed. Some information is included regarding the influence of landing-gear location on effective masses. Preliminary experimental results on the slide-out phase of landing include sliding and rolling friction coefficients that have been determined from tests of various skids and all-metal wheels. From authors' summary

6625. Shanks, R. E., Investigation of the low-subsonic flight characteristics of a model of an all-wing hypersonic boost-glide configuration having very high sweep, NASA TN D-369, 21 pp., June 1960.

Flight tests were made over an angle-of-attack range from 15° to 30° with and without artificial roll damping added. Force tests were made at angles of attack from 0° to 32° to determine the static stability and control characteristics.

From author's summary

6626. Harrin, E. N., Investigation of tandem-wheel and air-jet arrangements for improving braking friction on wet surfaces, NASA TN D-405, 20 pp., June 1960.

An investigation was made on a tire treadmill to determine the effectiveness of two methods of clearing away water ahead of a braking wheel in improving braking friction. One method consisted of mounting an idling wheel ahead of the braking wheel and the other method consisted of directing an air jet on the water-covered surface. The depth of the water was 0.09 inch for all tests. Measurements of maximum braking, full-skid braking, and free-roll friction coefficients were obtained for both methods while operating smooth and diamond-treaded 3.00×7 tires at speeds up to 93 feet per second. From author's summary

6627. Mewes, E., Diagrams for oscillation computations, especially for the determination of loads on control surfaces in the incompressible region (in German), *Z. Flugwiss.* **6**, 7, 203-207, July 1958.

Author presents a series of diagrams giving the solution in nondimensional form of the standard differential equation for the response of a damped one-degree-of-freedom linear system with a prescribed forcing function, applicable, for example, to the short-time longitudinal motion of an airplane due to a sudden elevator deflection. Title notwithstanding, control surface loads are not calculated. M. T. Landahl, USA

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(See also Revs. 6154, 6171, 6183, 6204, 6205, 6338, 6369, 6538, 6539, 6573, 6583, 6584, 6616, 6619, 6657, 6721)

6628. Moe, Mildred M., Solar-lunar perturbations of the orbit of an earth satellite, *ARS J.* **30**, 5, 485-487 (Tech. Notes), May 1960.

For the case when the angular velocity of the satellite is much greater than that of the Sun or Moon, the disturbing body is held fixed by the author for a satellite revolution. This enabled him to integrate Moulton's equation of the osculating ellipse yielding expressions for the change of perigee, inclination, longitude of the node and argument of perigee per revolution as functions of the relative position of the perturbing body.

To estimate the errors made by fixing the disturbing body, the case of co-planar motion was solved by more rigorous methods and it was found that these errors are of the order of the angular velocity ratio (disturbing body versus satellite) which is usually less than 0.03.

Explorer VI was used as an example. The influence of the moon on the perigee was found to be −0.06 n.mi per revolution. The order of magnitude of solar perturbation was found to be of the tenths of a mile per revolution decreasing or increasing the perigee depending on the relative position of the three bodies. The combined solar-lunar perturbations can produce an important lowering of the perigee. G. S. Gedeon, USA

6629. Mickelwait, A. B., Lunar trajectories, *ARS J.* **29**, 12, 905-914, Dec. 1959.

Author reviews the analysis of three-dimensional lunar trajectories. The approximate theory is presented in some detail although curves of the relationships between flight parameters include the results of more exact theories. Considerable attention is paid to allowable dispersions in velocity, angle, time of firing, etc. As with most reviews this one requires a previous knowledge of the problem, or else quite a bit of study of the numerous references given will be necessary.

R. E. Street, USA

6630. Riddell, W. C., Initial azimuths and times for ballistic lunar impact trajectories, *ARS J.* **30**, 5, 491-493 (Tech. Notes), May 1960.

Approximate formulas are given to indicate the time of launch and initial azimuth for which an Earth-launched space craft will intersect the moon. These equations were obtained from a simple system, consisting of the Earth-Moon space and a ballistic body under the influence of a potential field due to a spherical Earth. Such a vastly oversimplified model gives surprisingly good first approximations, which can be then used as initial conditions for the numerical integration of the ballistic trajectories leading to the vicinity of the moon. G. S. Gedeon, USA

6631. Grobner, W., and Cap, F., The three body problem earth-moon-spaceship (in English), *Astronaut. Acta* **5**, 5, 287-312, 1959.

Astronautics has become fashionable in the relatively recent past. With this activity has come the clear recognition that many parts of classical celestial mechanics are now parts of engineering science. Although new problems have been created and old ones have been reemphasized, there is continuing need for careful studies in all areas, and this is particularly true in precision orbit analysis.

Since many journals are now filled with solutions to problems that have been treated in the literature of celestial mechanics, it is essential that new contributions be judged carefully. There are clearly many levels to consider, from the informed survey article to the ably written article which clarifies methods which may be well known but are inaccessible to modern students. However, many current articles in this field must be strongly criticized for

(1) ignorance of the literature which applies to the problem and (2) re-working of perfectly well-known problems, poorly done. The present paper is unfortunately a strong example to which both criticisms apply.

The paper considers the standard three-body problem of celestial mechanics and is related to a previous paper by F. Cap presented at the IXth International Astronautical Congress of 1958 (pt. I, p. 62). It was stated that "the closed solution of the n-body problem is presented" and in the present papers the authors assert that "the closed solution of the three-body problem is given". Neither statement is correct nor justified in any sense acceptable to those working in the field of celestial mechanics.

Since no literature references have been given by the authors except to their own published (and forthcoming) work it is not out of place to note that this literature is both vast and detailed on this problem. Furthermore the eminent mathematician C. L. Siegel wrote as recently as 1955 in "Vorlesungen über Himmelsmechanik" (Springer-Verlag 1956) (p. 20), "Das n-Körperproblem besteht in der Beschreibung des Gesamtverlaufes aller Lösungen der Bewegungsgleichungen für beliebig vorgegebene Anfangswerte. Trotz der Bemühungen hervorragender Mathematiker seit 200 Jahren ist dieses Problem für $n > 2$ bis heute ungelöst geblieben." That is, the "solution" of the n-body problem must consist in the description of all possible orbits with arbitrary initial conditions; and despite the most intense efforts of eminent mathematicians, the general problem remains unsolved in this general sense.

Siegel then continues to explain the mathematical nature of the difficulties. It was conjectured by Dirichlet, for example, that a general series solution valid for all times t was possible, and Weierstrass encouraged S. Kowalevski and Mittag-Leffler to investigate the possibility. The famous prize offered by the king of what was then Sweden and Norway for a final solution was awarded to Poincaré in 1889 despite the fact that he did not solve the problem as stated. However Sundman was awarded the prize for $n = 3$ some 20 years later. The outstanding difficulty of the problem consisted in the possibility of collisions of 2 or more bodies and how to continue the solutions analytically beyond such singularities. Sundman's great achievement was in his introduction of a new time variable ω in terms of which the solutions could be written as an infinite series valid for all times and valid in the presence of a double collision, the case of a triple collision alone being excepted. Despite its great theoretical importance the method used by Sundman has not found practical application and no comparable theoretical results are known for $n > 3$.

Sundman's work showed that the singularities introduced by collisions are an essential part of the difficulty in a general solution, in addition to the known restrictions due to the general theorem of Bruns-Poincaré on the non-existence of additional algebraic integrals of the n-body problem, other than the 10 known ones. The same difficulties are ably outlined in the recent book by E. Finlay-Freundlich entitled "Celestial mechanics" (Pergamon Press, 1958), Chapter I.

The previous brief summary is essential for an understanding of what is meant by a "solution" of the n-body problem. The solution must be valid for all times t. Now it is a serious matter indeed to assert that such a solution, closed or otherwise, has been found, and a serious criticism to assert that it has not been found. Clearly a study of the literature shows that this central problem of celestial mechanics has not been solved, or properly appreciated, by the present authors.

What then has been accomplished in this paper? The series used are named after the mathematician S. Lie and at first it is not obvious what these series represent. However an examination of the analytical results shows that the final form of the solution is nothing more than a power series in the time t about $t = 0$. The solutions, then, are purely and simply power series in t, valid at most for a finite time $t \leq T$, i.e. up to a collision, and this is admitted on p. 289. Thus the work offers nothing at all new or revealing. The technique is applied to the two-body problem, with all the usual results abundantly known in standard treatises (for example, see Moulton's "Celestial mechanics"). The plane restricted problem of 3 bodies is considered next and even poor Kepler's equation is solved again! The computation formulas (121) clearly show that the solutions are power series in (Δt) and Δt must be kept small. No proper attention to numerical difficulties (round-off) are discussed.

Thus the bulk of the paper simply repeats known results under a formalism which effectively obscures the familiarity of the approach. It should not be assumed that even a simple power series will be the most appropriate tool for orbital studies and, although not "wrong" in principle, it may well be entirely inappropriate as a general procedure—and classical celestial mechanics is devoted to discussions of alternatives to it.

F. V. Pohle, USA

6632. Oertel, G. K., and Singer, S. F., Some aspects of a three-body problem (in English), *Astronaut. Acta* **5**, 6, 356–367, 1959.

The problem of three bodies is the central problem of classical celestial mechanics. A highly restricted problem in this area is that of the motion of a particle in the presence of two *fixed* centers of attraction. The fact that the motion is integrable was discovered by Euler in 1760 and is a consequence of Liouville's theorem [see Whittaker's "Analytical dynamics," 4th ed. p. 67]. The problem was also discussed and generalized in Jacobi's famous lectures (1842-3) in dynamics, but its most complete modern treatment is to be found in Charlier's "Die Mechanik des Himmels," (Vol. 1, Section 3, pages 115–163). Charlier presented qualitative discussions of the orbits in the following cases (p. 152f): (1) Straight line motions, (2) lemniscate motions, (3) satellite motions, (4) planetary motions, (5) divergent pendulum motions, (6) hyperbolic sinusoidal motions, (7) diverging spiral motions, (8) converging spiral motions, (9) convergent pendulum motions, (10) asymptotic straight-line motion, (11) elliptic motion, and (12) hyperbolic motion. Charlier (p. 156f) also discussed examples where the analysis might be applied; this will be discussed later.

The present paper, with very few exceptions indeed, is all to be found in various sections of Charlier cited work, both as to classification and to method of solution. However, only bound orbits are included in the authors' analysis and the material that is added refers primarily to corresponding and known results in the Kepler problem. The first three appendices again repeat known material. However, no numerical results are given for any orbits.

In view of the known nature of the results in this paper, interest must center on the illustrative example in appendix IV where it is stated that the theory may be applied to discuss the motion of a satellite with a large bulge or "bump". However, no detailed calculations are given in support of the statement nor are any restrictions made as to the relative motion of the satellite.

It must be recalled that the two attracting centers are *fixed* in inertial space. Therefore, only in some clearly defined approximate sense could an orbit furnish information as to the motion of a small mass in the presence of two moving force centers (restricted problem of three-bodies). That Charlier clearly had such notions in mind is made abundantly clear in Section 8 (p. 156f) where it is asserted that although nature does not provide any examples of a particle motion in the presence of two fixed centers, neverthless it is to be hoped that such orbits could be used as starting approximations to the actual orbits. One such case is mentioned by Charlier (p. 157): the motion of a small body which is moving with great speed through a double-star system. However, a case of more direct interest would be to the lunar theory. Charlier's calculations (p. 158–163) lead to the conclusion that the orbits based upon two fixed centers of attraction cannot be used even as a first approximation to the actual motion. The illustration shows how important it is to work out specific examples in order to sup-

port the statement that the simplified theory can be applied in the way the authors have asserted. This has not been carried out in any detail.

It is also suggested (p. 363 of present paper) that since celestial bodies often have shapes which differ from that of a sphere, the resulting gravitational field can be approximated. "By using two point masses as sources of the field and adjusting their masses and distance so that an optimum fit of this model to the actual field is obtained, it is possible to find (at least approximately) the influence of the deviations from spherical shape on the field and small masses moving in it." In view of the delicate nature of carrying out such comparisons it would have been of interest to illustrate this in numerical detail; however, this has not been done even approximately. F. V. Phole, USA

6633. Lawden, D. F., Optimal escape from a circular orbit (in English), *Astronaut. Acta* **4**, 3, 218–233, 1958.

The general equations for the optimal escape of a rocket from a circular orbit in an inverse square law field of attraction are set up and two cases are considered. First, that in which the rocket thrust is large and produces constant acceleration at a small angle to the direction of motion. The time program for the angle is obtained and the characteristic velocity of the maneuver is shown to be in good agreement and having a small improvement over corresponding previous results obtained by Tsien [*Jet Propulsion* **23**, p. 233, 1953] and Benney [*Jet Propulsion* **28**, p. 167, 1958]. Second, when the thrust is small; and here the result obtained is that in the maneuver the direction of thrust should bisect the angle between the direction of motion and the radius vector. The program is thus the mean of the programs of the previously mentioned authors. R. C. Knight, England

6634. Okhotsimsky, D. E., Eneyev, T. M., and Taratyrova, G. P., The determination of the period of existence of an earth satellite and an investigation of the secular perturbations of its orbit (in Russian), *Uspekhi Fiz. Nauk* **63**, la, 33–50, 1957; *Ref. Zh. Mekh.* no. 8, 1958, Rev. 8371.

The problem is examined of the life (period of existence) of an artificial Earth satellite for the general case of motion. For the solution of this problem the Lagrange differential equations of motion are integrated for the case of osculating elements. These equations are transformed for a new, independent variable—the argument of the latitude—and are integrated for the following, simplifying assumptions: the satellite moves in a terrestrial atmosphere, the rotation of which with the Earth and its diurnal motion can be neglected; the gravity field of the Earth is assumed to be central. The acceleration of the force of atmospheric resistance is put in the form:

$$R_x = \frac{c_x F}{m} \frac{\rho v^2}{2}$$

where c_x is the coefficient of aerodynamic resistance (drag coefficient), F the area to which the coefficient c_x is referred; m mass of the satellite; v relative velocity of the satellite in the atmosphere; ρ density of the atmosphere, approximated for altitude by the equation:

$$\rho = \rho_1 \chi \left(1 + \frac{y - y_0}{\alpha}\right)^{-k}$$

(ρ_1 is density of the atmosphere at a particular altitude; χ, α and k constants).

Assuming that, during a single orbit of the satellite around the Earth, the parameter p, eccentricity e, and distance of the perigee from the node ω are constant, author reduces the problem of determining the period of existence of the satellite to the integration of a system of two ordinary differential equations of relatively simple form. This has enabled a family of integral curves to be plotted, depending on only one parameter.

Relationships are established between the fundamental parameters of the osculatory ellipse, depending only on the law of distribution of atmospheric density with altitude.

The equations have been integrated by a BESM electronic computer; the results of the calculations are tabulated and charted. In conclusion, such factors as the deviation of the terrestrial gravity field from the central form, and the co-rotation of the atmosphere with the Earth, are discussed. The numerical data given on the life of artificial satellites are applicable only for purposes of orientation, since, when these researches were being made, reliable information on the parameters of the upper atmosphere was not available. P. P. Lavrinenko

Courtesy Referativnyi Zhurnal, USSR

6635. Currie, M. M., Generalized tables for the calculation of trajectory curves for bodies moving in air, Nat. Res. Counc., Canada, LR-277, 52 pp., Apr. 1960.

Tables are presented from which the Cartesian coordinates of a trajectory can be readily derived for any initial conditions if the terminal velocity of the body is known.

From author's summary

6636. Miele, A., Theorem of image trajectories in the earth-moon space, Boeing Scient. Res. Labs., Seattle, Wash., Rep. DI-82-0039, 20 pp., Jan. 1960.

The motion of a small vehicle in the Earth-Moon space is considered using the mathematical model of the restricted three-body problem. Two theorems associated with this motion are established: the Irreversibility Theorem and the Theorem of Image Trajectories. The Irreversibility Theorem states that, if a trajectory is physically possible in the Earth-Moon space, the reverse trajectory is not physically possible. The Theorem of Image Trajectories states that, if a trajectory is physically possible in the Earth-Moon space, three image trajectories are also physically possible: (a) the image with respect to the plane which contains the Earth-Moon axis and is perpendicular to the axis of rotation of the Earth-Moon system; (b) the image with respect to the plane which contains the Earth-Moon axis and the axis of rotation of the Earth-Moon system; and (c) the image with respect to the Earth-Moon axis. The first of these image trajectories must be flown in the same sense as that of the basic trajectory, while the other two must be flown in the opposite sense. As a conclusion, the time required for the parametric study of lunar trajectories is reduced considerably, since, once a basic set of trajectories is calculated, three additional sets can be obtained by simple transformations of coordinates. From author's summary

6637. Deutsch, A. J., Orbits for planetary satellites from Doppler data alone, *ARS J.* **30**, 6, 536–542, June 1960.

When an artificial satellite is put in orbit around another planet, it will be difficult or impossible from terrestrial observations to obtain its direction with sufficient precision to define its orbit. However, if the satellite carries a suitable transmitter, Doppler observations are capable of giving precise values for the line of sight component of the velocity vector. From such data alone, it is possible to obtain all the orbital elements and the mass of the planet. The analytical apparatus used by astronomers for the discussion of spectroscopic binary stars is here adapted to the interpretation of Doppler observations of a planetary or lunar satellite. An example is given of the orbit determination for a satellite of Venus. From author's summary

6638. Gedeon, G. S., and Greer, B. J., Elliptical orbit characteristics, *ARS J.* **30**, 6, 570–572 (Tech. Notes), June 1960.

To expedite satellite feasibility studies, the well-known orbital equations are presented in the form of convenient charts. These

charts permit the establishment of the permanent characteristics of the orbit and also allow following the change of the variable elements along the orbit. From authors' summary

6639. Bonomo, P. J., Curves for rapid determination of orbital transfer requirements, *ARS J.* **30,** 4, 371–372, 421 (Tech Notes), Apr. 1960.

Nondimensional curves are presented which permit the determination of minimum energy requirements for orbit-to-orbit transfer in a central force field. The curves are general and apply to minimum energy transfers with any body (Earth, Sun, etc.) as the primary gravitational source. Four examples are given to illustrate the utility of the curves: Transfer from a 1000-mile Earth orbit to an orbit in the vicinity of the Moon, successive transfers from a 200-mile Earth orbit to a 22,4000-mile "stationary" orbit, and interplanetary transfer from Earth to Mars and from Earth to Venus (Hohmann transfer ellipses). From author's summary

6640. Licher, R. M., A class of optimum trajectory problems in non-central gravitational fields, Douglas Aircr. Co. Rep. SM-23962, 14 pp., May 1960.

The transfer of a rocket vehicle from one point to another in centrally directed gravitational fields under the requirement of minimum fuel expenditures has been studied by several authors. One method has been developed for determining certain cases for which the optimum fuel expenditure is achieved by using terminal impulses only with a coasting trajectory between terminals. A dimensionless parameter P containing the strength of the gravitational field and the prescribed transfer time is introduced to measure the nonuniformity of the gravitational field and to indicate when terminal impulses only are optimum.

The present report contains a simple extension of the above method to cases in which the gravitational field is produced by two or more bodies. It is assumed that there is no atmospheric resistance and that the transfer time and terminal velocity vectors are specified. From author's summary

6641. Struble, R. A., Some new satellite equations, *ARS J.* **30,** 7, p. 649 (Tech. Notes), July 1960.

6642. Arthur, P. D., Karrenberg, H. K., and Stark, H. M., Simple method for approximating the characteristics of low thrust trajectories, *ARS J.* **30,** 7, 649–652 (Tech. Notes), July 1960.

6643. Wang, K., and Ting, L., An approximate analytic solution of re-entry trajectory with aerodynamic forces, *ARS J.* **30,** 6, 565–566 (Tech. Notes), June 1960.

6644. Sung, K. S., and Park, C., Two analytical results of fin-stabilized rocket trajectory under quadratic drag law, *J. Aero/Space Sci.* **27,** 5, p. 388 (Readers' Forum), May 1960.

6645. Lavrent'ev, M. A., Puncture problem at cosmic velocities, *ARS J.* **30,** 4, 386–388, Apr. 1960. [Translation of Iskusstvennye Sputniki Zemli (Artificial Earth Satellites), no. 3, USSR Acad. Sci. Press, Moscow, 1959, 61–65.]

Author considers the problem of "formation of the moon's craters." High velocities of impact, and incompressible-medium model are considered.

The paper contains two parts. In the first the one-dimensional case of the impact between a rod of length l and a plate of thickness a is considered. The problem of determining the momentum which the rod acquires as a result of the impact is calculated for the following two cases: (1) assuming that all the scattered particles of the gas cloud (the part of the rod vaporized due to the impact) acquire an identical velocity; (2) assuming that each layer of the gas cylinder scatters (in the direction of the rod axis) independently of the other elements.

In the second part, analogous calculations for the case when a ball strikes a hemispherical plate are made. The simplifying assumption that all the velocities of the points are radial is used.

N. E. Cristescu, Roumania

6646. Kornhauser, M., Satellite pressure losses caused by meteroid impacts, *ARS J.* **30,** 5, 475–479, May 1960.

In order to predict the frequency of hull penetration of a satellite capsule by meteoroids, estimates are made of the frequency of encounters with meteoroids, and the cratering effect of each impact. The cratering effects are based on the correlation of recent laboratory experimentation using hypervelocity particles, and very conservative estimates of impact frequency are employed. The resulting calculations of percentage of hull area covered by holes, and loss of internal pressure versus time, are expected to be conservative for design purposes. Application of this method to a manned satellite capsule having a given surface area and time of exposure to meteoroids results in predictions of: Probability of penetration of several hull designs, time required for air pressure to drop to 1/3 atm, and weight of reserve air needed to replace leakage and maintain internal pressure at 1 atm.

From author's summary

6647. Hewitt, M. H., Electrically charged missile in vertical descent, *ARS J.* **30,** 5, 505–506 (Tech. Notes), May 1960.

The Lorentz deflection of an electrically charged missile in vertical descent from low altitude is determined and compared in nature with the Coriolis displacement in the northern hemisphere. It is found that a 10^2-kg missile having an initial altitude of 10 km at latitude 60 deg N and an initial speed of Mach 10 is negligibly displaced. From author's summary

6648. Heller, G., Thermal control of the Explorer satellites, *ARS J.* **30,** 4, 344–352, Apr. 1960.

Thermal control of the Explorer satellites is discussed. The theoretical studies that were made prior to the launching of these satellites are described, and examples of relationships are presented in graphs. The lower limit of the instrument temperature was zero deg C (determined by the efficiency of the chemical batteries). The upper limit was specified as 65°C (based on long-time temperature limits of transistors in the electronic package). This paper relates some of the studies, describes the measuring results of telemetered temperatures and evaluates expected and obtained data. From author's summary

6649. Huth, J., Relativistic theory of rocket flight with advanced propulsion systems, *ARS J.* **30,** 3, 250–253, Mar. 1960.

Author develops a general approach to a wide variety of potential propulsion systems, by nuclear fission, fusion, or by photons, indicating the limitations imposed by fuel energetics and specific power for constant-thrust rockets.

Anticipating the attainment of velocities of electromagnetic order, the mechanics of propulsion is treated relativistically. In the opinion of the reviewer, though, caution should be exerted at that point, because the present-day set-up of relativity mechanics still happens to be in a controversial phase of development and cannot be taken for granted. The paper is of a highly specialized and rather speculative nature. J. Mandelker, USA

6650. Sanger, E., Atomic rockets for astronautics (in German), *Astronaut. Acta* **6,** 1, 3–15, 1960.

In this paper of a review nature, author derives first expressions for the rocket velocity as a function of the ratio of the fuel weight to the total weight of the rocket and as a function of the relativistic energy utilization contained in a given mass of fuel. Results are plotted in a graphical form with additional information indicating for what kind of missions a given set of parameters could be used.

Using the results contained in this graph, author surveys potential performance of chemical, fission and fusion rockets. In particular, he compares potential merits of pure adiabatic fission rockets, photonic rockets using the thermal radiation of fission reactors, convective fission rockets using heat exchangers, hot-water fission rockets where a water blanket is used for thermal insulation of walls and is being dissociated into propulsion plasma. He also discusses electrostatic ion rockets and electromagnetic plasma rockets.

Similar types of rockets are compared for the case of fusion reaction. Finally, an ideal photonic rocket resulting from the radiation of the matter annihilated by the anti-matter is briefly discussed.

Author's conclusion seems to be that the most promising performance in the present state of the art could be expected from the photonic rockets and from the hot-water fission rockets.

B. Zarwyn, USA

6651. Mohanti, H. B., Application of electronic differential analyzer to a guided missile, *J. Sci. Engng. Res., India* **3**, 2, 409–420, July 1959.

6652. The third Soviet artificial satellite of the Earth (in Russian), *Pravda* no. 138, 3–4, May 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2182.

The article describes the third Soviet artificial satellite of the Earth and the instruments it carried. The fundamental problems to be scientifically investigated by the Sputnik are enumerated: The study of the upper layers of the atmosphere and the influence of cosmic factors in the processes in these layers. The launching of the satellite took place on the 15th of May, 1958. The weight of the Sputnik was 1327 Kg, the general weight of scientific and measuring apparatus together with fuel supplies being 968 Kg. The satellite in form approaches that of a cone, with a length of 3.57m and a maximum diameter (not counting the antennae) of 1.73m. The satellite was furnished with radiotechnical, radiotelemetrical apparatus and a course indicator device on semiconductors; the energy sources consisted of electrochemical (based on silver-zinc accumulators and oxides of mercury elements) and of solar silicon batteries. It is shown that the possibility is available right here and now to launch a rocket beyond the limits of the earth's gravitational pull. The Sputnik's apogee was 1880 Km, the initial velocity more than 8000 m/sec, the period of orbit round the Earth initially was 105-95 min. The plane of inclination of the orbit to the plane of the earth's equator was 65°. The velocity of regression of the orbit was $\sim 4^\circ$ in 24 hours. Radiotransmitters on a frequency of 20005 M Hertz gave a continuous transmission of telegraphic radio signals with a duration of 150–300 m sec. Descriptions are given of the organization and methods employed in making the radiotechnical and optical observations on the satellite. The hermetically sealed shell of the satellite was made from aluminum alloys, the surface being polished and specially treated in order to obtain the determined values for the coefficients of the radiation and absorption of radiation.

The various appliances of the Sputnik are described in comparison with the former ones and the lay-out and purpose of the scientific devices are indicated. Changes in the thermal regime were made by controlled forced circulation of gaseous nitrogen in the satellite and also by alterations in the coefficient of radiation of the satellite's surface, for which purpose regulatable shutters consisting of 16 sections were fixed on the satellite's surface. Study of the ionosphere proceeded by means of observations on the propagation of radio waves radiating from the Sputnik and by the determination on appliances in the Sputnik of the concentration of charged particles in the ionosphere and of the spectra of the mass of positive ions. Descriptions are also furnished of the methods employed for the measurement of the concentrations of charged particles, for the investigations on the constitution of the ionosphere, for the examination of the electrostatic fields, the measurement of the Earth's magnetic field, for the study of cosmic rays, the corpuscular irradiation of the Sun, and adopted on the Sputnik. Manometers were fitted on the outside of the satellite; magnetic, to measure the pressure within the limits 10^{-5}–10^{-7}mm; mercury and ionizational, within the limits 10^{-6}–10^{-9}mm of mercury. For the calculation of the number of collisions with micrometeors piezoelectrical transmitters were used.

V. A. Prokof'ev

Courtesy Referativnyi Zhurnal, USSR

6653. Discoveries widen knowledge of the universe. Some items of information on the scientific investigations on the third Soviet artificial satellite of the Earth (in Russian), *Pravda* no. 278, p. 2, Oct. 1958; Ref. Zh. Mekh. no. 3, 1959, Rev. 2183.

The fact is mentioned that in conformity with the MGG (World Geophysical Year) program, a very large amount of work has been done in the U.S.S.R. on investigations of the upper layers of the atmosphere. An analysis of the radiotelemetric recordings shows that the scientific apparatus on the third artificial Sputnik worked normally and that the program set for scientific measurements was completed. The assigned temperature regime (from 15 to 22°) for the work of the apparatus was adhered to. In the paragraph dealing with "The dynamic effects and motions of artificial satellites of the Earth" it is shown that the magnitude of the ascending angle for the first, second and third Soviet Sputniks altered in the initial stages of their motion by 3.157, 2.663 and 2.528 degrees in 24 hours from the East to the West, the position of the perigee changed by 0.432, 0.407 and 0.326 degrees in 24 hours to the South and the period of rotation by 1.8, 3.08 and 0.75 seconds in 24 hours. The second satellite effected precession near the axis, making an angle of 86° with the longitudinal axis of the satellite with a period of 206 secs. The regime of the motion of the third Sputnik was close to the regime of regular precession, when the axis of precession was inclined by 84° to the longitudinal axis of the Sputnik, the period of precession being 140 secs, the period of rotation near the longitudinal axis of the Sputnik being near 18 min.

By the braking action on the Sputniks it was possible to determine the mean value of the density of the air at perigee height for the orbit (226 to 228 Km) which was found to be 3×10^{-15} g/cm^3; in transition of the perigee of the orbit from the night to the day period the product of the density of the atmosphere and the square root of the height of the homogeneous atmosphere increased for the first two Sputniks by 20 to 30%, and for the third by more than one and a half times. The measurement of pressure on the third artificial satellite was carried out with the aid of electromagnetic discharging manometers in the limits of 10^{-5} to 10^{-7} mm of mercury, and thermo-ionization manometers in an interval from 10^{-5} to 10^{-9} mm of mercury. Their operation is described. The manometer readings showed that the density at a height of 266 Km is 109 less than at the Earth's surface, while an increase in height by an additional 100 Km produces a still further decrease of 10 to 12 times; the results agree with those obtained by braking the satellites. It was discovered that in the ionosphere at heights of from 230 to 950 Km prominence is assumed by the ions of atomic oxygen, the ions of atomic nitrogen constituting 3 to 7% of the quantity of oxygen ions. Ions of molecular oxygen and nitrogen were not detected. A significant number of ions was detected at a height of 1000 Km. The third satellite was equipped with traps for the positive ions. The temperature of the electrons in the ionosphere was found to be much higher than the temperature of the neutral particles at those heights. The measured negative potential of the Sputnik at a height of 795 Km, measured in day time, was equal to 6 v, the concentration of positive ions was 160,000 ions/cm^3; the corresponding figures for a height of 242 Km were, respectively, 7 v and 500,000 ions/cm^3. The electrostatic field intensity in the upper layers of the atmosphere was 10 to 100 times more than the expected value.

The third Sputnik was also used to carry out measurements of the spatial distribution of the constant magnetic field of the Earth at great heights. Piezo transmitters having a general area of 840 cm^2 were used to record the impulses of meteoric particles in the range of 0.1 to 1000 g sec/cm^2 and recorded one impact every 100-sec interval. Observations were also made on the great increase of impacts of short duration reaching tens to the square meter in 1 sec. The meteoric particles possessed energies of ~10,000 erg. Results were given of the study of cosmic rays and corpuscular irradiation on various types of apparatus set up in the satellite, and also some conclusions reached regarding the successful adoption of solar batteries are stated. The conclusion is reached that the results of the investigations obtained from the third Sputnik have led to changes in the presentation of the structure of the upper layers of the atmosphere and of the processes proceeding in them. V. A. Prokof'ev

Courtesy Referativnyi Zhurnal, USSR

6654. Boni, A., Observation of a satellite near its culmination (in English), *Astronaut. Acta* **4**, 3, 188-217, 1958.

The problem of the tracking of a satellite with instruments of relatively low accuracy is discussed in detail. It is shown that if the tracking is performed when the satellite is at a point of culmination with respect to the observer, the orbital elements can be obtained to a satisfactory accuracy. A full mathematical treatment is given of the parameters involved, including step-by-step methods of approximation. Particular attention is paid to the use of a theodolite. R. C. Knight, England

Ballistics, Explosions

(See also Revs. 6214, 6235, 6303, 6476, 6548, 6563, 6629, 6649)

6655. Kisler, V. M., On explosions above surfaces of liquids, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24**, 1, 496-503, 1960. (Pergamon Press, 122 E. 55th St., New York 22, New York.)

The effect of an external explosion on the motion of the free surface of a liquid is simulated by prescribing an unsteady radially symmetric pressure distribution $p_0(r,t)$ over the surface, specifically in the form $\sum_n^\infty \lambda_n(t)r^{2n}$ for $r \leq r_0$ and zero for $r > r_0$. In the framework of the theory of linearized gravity waves, use of Fourier transforms leads to an expression for the velocity potential in terms of products of integrals which in turn involve products of Bessel, hyperbolic, trigonometric and power functions. The bulk of the paper is concerned with the difficult evaluation of those products of integrals which give the shape of the free surface for the case of infinite liquid depth. Within the radius of action, r_0 of the applied pressure, the computations appear prohibitive so that the more general formulas are simplified to the asymptotic case of a weightless liquid. This solution is intended to give an indication of the surface motion in the early stages of the explosion when the pressure effects dominate.

M. V. Morkovin, USA

6656. Brossard, J., and Manson, N., Application of the theory of the double discontinuity; characteristics of detonations of gaseous mixtures (in French), *C. R. Acad. Sci., Paris* **249**, 12, 1033-1035, Sept. 1959.

6657. Wheelon, A. D., Free flight of a ballistic missile, *ARS J.* **29**, 12, 915-926, Dec. 1959.

This excellent discussion of the flight equations takes into account the earth's rotation and oblateness. The complete equations are given but only the first-order effects are presented numerically and graphically. However, methods of numerical solution of the more accurate equations are indicated. The effects of gravitation anomalies, uncertainties in the physical constants, attraction of the sun and moon and relativistic effects are shown to be calculable and quite small. R. E. Street, USA

Book—6658. Pokrovsky, G. I., and Fedorov, I. S., The action of shock and explosion in plastic media (in Russian), Moscow, Promstroyizdat, 1957, 276 pp.; *Ref. Zh. Mekh.* no. 8, 1958, Rev. 8483.

Authors discuss the applied theory of the action of shock and explosion in solid, plastic media—soils, rocks, and building materials. The book contains five chapters. In the first chapter, "Fundamental characteristics of shocks and explosions," a short account is given of the different forms of explosion, e.g., detonation, cumulative and directed explosion. The second chapter, "The action of an explosion in a medium," first examines the qualitative characteristics of the propagation of explosions in solid media, and then deals with the process of expansion of a gaseous cavity in a medium. There is a particularly interesting and original explanation of the characteristic features of air and gas explosions in mine galleries and pipe conduits. The investigation of the propagation of an explosion wave through the ground is also of great interest. The most important chapter is the third, "Calculation of charges in a medium," which gives methods for calculating charges for different circumstances of rock caving and knockout (crater) explosions. Special attention is given to large-scale explosions. The question of the motion and flight of rock fragments after the burst is given individual consideration. The fourth chapter, entitled "The theory of similarity and analogue methods for dynamic processes," gives brief "prescriptions" for modelling large-scale explosions. The fifth chapter, "Practical applications of analogue techniques to dynamic processes," describes the results of model tests, the equipment used, and discusses individual problems in the physics of explosions and the action of explosions on building structures, not covered by the earlier chapters. K. P. Stanyukovich

Courtesy Referativnyi Zhurnal, USSR

6659. Strehlow, R. A., and Cohen, A., Shock-initiated detonations, *Physics of Fluids* **3**, 2, 319-320 (Letters to the Editor), Mar./Apr. 1960.

6660. Dremin, A. N., and Adadurov, G. A., Shock adiabatic for marble, *Soviet Phys.-Doklady* **4**, 5, 970-973, Mar./Apr. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **128**, 2, 261-263, Sept./Oct. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

6661. Webb, F. H., Jr., Bingham, H. H., and Tollestrup, A. V., High-energy densities before dwell in electrically exploded wires, *Physics of Fluids* **3**, 2, 318-319 (Letters to the Editor), Mar./Apr. 1960.

Acoustics

(See also Revs. 6296, 6334, 6470, 6476)

6662. Ankerman, P. W., and Rubega, R. A., Bistatic scattering from totally reflecting flat plates, *J. Acoust. Soc. Amer.* **32**, 4, 478-481, Apr. 1960.

This paper presents theoretical computations of the bistatic scattering of sound by a flat plate five wavelengths long. The computations are compared with experimental data obtained in air. Although agreement is quite good there are significant deviations which the author ascribes to the difficulty of experimentally satisfying all the theoretical assumptions. The reviewer notes that while this is probably the case the lack of any experimental detail makes it difficult to estimate the confidence one can place on the-

oretical bistatic scattering computations when the conditions are less than ideal. M. S. Weinstein, USA

6663. Khaskind, M. D., Diffraction and irradiation of acoustic waves in liquids and gases (in Russian), *Akust. Zh.* **3**, 4, 348–359, 1957; **4**, 1, 92–99, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2393. See also the two following reviews.

Derivation of formulas for the hydrodynamic forces and moments acting on a solid body of arbitrary form, situated in some given acoustic field and carrying out an arbitrary periodic motion, forms the first part of the paper; the period of this vibration coincides with that of the given basic field. The magnitudes of the forces and moments can be expressed through the asymptotic characteristics of the functions, describing the irradiating body of the acoustic field; this occurs in the case where the basic field is represented by a sinusoidal plane wave; in the general case the forces and moments are determined through the double integrals of the irradiation functions, spread over the vibrating body. The second part of the paper deals with the preparation of formulas for the calculation of the mean values for the period of vibrations of forces and moments applied to the body, with the same general assumptions regarding the solid body and its motions. Specially simple results are established for the case where the basic acoustic field is made up of plane waves or of spherical waves. The formulas obtained in both parts of the paper are illustrated in a series of special examples. L. N. Sretenskii

Courtesy Referativnyi Zhurnal, USSR

6664. Khaskind, M. D., Diffraction and radiation of acoustic waves in fluids and gases. Part I, *Soviet Phys.-Acoustics* **3**, 4, 371–384, July 1958. [Translation of *Akust. Zh., USSR* **3**, 4, 348–359, Oct./Dec. 1957 by Amer. Inst. Phys., Inc., New York, N. Y.]

Paper is concerned with the linearized calculation of the resultant force and moment acting upon a solid body which diffracts and/or radiates acoustic waves. Several general relations for force and moment are derived and then specialized for spherical and plane waves. For the diffraction by plane waves, force and momentum are expressed by the farfield (a result that could also be derived from the reciprocity law of acoustics). The sphere, the cylinder and the infinitely long ribbon are treated numerically and the results presented in graphical form. Finally, the general results are applied to a random noise field and the statistical characteristics of the resultant random force acting upon a sphere determined. H. L. Oestreicher, USA

6665. Khaskind, M. D., Diffraction and radiation of acoustic waves in liquids and gases. Part. II. *Soviet Phys.-Acoustics* **4**, 1, 91–99, Oct. 1958. [Translation of *Akust. Zh. USSR* **4**, 1, 92–99, Jan./Mar. 1958 by Amer. Inst. Phys., Inc., New York, N. Y.]

This is a continuation and extension of the preceding paper, now taking into account the nonlinear effects up to the second order. The time average of the resulting force and moment acting upon a diffracting and/or radiating rigid body are calculated, including quadratic terms.

Formulas for both the three- and two-dimensional case are derived and the eccentric rotation of a cylinder and a sphere in a compressible gas is given as numerical example.

H. L. Oestreicher, USA

6666. Lysanov, Yu. P., The dissipation of sound on a heterogeneous surface (in Russian), *Akust. Zh.* **4**, 1, 47–50, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2394.

The full value of a sound pressure $p(x, z)$ above a dissipating plane heterogeneous surface with a periodically altering acoustic conductivity during the normal fall on to the surface of a plane monochromatic sound wave can be depicted in the form of a series

$$p(x, z) = e^{-ik_0 z} + \sum_{m=-\infty}^{m=+\infty} A_m e^{ik_0 (\sin \vartheta_m x + \cos \vartheta_m z)}$$

Here A_m are the complex amplitudes of the dissipated waves which are determined from the solution of some infinite system of algebraical equations. The following method is proposed for the solution of the foregoing system of equations. If from the infinite system of equations one, two and so on equations are separated out, leaving in them one, two and so on first unknowns respectively, then a series of successive approximations can be found for the amplitudes of the dissipated waves. In this process it was found that commencing from some value $n = l$, two successive approximations will differ from each other to a smaller extent than any assigned magnitude however small it may be. It is shown that the values of the coefficients obtained from the solution of a system of l equations will differ from the exact values of the coefficients corresponding to the infinite system of equations by magnitude of the same order. Criteria are indicated which determine for each concrete case the required number of equations. That number depends on the parameters of the heterogeneous surface, the length of the sound wave and the given precision of the determination of the amplitudes of the dissipated waves. The proposed method is applicable to arbitrary divergencies of acoustic conductivity from their mean values and opens up the possibility of making calculations for the amplitudes of dissipated waves to any degree of accuracy. Because of the presence of recurrent correlations the calculations for the amplitudes of the dissipated waves even when there is a comparatively large number of equations involved can be performed in relatively simple fashion.

L. M. Lyamshev

Courtesy Referativnyi Zhurnal, USSR

6667. Ivanovskii, A. I., Investigation of acoustic flows (in Russian), *Nauchn. Dokladi Vyssh. Shkoly. Fiz.-Matem. Nauk* no. 1, 143–148, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6105.

By making use of the equation from his own study [*Akust. Zh., USSR* **4**, 6, 143–152, 1958] for small Mach numbers the author obtained the following correlation for the velocity of a stationary acoustic flow in a long cylindrical tube, covered on one side by a sound absorber

$$u = \frac{2I_0}{\eta c} \frac{1 - e^{-\alpha L}}{L} \Phi(R) \tag{1}$$

Here I_0 is the intensivity of the sound, η is the shearing viscosity, α is the absorption coefficient, L is the length of the tube, Φ is a factor depending on the dimensions of the tube and the sound irradiator. With $\alpha L \ll 1$ this correlation agrees with the correlation obtained by Eckart [C. Eckart, *Phys. Rev.* **73**, p. 68, 1948]. The results of an experimental verification of correlation [1] are given. The measurements of the intensity of sound and the velocity of flow (by means of measuring the velocity of motion of the suspended particles) were carried out for the absorption along the length of the vessel $\alpha L \ll 1$; the values for the coefficient of absorption obtained by means of [1] do not differ from the tabular values by more than 10–15%. The results of measurements with $\alpha L \gg 1$ qualitatively confirm [1]. The measurements were carried out in a vessel 14 cm long and with a square section of 4×4 cm; the velocities of the flows were ~ 2 cm/sec.

L. K. Zarembo

Courtesy Referativnyi Zhurnal, USSR

6668. Shimanskii, Yu. I., Kobiichuk, N. M., and Golik, O. Z., The compressibility of isoviscous substances (in Russian), *Ukr. Fiz. Zh.* **3**, 4, 537–541, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6575.

A study is carried out on the velocity of the sound and compression of pure normal alcohols and of their binary solutions. It was established that substances with identical viscosity have identical compressibility. From authors' summary

Courtesy Referativnyi Zhurnal, USSR

6669. Guthrie, A. N., Tolstoy, I., and Shaffer, J., Propagation of low-frequency cw sound signals in the deep ocean, *J. Acoust. Soc. Amer.* **32,** 6, 645-647, June 1960.

The acoustic field of a 10-cps cw source, towed at slow speeds at a depth of 24 m, was measured by a stationary hydrophone out to ranges of about 70 km. The depth of the hydrophone was approximately 450 m. The records obtained are compared with theoretical calculations of the wave field, using a method developed by one of the authors. The degree of agreement is fair enough for one to feel that further work along these lines should lead to a quantitative understanding of long-range sound propagation in the oceans.

From authors' summary

6670. Yasunaga, T., Effect of pressure on the structure of and sound velocity in water, *J. Acoust. Soc. Amer.* **32,** 6, 713-715, June 1960.

A simple model was used to describe the propagation of sound in water. This model assumed that sound waves travel at infinite velocity within the molecules and at gas kinetic velocities through the spaces between the molecules. Resulting calculations showed that the latter velocity increases with pressure. From the change of sound velocity and acoustical properties for normal water and from those for nonassociated water, which were obtained from acoustical data for mixtures of alcohol and water, the change of the water structure with pressure was calculated and compared with the results of Gierer and Wirtz.

From author's summary

6671. Smith, M. W., and Lambert, R. F., Acoustical signal detection in turbulent airflow, *J. Acoust. Soc. Amer.* **32,** 7, 858-866, July 1960.

Improvement in detected signal-to-noise ratio is obtained for a periodic signal masked by additive noise and turbulent noise backgrounds. Comparisons are made between autocorrelation, crosscorrelation, and a combination of frequency filtering and crosscorrelation. Although the latter method provided the greatest improvement, the crosscorrelation technique was the most successful single method. It turned out that the maximum improvement obtainable was limited by the dynamic range of the correlator computer and not by errors due to finite averaging time and scanning the delay. The improvement for signals masked by turbulent noise was found to be about 5 db less than that obtained for additive noise.

From authors' summary

6672. Wieber, P. R., and Mickelsen, W. R., Effect of transverse acoustic oscillations on the vaporization of a liquid-fuel droplet, NASA TN D-287, 25 pp., May 1960.

Application of energy at audio frequency in a direction normal to the travel direction of the droplet accomplishes two effects. The residence time for the droplet in a specific linear distance is increased. The shape of the droplet is altered, increasing the ratio of surface to mass. We suspect the latter effect is of greatest importance in acceleration of vaporization.

Acceleration of combustion is thought to be due to greatly enhanced condition of turbulence set up by the cyclic displacement of the atmosphere normal to the direction of travel of the droplet. Oxygen stagnation or deficiency is avoided in the burning zone.

R. D. Reed, USA

6673. Jackson T. W., and Johnson, H. L., Convective flow due to acoustic vibrations in horizontal resonant tubes, AFOSR TR 60-52 (Georgia Inst. Tech., Engng. Experiment Station), 35 pp., Mar. 1960.

Investigation shows that acoustically excited flow patterns in horizontal resonant tubes are not normally of type predicted by Rayleigh and demonstrated by Andrade, unless rigorous precautions are taken to avoid convection effects. Flow visualization was by flash illumination of smoke particles together with photographic recording. In addition to interference by convection and exterior sound fields, vortex and thermal circulations take place and render results not readily amenable to analysis. Sound-induced flows round various shaped objects were also photographed.

J. K. Kilham, England

6674. Sveshnikov, A. G., Waves in bent tubes (in Russian), *Radiotekhn. i Elektronika* no. 5, 614-698, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2386.

An approximate method is offered for the solution of the problem on the propagation of steady acoustic vibrations in an arbitrarily deforming wave carrier, for the case where there is a slow change in the direction of the axis of the wave carrier and where there is only a small deformation of its transverse section. The proof for the method: it is known that the problem on the propagation of vibrations in a wave carrier with absolutely rigid walls merges with the solution of a homogeneous wave equation with homogeneous boundary conditions (Neumann's internal problem). An irregular wave carrier is assigned and a new polar system of coordinates is introduced by means of some general method. In the new system of coordinates both the wave equation and the boundary conditions are heterogeneous. The right hand parts of the wave equation and, correspondingly, of the boundary conditions are functions of the parameters determining the irregularity of the wave carrier. The obtained heterogeneous equation is solved by the method of small perturbations. The solution being sought is presented in the form of a resolution along the small parameter ε, which characterizes (when $\varepsilon \ll 1$) the requirements for a slow change in the direction of the wave carrier's axis and for a small deformation of its transverse section. The solution of the equation obtained in conditions where the right hand part is equal to zero, both in the actual equation and in the boundary conditions, is taken as representing the zero approximation. The determination of the term of the series in the resolution describing the first approximation merges with the solution of a heterogeneous equation with heterogeneous boundary conditions, the right hand parts of which are determined by the solution obtained in the zero approximation. The solution of this equation is carried out by introducing functions of origin. The results of the calculations are restricted to the first approximation. Concrete examples are examined. The propagation of acoustic vibrations is investigated in a regular cylindrical wave carrier with a three-dimensional curvature and in the so-called "pulsating" wave carrier, in which the form of a portion of a regular cylindrical wave carrier changes in sinusoidal fashion along the generatrix and along the perimeter. The above approximate method is also used for the investigation of the propagation of electromagnetic waves in feebly irregular wave carriers, in particular for solving the problem of torsion of a rectangular wave carrier of constant transverse section.

L. M. Lyamshev

Courtesy Referativnyi Zhurnal, USSR

6675. Fand, R. M., and Kaye, J., Acoustic streaming near a heated cylinder, *J. Acoust. Soc. Amer.* **32,** 5, 579-584, May 1960.

A photographic study employing smoke as the indicating medium has shown the existence of a new type of streaming near a heated horizontal cylinder in the presence of a horizontal transverse sound field. This phenomenon, called "thermoacoustic streaming," is characterized by the development of two vortices above the cylinder; the fluid pattern resembles vortex shedding behind a cylinder in forced flow normal to its axis. In the presence of sound waves whose half-wavelength is six or more times greater than the diameter of the heated cylinder, the formation of the vortex flow is a function of the sound intensity only; for such wavelengths the vortices begin to appear at 140 db (*re* 0.0002 μ bar) and become fully developed at 146 db. This type of streaming is a flow phenomenon which is much stronger than isothermal streaming for the same geometry and sound intensity. It appears that thermo-

acoustic streaming will have important practical applications, particularly in the field of heat transfer.

From authors' summary

6676. Howes, W. L., Similarity of far noise fields of jets, NASA TR R-52, 44 pp., 1960.

The material presented in this report permits the prediction of the noise characteristics of circular jets under conditions of subsonic and supersonic flow in terms of the geometry and fluid properties of the jet. The similarity relations specifically discussed are the total acoustic power level, power spectrum, directivity, local mean-square-pressure spectrum, and pressure probability-density. These relations are first derived and then correlated with experimental data. These data were obtained from noise studies of various sizes of cold and hot air jets as well as turbojet-engines. Good agreement between the predicted and experimental values of these parameters was obtained for subsonic flow and for all but the total acoustic power for supersonic flow. In the case of the latter, the author questions the validity of the derived expression.

C. F. Speich, USA

6677. Hilton, D. A., Mayes, W. H., and Hubbard, H. H., Noise considerations for manned reentry vehicles, NASA TN D-450, 13 pp., Sept. 1960.

Noise measurements pertaining mainly to the static firing, launch, and exit flight phases are presented for three rocket-powered vehicles in the Project Mercury test program. Both internal and external data from onboard recordings are presented for a range of Mach numbers and dynamic pressures and for different external vehicle shapes.

From authors' summary

6678. Igarashi, T., Noise of a ball bearing (1st report: Case of a simple ball bearing), *Bull. JSME* **3,** 10, 220-227, May 1960.

There have been many researches on the noise of a ball bearing, and the conclusions about noise source can be divided into two main opinions; one attributes it to elastic vibrations of the outer ring, the other to vibrations of the cage or ball. However, these conclusions are only based on a hypothesis.

Author measured the noises of three kinds of simple single-row deep-groove ball bearing, analyzed the peak values of their frequency spectrums, and obtained these results:

(1) The noise of a simple ball bearing is due to vibrations of the outer ring.

(2) In the above vibrations, there are radial and axial bending ones which have the first and the second modes.

(3) In the case of axial load, another vibration exists, and its vibratory system, which was found by Mr. Tanaka, consists of a mass of outer ring and an axial stiffness of ball bearing.

From author's summary

Micromeritics

(See also Revs. 6544, 6545, 6547)

6679. Badzioch, S., Correction for anisokinetic sampling of gas-borne dust particles, *J. Inst. Fuel* **33,** 230, 106-110, Mar. 1960.

Dust content of a flowing gas stream is measured by aspirating a sample of dusty gas into a nozzle facing upstream, and afterwards separating the dust particles from the gas sample by a collecting device. The sample will be representative of the gas stream, provided the velocity of aspiration into the nozzle (sampling velocity) is equal to the velocity of the gas stream at that point; this is termed isokinetic sampling. Under these conditions there is no disturbance of the gas streamlines, and all the particles approaching the nozzle, and only those, will enter it. In anisokinetic sampling the approaching streamlines are deflected, causing some particles to become deflected from their original direction of motion so that the quantity of particles entering the sampling nozzle per unit time will differ from that entering under isokinetic conditions. Concentration of coarse particles (which follow their original direction of motion) in the sample will be little affected, that of fine particles (which follow the deflected streamline of gas flow) will be more affected by anisokinetic sampling. An expression is given for the effect of anisokinetic sampling upon the relative quantity of dust entering the sampling nozzle.

Two methods of calculating the dust concentration are considered and a criterion is provided for deciding, depending on sampling conditions, which method gives a closer approximation to the true dust concentration. Procedures are given for determining the true concentration and true size distribution of dust from an anisokinetically collected sample. Sampling at fixed velocity is considered. In case of very fine and very coarse particles the errors arising from sampling at fixed velocity, irrespective of the value of the gas velocity, are small providing the appropriate method of calculation is employed.

This is a paper of great practical importance for the dust sampling of flue gases, based on authoritative researches of the author and others, sponsored by the British Coal Utilization Research Association.

K. J. DeJuhasz, USA

6680. Berg, B. R., Development of a new horizontal-flow, plate-type precipitator for blast furnace gas cleaning, *Iron Steel Engr.* **36,** 10, 93-101, Oct. 1959.

Paper is in two parts: analysis of precipitator performance in general, and description of improved water washing design.

Analytical approach shows that performance can be represented on plot of migration velocity against function of gas velocity; collection efficiency increasing with migration velocity. Tests on pilot-scale precipitator showed that higher migration velocities could be obtained when collecting electrodes were flushed with water continuously and discharge electrode intermittently.

Design of full-scale units is described and operational difficulties outlined. Major difficulties were overcome by spraying water into precipitator with and across gas flow, good water coverage being obtained by its even precipitation.

Adaptation of the system to future high-pressure (14 psig) plant operation is outlined.

Reviewer wishes that comparison of wet and dry migration velocities had been quoted so that judgment could be made of advantages of wet systems in view of the increased effluent handling problem.

D. B. Leason, England

6681. Zebel, G., Theory of coagulation of dispersed systems of electric or magnetic dipoles (in German), *Staub* **19,** 11, 381-387, Nov. 1959.

6682. Todorov, I., and Sheludko, A., The deposition of aerosol particles on the walls of an enclosed space (in Russian), *Kolloidn. Zh.* **19,** 4, 496-504, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2776.

An investigation is made of the deposition of aerosol particles from the bulk of a sphere on its walls because of diffusion and sedimentation in the absence of convection and with an adhesion coefficient of $A < 1$. It was shown that the velocity of deposition is at a minimum for particles of a certain optimum weight P_*. If the coefficient A is not too small then its influence on the magnitude P_* is inappreciable, and the correlation $P_* = 4kT/a$, (where a is the sphere's radius, K Boltzman's constant, and T the absolute temperature) is approximately fulfilled.

V. Dunskii

Courtesy Referativnyi Zhurnal, USSR

6683. Radchenko, G. A., and Beloboradov, P. V., Distribution of the concentration of dust according to height in diagonally

connected air ducts (in Russian), *Trudi In-ta gorn. Dela. Akad. Nauk KazSSR* **2**, 173–188, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2783.

This investigation was carried out on a model of diagonally connected air ducts. A plan is given for the corresponding experimental assembly with notes regarding the basic dimensions and locations of the measuring stations and grids distributing the air. The experiments were carried out at magnitudes for the mean velocity at the inlet portion equal to 3.3 and 18.2 m/sec (corresponding to Reynolds numbers from 70,000 to 398,500). The experimental results are presented in the form of graphs of relative velocities (local velocity, referred to the mean in the section) and of graphs showing the distribution of the relative concentrations of dust according to height in the different sections of the experimental assembly with variable consumption of air. According to the authors' views the analytical presentation of the curves for the distribution of the concentrations of dust in the assembly with diagonal connections of the air ducts, because of the large number of data exercising definite influence (in addition to the customary ones—turbulent mixing and settling of the dust because of gravity—influence is also being exerted, as shown, by the introduction of dust into the flow, by the increase in size of the small particles by turbulent pulsations, by the alteration of the flows structure due to the action of local resistences: bends, branching, and so forth), cannot apparently be given in the form of some single general relation. It is therefore quite clear that the equations obtained in the paper being abstracted can only be used in practice in conditions prevailing in sufficiently smooth unstrengthened mine workings (which conform to the greatest extent to the conditions of the experiments). The agreement of some of the results obtained with the data of other authors is referred to.

G. E. Khudyakov
Courtesy Referativnyi Zhurnal, USSR

6684. Szonyi, J., and Papai, L., Velocity characteristics in the pneumatical conveyance of grainy material (in Hungarian), *Jarmuvek Mezogazdasagi Gepek* **6**, 8, 225–235, 1959.

6685. Pyshkin, B. A., Spiral motion of a liquid containing suspended sediment in a straight tube of square section (in Russian), *Izv. In-ta Gidrol. i Gidrotekhn. Akad. Nauk USSR* **12**, 46–57, 1955; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2772.

The paper describes the theoretical investigation of the kinematic properties of a homogeneous spiral flow of an inviscid incompressible liquid in a straight tube of square section. An analysis is then carried out of the motion of the suspended sediment in this type of flow, made on the basis of the kinematic relationships obtained. Here the motion of the suspended particles is investigated by means of the plan evolved by M. V. Potapov [*Izv. Akad. Nauk, SSSR Otd. Tekh. Nauk* no. 8, 1947].

Remarks by the abstractor: The formulation of the boundary conditions for the examination of the kinematics of the flow gives rise to some doubts. In addition, the author states incorrectly that I. S. Gromeko investigated "only a particular case of the spiral motion of a liquid in which the longitudinal velocity v_x attains its maximum in the center of the section, while on the contour this value is equal to zero." Actually, Gromeko in his known work "Some cases of motion of an incompressible liquid" [Kazan', 1881; Collection of works, Izd-vo Akad. Nauk SSR, 1952] investigated a much more general case in which $v_x \neq 0$ on the contour.

O. F. Vasil'ev
Courtesy Referativnyi Zhurnal, USSR

6686. Kostyuchenko, E. V., The dissipation of the particles of a noncohesive soil falling into a flow of liquid (in Russian), *Trudi In-ta Vodn. Kh-va i Energ. Akad. Nauk KirgSSR* no. 4 (7), 85–99, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2774.

Author examines the dissipation of soil particles moving in a flow, and investigates the practical problems in calculations of settling tanks and the dumping into riverbeds of sediment of the return filtration of earthen dams. From the analysis of the experimental material it was established that the asymmetry of the curve for the distribution of the particles on a certain plane is determined by the angle between the mean path of the particle motion and the plane being investigated. A derivation is given for the theoretical relation of the distribution curve and data are furnished for the experimental verification of the theory; also given are examples of the calculation. It is shown that the normal Gauss principle for distribution holds good only for a plane perpendicular to the mean trajectory of the particle motion. At the plane of the bottom the dissipation of the particles, the form and the granulometric composition of the layer of settled-out particles are determined by means of relations obtained by the author.

S. Ya. Vartazarov
Courtesy Referativnyi Zhurnal, USSR

6687. Mani, J. V. S., Kaura, N. N., and Rao, M. N., Flow pattern of liquids on the vanes of a disc atomizer, *J. Sci. Indust. Res., India* **19** B, 1, p. 28 (Letters to the editor), Jan. 1960.

6688. Dodu, J., Influence of the Weber and Reynolds numbers on the dispersion of high velocity liquid jets (in French), *C. R. Acad. Sci., Paris* **249**, 4, 499–501, July 1959.

6689. Borneas, M., and Babutia, I., Is the rotation of fluid particles of influence on the surface tension? (in French), *C. R. Acad. Sci., Paris* **249**, 12, 1036–1038, Sept. 1959.

Porous Media

(See also Revs. 6214, 6532)

6690. de Witte, A. J., A kinetic theory of liquid displacement, *Physics of Fluids* **3**, 2, 197–204, Mar./Apr. 1960.

A theory is formulated for the displacement of a liquid in a porous medium by another liquid miscible with the first; the theory also applies to nonmiscible liquids. Molecular diffusion is not taken account of specifically in the derivation, and the flux and concentration is found to be governed by an equation of hyperbolic type. A comparison is made between present theory and earlier ones which lead to parabolic equations.

R. E. Gibson, England

6691. Madonna, L. A., and Lama, R. F., How to calculate pressure drop in spouted beds, *Indust. Engng. Chem.* **52**, 2, 169–172, Feb. 1960.

Equations are given for the pressure drop in the case of fluid flow upward through a bed of granular noncohesive particles for (1) no displacement of particles and (2) displacement of particles by flow (spouting). The pressure drop is expressed in terms of voids, particle diameter and surface area, fluid viscosity, etc.

Experiments were performed using 6-inch and 4-inch diameter columns made of glass and aluminum with both flat bottoms and 60-degree cone angle bottoms. Granular materials used were wheat, barley, rape seed, etc., and an air jet (instead of liquid) was applied at the bottom of the columns.

The results indicate (1) linear increase of pressure drop with rate of air flow up to a maximum drop, (2) linear variations of pressure gradient with Reynolds number for the different media for both gradients below spouting and at spouting (transitional flow).

E. Vey, USA

6692. Ytshar, A., Fundamentals of hydraulic-potential conception in soil hydraulics (in German), *Bauingenieur* **34**, 12, 469–471, Dec. 1959.

The hydraulic-potential concept in soil hydraulics has already been presented and discussed at the Fourth Congress of the International Society for Water Supply, Brussels 1958. It has also been described in earlier publications by the author.

The present paper is a summary and revised treatment of the same problem. Its purpose is to establish firm fundamentals and definitions for the hydraulic-potential concept, so that misunderstandings and erroneous interpretations may be avoided. This appears necessary because the new concept in several respects disagrees with the classical theories and, furthermore, limits the application of the flow net concept by Forchheimer.

From author's summary by M. J. Hvorslev, USA

6693. Gerber, S., and Pilod, P., Proving the existence of two types of seepage of a fluid through a dam with vertical walls, by means of electric analogy (in French), *C. R. Acad. Sci., Paris* **249**, 20, 2006-2008, Nov. 1959.

6694. Priazhinskaia, V. G., The problem of plane unsteady motion of ground waters, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 5, 1358-1364, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The problem is reduced to a certain integrodifferential equation, and thence to a system of nonlinear integral equations. Under specified conditions the existence of a solution of this system is established. (From author's summary)

This highly mathematical paper is likely to be understood only by those well versed in analytic function theory.

A. H. Armstrong, England

6695. Krischer, O., and Mahler, K., On the determination of the vapour and liquid moisture diffusivities from steady and transient processes (in German), *VDI Forschungsheft* **25**, 473, 40 pp., 1959.

Liquid and vapor diffusivities are determined for water movement through two types of building materials (with porosities of 31% and 76%) by means of an analysis of steady-state moisture movement and of transient drying experiments. Liquid diffusivities are also related to measured moisture distributions after subjecting samples to a centrifugal force field. Moisture contents are determined by weighing and by measuring the dielectric constant of samples at 10 megacycles.

Results obtained by the various methods are in good agreement. They are of the same order of magnitude as those found by a number of other investigators for similar materials, the range for the liquid diffusivities being 10^{-2} to 10^{-6} m^2/h.

Results illustrate *inter alia* that so-called "shape factors" in semi-theoretical formulas for calculating conductivities or permittivities of porous media depend not only on the geometry of the material, but also on the conductivities or permittivities of its constituents.

Reviewer considers paper to represent a thorough and well-designed experimental study. However, he misses an analysis of the influence of temperature gradients in the samples on the results [cf. DeVries, AMR **12**(1959), Rev. 5232].

D. A. DeVries, Holland

6696. Belash, P. M., The structure of filtration flows in homogeneous petroleum strata (in Russian), *Trudi Vses. Neftegaz. N.-i In-ta* no. 12, 233-241, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2799.

The known method of graphical presentation of hydromechanical networks for homogeneous regions is generalized to cover the case for heterogeneous layers, the filtration principles of which are described by the general equation of an elliptical type. The results are applied to the graphical solution of problems of the filtration of petroleum in heterogeneous strata. This procedure also gives the possibility of determining approximately the percentage of waterflooding in the wells and the situation arising therefrom in regard to water-petroleum contacts. A concrete example is investigated. More complicated examples of solutions for the problem are given in the article by P. M. Belash, M. I. Maksimov, *Trudi Vses. Neftegaz. N.-i. In-ta* no. 12, 233-241, 1958.

P. F. Fil'chakov

Courtesy Referativnyi Zhurnal, USSR

6697. Golubev, G. V., Determination of the field of pressure in a partially homogeneous stratum consisting of m "Sofokusnykh" ellipses when there is a feed contour present (in Russian), *Uch. Zap. Kazansk. In-ta* **117**, 9, 84-89, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2803.

The problem concerns a horizontal stratum of constant power divided by "Sofokusnykh" ellipses L_i into portions with given constant permeability. The boundary L_n of the stratum is also an "Sofokusnykh" ellipse with L_i. The stratum contains a finite number of wells distributed symmetrically relative to the axis x and having assigned yields which do not disturb the symmetry. The problem is to ascertain the pressure in the stratum on the assumption that the pressure L_n is known and constant. The problem is converted by conforming reflection to the analogous problem with peripheries and is solved by merging with a system of Fredholm integral equations of the second order. The sought functions of this system are resolved into Fourier series and are replaced by segments from these series, which results in a system of algebraical linear equations being found for the determination of the coefficients.

N. V. Lambin

Courtesy Referativnyi Zhurnal, USSR

6698. Molokovich, Yu. M., The reestablishment of the functions for pressure in strata of variable permeability when taking into account the difference in viscosity of water and petroleum (in Russian), *Uch. Zap. Kazansk In-ta* **117**, 9, 127-132, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2804.

The determination of the field of pressure is the problem investigated; the field is constant as regards power and variable as regards permeability of the stratum with the condition that the pressure $p_0(t)$ is maintained on the feed contour. The stratum is saturated with water and petroleum; the boundary between them (the water-petroleum contact) changes position in the process of working the stratum in accordance with the principle $F(x, y, t) = 0$. The stratum is worked by means of n exploitation and l pressure wells. The problem is solved with the following assumptions: the filtration principle is laminar, the exploitation regime of the stratum is a water-pressure one, the liquids are incompressible, the coefficient of permeability of the stratum $k(x, y)$ is continuous and the function is delimited together with its derivatives up to the second order inclusively, while in this process $k(x, y)$ does not change to zero value at any point in the stratum. The problem set in the paper had been solved previously by R. M. Nasyrov [*Uch. Zap. Kazansk. Un-ta* **116**, 5, 45-49, 1956] when the assumption was made that the function $k(x, y)$ satisfied Helmholtz's equation $\Delta\sqrt{k} + a^2\sqrt{k} = 0$. The author generalizes Nasyrov's results and, by adopting the method of successive approximations, obtains in a closed form a formula for the determination of the pressure of the stratum.

P. F. Fil'chakov

Courtesy Referativnyi Zhurnal, USSR

6699. Merkulov, V. P., Filtration to a horizontal well of finite power (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Neft'i Gaz* no. 1, 73-80, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6540.

A solution is furnished for the problem on the pressurized steady filtration of an incompressible liquid in a horizontal well, disposed symmetrically in relation to the roofing and foundation of a horizontal homogeneous stratum (infinite and bounded). In the investigation the well is replaced by a linear flow of constant

specific yield, and the surface of the well is replaced by one of a number of equipotential surfaces, which will approximate to ellipsoids of rotation when near the linear flow. The equipotential surface replacing the surface of the well has a large axis, equal to the length of the well and a small semi-axis, equal to 1.22 of the radius of the well. The solution obtained contains a rapidly converging series, which is not summed in the paper. It is proposed to determine this summation by indirect means, for instance by experiment. S. N. Numerov

Courtesy Referativnyi Zhurnal, USSR

6700. Kel'tsev, N. V., Hydraulic resistance of activated carbon with gas passing through (in Russian), *Gaz. Prom-st'* no. 12, 31–35, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6549.

The hydraulic resistances are investigated when commercial grades of activated carbon are subjected to a flow of various gases: nitrogen, carbon dioxide, methane, hydrogen. The relation between the coefficient of resistance f and the Reynolds number R was examined

$$f = \frac{g \Delta P D \rho}{2 L G^2}, \qquad R = \frac{DG}{\mu}$$

where $\Delta P/L$ is the pressure gradient, G the gravimetric velocity of the gas, D the equivalent diameter of the particles. Analysis of the experiments confirms the truth of the relation demonstrated by E. M. Minskii: $f = (aR) + b$. The author carried out a series of experiments with activated carbon in a glass tube 25 mm in diameter and 2 m long, through which a gas was passed. A graph was drawn to show the relation of the resistance coefficients to Reynolds numbers, to cover the cases of an absorbent layer, a stationary layer, or a layer in motion, enabling calculations to be made for the falls in pressure. A. P. Shkirich

Courtesy Referativnyi Zhurnal, USSR

6701. Hammad, H. Y., Seepage flow problem in the Nile valley with application to the ground water scheme (in English), 9th Congrès Intern. Mecán. Appl., Univ. Bruxelles, 1957; **4**, 307–315.

Analytical solution is presented for lateral ground water from a river into and out of a two-layered aquifer (a thin clay layer underlain by thick sand layer, both intersected by the river). Annual flood waves on rivers such as Nile cause similar damped waves with time lag in the ground water table. Also treated is the case where there is uniform valley pumping, either continually, or for part of the dry season only. Such pumping at the end of dry season causes a net increase in available water because the reduction of dry weather return flow to the river is less than amount pumped.

Reviewer believes the simplifying assumption [Eq. 17] used in deriving solution with pumping is not adequately justified, and is of limited validity.

Recent work on same subject (but without pumping considered) is "River seepage investigation" by D. K. Todd and J. Bear, University of California, Berkeley, Sept. 1, 1959.

N. H. Brooks, USA

6702. Dallos, I., Investigation of flow of artesian waters by radioactive isotopes (in Hungarian), *Hidrológiai Közlöny* **39**, 3, 199–204, June 1959.

Geophysics, Hydrology, Oceanography, Meteorology

(See also Revs. 6296, 6300, 6393, 6472, 6542, 6601, 6669)

Book—6703. Panov, B. P., Winter regime of rivers of the USSR [Zimnii rezhim rek SSSR], Leningrad, University of Leningrad, 1960, 240 pp. $1.60.

A comprehensive hydrologic study of great practical importance for northern countries. Its contents: Definition of the term "winter season," river classification according to that term, river regime before freezing, freezing-up period, ice-breaking period, ice cover and water temperature in spring, break-up progress on large rivers, terms of freezing and break-up of Russian rivers, river runoff during winter period. Book demonstrates a great amount of experience gained by Russian hydrologists in their severe climatic conditions. Problems of growth of ice cover, measurement and computation of water flow under ice, are of particular interest. A list of 183 references, mostly Russian, emphasizes the trend of development of hydrology in Russia. S. Kolupaila, USA

Book—6704. Lebedev, V. V., Hydrology [Gidrologiia], Leningrad, Rechnoi Transport, 1959, 192 pp. $0.85.

A textbook for river navigation technical colleges, containing brief survey of continental hydrology and dealing mostly with runoff and river regime. S. Kolupaila, USA

Book—6705. Luchsheva, A. A., Practical hydrology [Prakticheskaia gidrologiia], 2nd ed., Leningrad, Gidrometeoizdat, 1959, 468 pp. $2.85.

This is a textbook for hydrometeorological technical colleges, revised and enlarged from the 1st edition of 1950. It shows the contents and extent of the course of hydrology as studied by Russian students. It consists mostly of practical problems on river runoff and floods, solved with real data and contemporary methods. Numerous tables and 10 maps and diagrams increase the practical value of this interesting book.

List of 51 references does not include any foreign publications.

S. Kolupaila, USA

Book—6706. Spengler, O. A., and Spengler, E. N., Hydrologic dictionary in foreign languages [Gidrologicheskii slovar' na inostrannykh iazykakh], Leningrad, Gidrometeoizdat, 1959, 214 pp. $1.60.

Dictionary consists of four parts: English-Russian, German-Russian, French-Russian, and combined Russian-English-German-French. Appendix gives abbreviations, conversion factors, etc. Dictionary was prepared by hydrologists for hydrologists, therefore contains all practically important terms. Authors did a magnificient work, which can be of substantial value for our translators. There are, of course, some omissions or errors, e.g., struia (jet, Strahl, le jet) is translated as filament of water. Surprisingly, some well-established Russian terms, like gidromodul' (unit discharge) are eliminated. Moreover, some very poor terms from our technical jargon, like hydrograph and unitgraph (i.e. hydrogram and unitgram) are adopted by Russian hydrologists, although "hydrograph" in correct Russian means hydrographer, a man, or a hydrographic instrument, but not a diagram.

S. Kolupaila, USA

Book—6707. Bibliography of hydrology for the year 1957 [Hydrologicka bibliografie za rok 1957], Prague, Vyzkumny Ustav Vodohospodarsky, 1957, 238 pp. (Paperbound)

6708. Ovsepyan, V. M., Possible forms for the curves of the free surface in nonprismatic river beds (in Russian), *Sb. Nauchn. Trudl Erevansk. Politekhn. In-ta* no. 9, 81–97, 1955; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2630.

The author, starting from the basic differential equation for the uneven motion of a liquid, investigates the possible forms that the curves could take when depicting the free surface of a flow in nonprismatic (widening and narrowing) open river beds, on the assumption that the "linear" principle is operating in regard to the change of area of the live section ω by the length of the flow s

$$\omega = \omega_0 + \beta sb$$

where β is the coefficient characterizing the angle of convergence or divergence of the borders of the riverbed. The given linear hypothesis for the change of ω has no actual physical meaning. However, the basic initial premise in the paper $\partial\omega/\partial s = \beta b$ is true when the change in width of the bottom along the flow of a rectangular or triangular riverbed is linear. The questions regarding the integration of the fundamental differential equation for the uneven motion of the liquid and the drawing of a profile for the free surface of the flow are not touched on in the paper. The author limits his investigation to the possible forms of the curves for the free surface of the flow in rectangular river beds, with the result that the types of curves for the free surface examined by him fail to incorporate all the possible forms. A full solution of the problem during the linear change of the width of the bottom along the flow (for a trapezoidal river bed) is given in the works of the abstractor [*Dokladî Akad. Nauk SSSR* **96,** 4, 705–707, 1954; *Izv. Vses. Nauki In-ta Gidrotekbn.* **54,** 38–53, 1955].

V. B. Dul'nev
Courtesy Referativnyi Zhurnal, USSR

6709. Konstantinov, A. R., Bases of the calculation procedure for evaporation in natural conditions (in Russian), *Trudî Gos. Gidrolog. In-ta* no. 48, 22–37, 1955; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2751.

A method is given for the calculations of turbulent flows of various substances, based on the analysis of experimental material for the pulsations of meteorological elements by means of physical concepts. It is shown that the main difficulty appears to be that in a thermally heterogeneous (on the horizontal plane) atmospheric mass there is an upward motion of relatively warm and a downward motion of relatively cold particles of air. This effect, and not the intensified results of mixing, is the dominant factor in the growth of turbulent flows with increases in the thermal instability. Formulas are given for the coefficients of turbulence and wind gradients, the temperature and humidity near the earth's surface. The divergence of the profiles from the logarithmic is taken into consideration by the introduction of a special coefficient.

D. L. Laikhtman
Courtesy Referativnyi Zhurnal, USSR

Book—6710. Frenkiel, F. N., and Sheppard, P. A., edited by, Advances in Geophysics, Vol. 6, Atmospheric diffusion and air pollution (Proceedings of a Symposium held at Oxford, Aug. 24–29, 1958), New York, Academic Press, Inc., 1959, xvii + 471 pp. $12.

The Symposium consists of 43 formal papers broken down as follows: General survey of atmospheric diffusion and pollution (3), Recent findings on atmospheric turbulence (5), Theory of turbulent diffusion (9), Diffusion of heavy or finite particles (2), Transfer through the troposphere and stratosphere (10), Effects of thermal stratification on diffusion (3), Pollution pattern from point and area sources (11), plus (4) review (of the Proceedings) articles. Contributors include meteorologists, mathematicians and fluid dynamicists. The countries represented are USA (18), England (9), Sweden (4), USSR (4), Australia (2), Mexico (2), Netherlands (2), and one each from Canada, France, Germany, India, Norway and Japan.

Papers vary greatly in quality, length and interest. The brief discussions following each paper are frequently illuminating, although the placement of articles in the various subsections is sometimes mystifying. Reviewer finds the theoretical articles of major interest. The summary of recent findings on the spectrum and correlation structure of turbulence near the ground is both convenient and valuable.

Atmospheric turbulence is characterized by turbulent elements on almost all time or space scales and the several articles dealing with methods of coping with such a system are of great importance. A further distinguishing characteristic of the atmosphere is the almost invariable presence of vertical potential temperature gradients which profoundly affect turbulence structure and diffusion. The few articles devoted to this subject are at least seminal in nature. As with all turbulent diffusion problems, the relationships between the observable Eulerian statistics and the virtually unobservable but pertinent Lagrangian statistics are fundamental. Several articles deal with this problem mainly from an empirical point of view.

There is little in the volume which is directly applicable to practical pollution problems. Reviewer believes book to be of interest and utility mostly to specialists in the fields of turbulence and atmospheric diffusion. Chief merit of the book is that it reflects accurately the current status of research and thinking in the field of atmospheric diffusion.

B. Davidson, USA

6711. Gliddon, J. E. C., Diffusion of ions in a static F_2 region, *Quart. J. Mech. Appl. Math.* **12,** 3, 340–346, Aug. 1959.

The vertical diffusion of ions under the action of gravity with a rate of electron loss which decreases exponentially with height is shown to correspond to heat conduction in a rod with a heat-source distribution and radiation from the lateral surface. Electron density is determined as a time-periodic function of the height and is expressed in the form of an infinite series involving parabolic functions. Although an asymptotic formula for terms of large order is derived, no computation of the solution is attempted.

R. P. Pearce, Scotland

6712. Gliddon, J. E. C., Use of Green's function in the solution of ionospheric diffusion problems, *Quart. J. Mech. Appl. Math.* **12,** 3, 347–353, Aug. 1959.

A Green's function is derived for the analogous problem of heat conduction in semi-infinite rod with a given heat-source distribution. Two cases are considered—one with a constant loss coefficient and the other with a loss coefficient decreasing exponentially with height—and shown to agree with solutions obtained previously by Ferraro and Ozdogen and the author.

R. P. Pearce, Scotland

6713. Strom, G. H., Scale-model wind-tunnel studies on atmospheric-diffusion phenomena, Proceedings of the Seventh Hydraulics Conference, Iowa Institute of Hydraulic Research, June 16–18, 1958; Iowa City, Iowa, State Univ. of Iowa, 1959, 91–110.

Paper describes wind-tunnel diffusion experiments which simulate the dispersion of industrial stack gases into the atmosphere. Criteria for choosing scale of model, and variables such as wind speed and stack gas density, are discussed using dimensional analysis. Control of atmospheric variables introduces problems distinguishing meteorological wind-tunnel modelling from usual aerodynamic experiments. Correlation of data from two industrial prototypes with data from wind-tunnel modelling is quite satisfactory. [AMR **8**(1955), Revs. 783, 3882.] Downwind diffusion of sulfur dioxide from point source in wind tunnel is demonstrated to be quite dependent on stability of vertical thermal gradient.

A. F. Presler, USA

6714. Reshetnikova, K. A., Determination of vertical currents by means of the equations for the dynamics of a turbulent atmosphere and analysis of their mean monthly values (in Russian), *Trudî Gl. Geofiz. Observ.* no. 71, 72–93, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6479.

By taking advantage of the equations for the motion of a turbulent atmosphere in a horizontal plane at any known distribution of atmospheric pressure determinations are made by means of successive approximations for the horizontal components of the wind's velocity (in the first approximation the wind's acceleration is disregarded); then follow determinations for the vertical velocity

by use of the equations of continuity. For the purpose of solving the equations of motion the lower boundary condition, emanating from the condition of the constancy of the turbulent stress in the underlayer next the earth, is assigned with approximate consideration for the irregularities of the earth's surface (following the theory of long waves). The assumption is made that at high altitudes the forces of turbulent viscosity are small and that the wind is akin to the geostrophysical. The calculation for the vertical velocities averaged by time is demonstrated. Tables are given and also diagrams to show the computed mean vertical velocities for January and July at an altitude of 1500m in the latitude range of 20° to 70°. The absolute magnitudes of the vertical velocities at 1500m altitude on mountain slopes varied from 1 to 70 mm/sec; above valleys and above seas, within the limits of 1 to 3 mm/sec. Calculations for the field of mean vertical velocities above ravines at 3000m altitude showed some increase in the absolute magnitude of the velocities (their maximum value equalled approximately 5 mm/sec). An attempt was made to ascertain the numerous correlational links between the vertical velocity, the relative humidity at the earth's surface and the precipitations. The influence was examined of vertical currents on the inflow of warmth in the atmosphere. Tables are published showing the results of calculations for the advection, convection and horizontal turbulent flows of heat at 1500m altitude.

G. P. Kurbatkin

Courtesy Referativnyi Zburnal, USSR

6715. Saltzman, B., and Fleisher, A., Spectrum of kinetic energy transfer due to large-scale horizontal Reynolds stresses, *Tellus* **12,** 1, 110-111, Feb. 1960.

An integral representing the rate of transfer of kinetic energy between the atmospheric mean zonal flow and the harmonic components of the eddy flow is applied to geostrophic winds at 500 mb for each day of 1951. A fifteen wave-number Fourier resolution was used along every five degrees of latitude from 15° N to 80° N of contour heights spaced every ten degrees of longitude. Results are shown in graphical and tabular form. The net transfer is found to be directed from the eddies to the zonal flow; the annual average transfer is 3.8×10^{20} ergs sec^{-1}, the winter average 5.8×10^{20} ergs sec^{-1} and the summer average 1.8×10^{20} ergs sec^{-1}. These values are smaller than, but of the same order as, those obtained by Starr for the same year using observed winds.

G. Thuronyi, USA

6716. Crapper, G. D., A three-dimensional solution for waves in the lee of mountains, *J. Fluid Mech.* **6,** 1, 51-76, July 1959.

This paper presents a three-dimensional small-perturbation approach to the problem of waves produced in a statically stable stratified air stream flowing over a mountain. The fundamental solution for a doublet disturbance in an air stream in which the parameter $l^2 = g\beta/V^2$ is constant is calculated, and then is extended to that for a disturbance due to a circular mountain in the same air stream. A simple approximation to the known two-dimensional flow over an infinite ridge is also given. The second ("upper") boundary condition for the solutions is determined in a rigorous analytical manner, assuming the presence of small friction forces, or, alternatively, of time dependence. It is hoped that this will settle the controversy which exists over the choice of this condition.

The results show that the behavior due to a doublet is peculiar and not truly representative of that due to a mountain. The latter shows waves which decay downstream and are contained in a strip, the width of the strip being determined by the radius of the mountain. An interesting result is that the circular mountain can give rise to waves which have greater amplitude than those produced by the infinite ridge under the same conditions.

In some previous papers the waves produced by the infinite ridge have been neglected, but the present paper shows that in many cases this procedure is not justifiable. The detailed solution for the waves behind a circular mountain has a form which emphasizes the importance for lee-wave production of "resonance" between the width of the mountain and the characteristic length l^{-1} of the air stream.

Reviewer considers this paper to be a significant contribution to the theory of mountain waves in the atmosphere, and an important development of the earlier work of Queney, Lyra and Scorer.

R. P. Pearce, Scotland

6717. Matveev, L. T., The part played by vertical currents and turbulent mixing in the formation and evolution of stratus cloud (in Russian), Investigations on clouds, precipitations and thunderstorm electricity (Issled. Oblakov, Osadkov i Grozovogo Elektrichestva), Leningrad, Gidrometeoizdat, 1957, 37-42; *Ref. Zb. Mekb.* no. 3, 1959, Rev. 2752.

Calculations were carried out for the vertical velocities in the boundary layer of the atmosphere (the relation of the coefficient of turbulence to the altitude was represented as conforming to the exponential principle) which were compared with the various characteristics of the cloudy state (the amount, the situation of the lower and upper boundaries, and so forth). The author notes, as the result of his analysis of the tables given, that the vertical velocity is an important though not the sole factor in cloud formation; in approximately one-third of the cases considerable cloudiness is observable in descending currents and a cloudy state of less than seven points in ascending currents. The results of comparison of the conditions causing the evolution of clouds, in the sense of the amount formed with change of the vertical velocity with time (for 12 hours), showed that the positive change in the vertical velocity is linked usually with an increase in the amount of cloud (repeatability 84 and 67% respectively); with negative change with time there is cloud dissipation (repeatability 91 and 85% respectively). The author gives a theoretical explanation for the principal differences and distribution of humidity content with altitude in clouds of the type Sc-Ac-St and Ns-As-Cs; this is effected by analysis of the solutions of the equations for the turbulent diffusion of aqueous vapor when different assumptions are accepted relative to the dependence of the coefficients of turbulence and vertical velocity on the altitude, and also with various boundary conditions which satisfy the function being sought (the specific moisture content). Formulas are put forward for the determination of the altitude of the lower cloud level. The author concludes by explaining a widely-occurring phenomenon, the formation of "dry" (cloudless) interlayers in cloud strata.

K. G. Abramovich

Courtesy Referativnyi Zburnal, USSR

6718. Tkachenko, A. V., On the power of convection and the use of convection in local prognoses (in Russian), *Trudi Gl. Geofiz. Observ.* no. 69, 36-40, 1957; *Ref. Zb. Mekb.* no. 3, 1959, Rev. 2754.

In order to compute the energy in convection motions in the atmosphere the so-called "method of the power of convection" is utilized [D. L. Laikhtman, *Trudi Gl. Geofiz. Observ.* no. 37, p. 99, 1952]. The power of convection for this purpose is determined as the work which is performed in a unit of time by the whole of the mass taking part in convection motions in a column of air of determined height b with a single transverse section. The author uses the expression for the mean value of the power of convection P in the form

$$P = -\frac{g\overline{\eta}}{T}\,\frac{1}{b}\int_0^b (T - T_a)\,d_z \qquad [1]$$

where g is the acceleration of the force of gravity, T and T_a are the temperatures at a given height z in the surrounding medium and in the vertically mixing particles, following the adiabatic

principle, $\overline{T}$ is the mean temperature of the layer, $\overline{\eta}$ is the mean coefficient of exchange for the layer. The mean power of convection is the magnitude proportional to the energy of instability, where the coefficient of proportionality is equal to the mean coefficient of turbulent exchange for the layer $\overline{\eta}$, divided by the thickness of the layer being investigated b. The author gets a working formula for the determination of the mean power of the convection in accordance with the data obtained from the wind and temperature soundings of the atmosphere. This he accomplishes by an investigation of a layer extending from the condensation level to the convection level and by introducing a number of supplementary assumptions and simplifications. The relationship is examined of the power of convection determined by the data from the morning soundings and the maximum intensity of convection in the daytime which characterizes the weather qualitatively (the amount and form of the convection clouds, torrential rain, thunderstorms above the point or near it). For this purpose results were used of a series of observations made by airplane and balloon pilots from the Principal geophysical observatory in the summer months of 1946, 1947 and 1948 on days when cumulus cloud was about. The daily values of the mean power of convection $\overline{P}$ and of the mean, the same layer, of the relative humidity values r, computed on the data of the morning soundings, were utilized for drawing up the diagram for the intensity of the convection. It was found possible to separate out three regions in this diagram, corresponding to the three steps of intensity of the convection in conditional gradations. The diagram is recommended for use in prognoses of the intensity of convection. It was shown that a similar diagram drawn for showing the energy of instability gives an error almost three times as large when used for prognoses.

K. G. Abramovich
Courtesy Referativnyi Zhurnal, USSR

6719. Shur, G. N., Determination of vertical velocities of turbulent gusts of wind in aeroplane flight investigations (in Russian), *Trudi Tsentr. Aerol. Observ.* no. 22, 9–16, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2755.

The relation $W_z = V_z + \Delta n$ was used to determine the vertical velocities of turbulent gusts of wind in airplane flight investigations, where W_z is the velocity of the vertical gust of the wind, V_z is the vertical component of the velocity of the center of gravity of the airplane, Δn is the overload (the vertical acceleration of the center of gravity of the airplane) recorded by accelograph, b is the coefficient depending on the flight parameters of the airplane. The magnitude of V_z in this calculation is obtained as the result of the graphical integration of the accelopraph record. The author built an apparatus, named by him "the electronic intergrator of overloads" (EIP Ts AO), which enables simultaneous registration of the vertical acceleration (overload) and of the vertical velocity of the center of gravity of the airplane. The basic center of the above apparatus is the electrointegrator. A description is furnished of the apparatus; the principle of action of the electro-integrator is demonstrated; and the method of calibration is indicated.

K. G. Abramovich
Courtesy Referativnyi Zhurnal, USSR

6720. Gaigerov, S. S., and Kastrov, F. G., Experiments for the measurement of the vertical turbulent flow of heat in the lower part of the troposphere (in Russian), *Trudi Tsentr. Aerol. Observ.* no. 23, 16–51, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2757.

Results are furnished of the measurements of the vertical turbulent flow of warmth in the lower troposphere obtained with the aid of the pulsation method. The measurements were carried out in apparatus placed in the basket of a free aerostat. The temperature pulsations were measured by means of a microthermograph, built to the design of S. I. Krechmer, while the pulsation of the vertical velocity was recorded on a winged vertical anemometer designed by V. S. Khakhalin and O. F. Rabinchuk. The possible errors of the apparatus used are the subject of analysis. A method for handling the data is given. In order to eliminate micropulsations, beyond the scope of measurement by the anemometer, the data were subjected to a sliding averaging with a period of 12 secs. The fluctuations of temperature connected with the oscillations due to the altitude of the aerostat were not taken into consideration. The statistical characteristics of the temperature fluctuations are given, as well as information on the magnitude of the vertical gradients of the temperature in different conditions, information regarding the values for the turbulent thermal flow (which in the majority of cases did not exceed the limits of ± 0.01 cal/cm^2 min) and their relation to the temperature gradient, and data for the possible values of the exchange coefficient. It is noted that the transfer of heat by large-scale vortices not registered in the measurements carried out can exceed by many times the measured values for the flow of heat due to small-scale vortices. A supplement gives information in brief form of the flights and the results of the observations made in these flights.

A. S. Monin
Courtesy Referativnyi Zhurnal, USSR

6721. Alekseev, P. P., Besyadovskii, E. A., Golyshev, G. I., Izadov, M. N., Kasatkin, A. M., Kokin, G. A., Livshits, N. S., Masanova, N. D., and Shvidkovskii, E. G., Investigation of the atmosphere by rocket (in Russian), *Meteorolog. i Gidrologiya* no. 8, 3–13, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2758.

Article is a study of the structural parameters of the atmosphere, carried out with the assistance of meteorological rockets. The head of the meteorological rockets, which breaks away at a given altitude from the body of the rocket and descends to earth by parachute contains a collection of scientific apparatus. A plan is given to show how this apparatus is placed in the rocket head. In order to reduce the effect of gas separation the manometers and temperature transmitters are placed in the front part of the rocket head. In order to determine the position of the rocket in space four photographic devices are placed in the rocket head which operate synchronously and are located in one plane at an angle of 90° between the axes of the lenses. The transmission of the data of the measurements is effected by radiotelemetry having a high frequency modulation. The measurements are made in ascent and descent. The determination of the rocket's trajectory and of its components is made by means of kinotheodolite base observations. The final formulas are examined, connecting the pressure inside the manometer with the pressure of the free atmosphere, the temperature recorded by the rocket head's thermometer with the atmospheric temperature; values are given (in graphs and in tables) of the pressure, density and temperature of the atmosphere for ranges between 0 to 80 km in height.

V. V. Mikhnevich
Courtesy Referativnyi Zhurnal, USSR

6722. Ogneva, T. A., The magnitude of the turbulence coefficient above reservoirs (in Russian), *Trudi Gl. Geofiz. Observ.* no. 69, 51–56, 1957; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6491.

A proposal is put forward, on the basis of the generalization of M. P. Timofeev's work [*Uch. Zap. LGU, Ser. Fiz. Nauk* **7**, no. 120, 1947], to determine the coefficient of turbulence k_1 by means of the data for temperature or humidity changes above a reservoir with the aid of the formula

$$\frac{T - T'}{T_0 - T'} = \left(1 + \frac{\gamma x}{T_0 - T'}\right) \frac{1}{\Gamma(p)} \int_{1/2L}^{\infty} e^{-t}\, t^{p-1}\, dt$$

$$L = \frac{k_1 x}{u_1}\, z^{-\frac{1}{1-2p}} \left[2(1-2p)^2\, z^{\frac{1-4p}{1-2p}}\right]^{-1}$$

where T, T^*, T_0 are temperatures of the air at points (x, z), of the air at point $(0, z)$, and the surface of the water at point $x = 0$ (where the reservoir begins); γ is the horizontal temperature gradient of the water's surface; u_1 is the velocity of the wind. An analogous formula is recorded to meet the case when the temperature of the water's surface changes with distance x not linearly but in accordance with the parabolic principle. It is shown how the coefficients change with the substitution of temperature by humidity. An evaluation is made of the relative error in the determination of parameter L by means of empirical data. Results are published for the determination of k_1/u_1 by measurements in the Balkash lake. The values of k_1/u_1 can be estimated on an average at 0.01–0.02 m, but there is a divergence of values obtained with different values for x and when employing data for the temperature or humidity. A. S. Monin

Courtesy Referativnyi Zhurnal, USSR

6723. Fortak, H., Different representations of isobaric wind vectors (in German), *Z. Meteorol.* **12**, 4/6, 163–172, Apr./June 1958.

This paper develops expressions for the vector wind field in terms of the kinetic energy K, the wind direction α, the divergence D and the vorticity ζ. A purely kinematic relation is

$$\underset{\sim}{V} = (\zeta^2 + D^2)^{-2}\,[D\,\Delta K + \zeta \underset{\sim}{k} \times \Delta K + 2K\zeta\Delta\alpha - 2KD\underset{\sim}{k} \times \Delta\alpha]$$

From the familiar equations governing the rates of change of these quantities, several similar expressions for the wind field can be derived with the time derivatives on the right hand side of the equation. For example,

$$\underset{\sim}{V} = (K/fK_g)(\dot{\alpha} + f)\underset{\sim}{V}_g\ (2fK_g)^{-1}\ \dot{K}(\underset{\sim}{k} \times \underset{\sim}{V}_g)$$

The research may thus be considered as a continuation of lines begun by Brunt, Douglas, Ertel, Hollman, and Pettersen, quite formal in itself but possibly useful.

It should be noted that the equations are uncomfortable to work with in any co-ordinate system but the Cartesian.

M. G. Wurtele, USA

6724. Malkus, J. S., and Riehl, H., On the dynamics and energy transformations in steady-state hurricanes, *Tellus* **12**, 1, 1–20, Feb. 1960.

A dynamic model of the inflow layer in a steady mature hurricane is evolved, relating wind speed, pressure gradient, surface shearing stress, mass flow, and convergence. The low-level air trajectories are assumed to be logarithmic spirals. With this hypothesis, properties such as maximum wind and central pressure are determined through choice of a parameter depending on the inflow angle: a moderate hurricane arises with inflow angles of about 20°, while 25° gives an intense ... storm. The heat transfer from the ocean and the release of latent heat in the core determine the pressure gradient along the trajectory, and this prescribes the particular trajectory selected by the air. This selection principle is evolved using recent work on "relative stability" of finite amplitude thermal circulations [AMR **12**(1959), Rev. 1506]. Of an infinite number of dynamically possible spirals, the one is realized which maximizes the rate of kinetic energy production under the thermodynamic constraints, here formulated in terms of the relation between heat release and pressure gradient.

Finally, rainfall, efficiency of work done by the storm, and kinetic energy budgets are examined in an attempt to understand the difference between the hurricane and the common sub-hurricane tropical storm.

From authors' summary by G. Thuronyi, USA

6725. Veigas, K. W., Miller, R. G., and Howe, G. M., Probabilistic prediction of hurricane movements by synoptic climatology, Travelers Weather Res. Center, Occasional Papers in Meteorology no. 2 (Travelers Insurance Co., 700 Main St., Hartford 15, Conn.), 54 pp., June 1959.

A statistical approach is made to the prediction of a hurricane track. The forecasting equation consists of the variables of surface pressures at several selected points around the storm and the position coordinates of the hurricane for prediction time together with those for 24 hours prior to it. For constructing the multiple linear regression equation, hurricane samples from 1928–1953 were selected and 447 developmental cases were utilized, each of the cases involving 95 gridpoint values and position coordinates. A test on independent data revealed that the average length of vector errors for 65 northern-zone storm forecasts is 150 n.mi. and for 60 southern-zone forecasts it is 95 n.mi. In the case of Betsy of 1956, which is one of three illustrated examples, the recurvature of track was well forecasted. Further, the method of qualitative estimation of probability is presented.

K. Miyakoda, Japan

6726. McDonald, J. E., Direct absorption of solar radiation by atmospheric water vapor, *J. Meteorol.* **17**, 3, 319–328, June 1960.

A review of Fowle's solar-absorption studies reveals that no pressure corrections must be applied to the familiar Fowle band-absorptivity curves since these were obtained by methods that automatically corrected them to sea-level pressures. Using these absorptivities, along with recent solar-energy-distribution data, total insolational absorptivities are obtained. These are fitted (with relative accuracy of about 1% over the range 0.3 to 8.0, precipitable water) to an exponential function similar to that used by Mügge and Möller to obtain working relations for use in calculating daily heating amounts due to insolational absorption by water vapor. It is found that variation of the pressure exponent from zero to unity in trial pressure-correction laws yields only a 6% variation in total columnar absorption for a concrete case. Estimates accurate to within about 10% with respect to the Fowle absorption data are attainable for levels below about 400 mb in humid regions. However, clarification of the method employed by Houghton in arriving at his roughly 30% greater absorptivities leaves doubt as to the status of the familiar Fowle absorptions, so it is concluded that we cannot now claim to be able to predict insolational heating rates to better than about 30% absolute accuracy. From author's summary

6727. Wagner, C. A., The available chemical energy in the upper atmosphere, Univ. Wisconsin, Theoretical Chem. Lab., Wis-OOR-28, Series 5, 89 pp., May 1960.

In this paper the total amount of available chemical energy in the atmosphere is determined for various altitudes. This is compared with approximations to the energy required to orbit a fuel-less recombination ramjet missile at these same selected altitudes.

Since thermodynamic, temperature, pressure, and especially composition data are extremely critical to these calculations, a discussion of sources is included. Details of the thermodynamic calculations and the computational techniques employed in conjunction with the Bendix G-15 high-speed computer are also discussed.

Tables of important data and results are presented for future reference and checking; new temperature, pressure, and composition data will undoubtedly be available soon for more accurate calculations. These calculations, using presently available data, indicate that operation of such a vehicle would be marginal around 120 kilometers and impractical elsewhere.

From author's summary

6728. Dubov, A. S., and Stolyarova, G. V., An attempt to make temperature forecasts by hydrodynamic methods (in Russian), *Trudi Gl. Geofiz. Observ.* no. 76, 30–39, 1958; *Ref. Zh. Mekh.* no. 6, 1959, Rev. 6478.

Results are published of experiments carried out by the computation group of the NW section of the UGMS during 1955-56 to forecast the night minimum temperatures with tentative durations of 12-15 hours and to make 24-hour predictions of the extremes of temperatures, more accurate than those for durations of 03 to 15 hours. The night minimum was computed when taking into account the initial conditions, the radiation and advective factors; consideration was also given to the possible changes in cloud conditions. The predictions of extremes of temperature with a 24-hour forecast period included the determination of temperature advection for the twenty-four hours (the transfer was made on the initial field AT_{850}) and the introduction of a correction for transformation in agreement with M. E. Berlyand

$$\Delta T_{tr} = b\Delta T_0 + a\Delta R$$

where ΔT_0 is the difference of temperature in the end and initial points of the trajectory of the particle in the initial moment of time, ΔR is the difference of radiation balances in the end point of the trajectory in the end moment of time and in the initial point of the trajectory in the initial moment of time, a and b are coefficients dependent on the velocity of transfer, the properties of the underlying surface and so forth. Results of the prognoses made are furnished in tabular form. Some conclusions are arrived at regarding the suitability of the methods used and the quality of the prognoses.

G. P. Kurbatkin

Courtesy Referativnyi Zburnal, USSR

Naval Architecture and Marine Engineering

(See Revs. 6284, 6390, 6485)

Friction, Lubrication and Wear

(See also Rev. 6678)

6729. Buckley, D. H., and Johnson, R. L., Use of less reactive materials and more stable gases to reduce corrosive wear when lubricating with halogenated gases, NASA TN D-302, 18 pp., Aug. 1960.

The gases CF_2Cl—CF_2Cl, CF_2Cl_2, and CF_2Br—CF_2Br were used to lubricate metals, cermets, and ceramics at temperatures to 1400°F. The use of cermets and ceramics decreased corrosive wear at high temperatures with these gases as lubricants. In friction and wear experiments, a hemispherically tipped rider under a 1200-gram load slid on a disk rotating at speeds from 75 to 8000 feet per minute. The gas CF_2Cl—CF_2Fl was found to be an effective lubricant for Al_2O_3 sliding on Stellite Star J at temperatures to 1400°F. The gas CF_2Br—CF_2Br provided effective lubrication for the cermet K175B (nickel-bonded metal carbide) sliding on Hastelloy R-235 (nickel-base alloy) at temperatures to 1200°F.

From authors' summary

6730. Flom, D. G., and Bueche, A. M., Theory of rolling friction for spheres, *J. Appl. Phys.* **30**, 11, 1725–1730, Nov. 1959.

A theory of rolling friction featuring the importance of elastic hysteresis losses is presented....

A prediction resulting from the theory is that the coefficient of friction for a relative hard sphere rolling on a softer base material should vary with speed so as to go through a maximum....

The results are not restricted to rolling, but also apply to well-lubricated sliding where shearing forces have been minimized. Although the theory is developed for a material with idealized physical properties it nevertheless affords a basis for comparing real materials and for predicting their frictional properties, in cases where deformation losses are predominant.

From authors' summary by E. Fliess, Argentina

6731. Kosikov, S. I., A portable friction gauge (in Russian), *Trudî In-ta Fiz. Khimii. Akad. Nauk SSSR* no. 6, 168–173, 1957; *Ref. Zb. Mekh.* no. 3, 1959, Rev. 2337.

A description is given of this gage, which is fitted with an electrical device for recording the forces of friction. This device for recording the static friction works with an accuracy for readings up to 0.5%, and was used in field conditions for measuring the coefficient of friction on the rails of a railway track.

N. A. Talitskikh

Courtesy Referativnyi Zburnal, USSR

6732. Ubakeev, S. U., and Mashanov, A. Zh., The theoretical and practical determinations of the coefficients and friction of mineral strata (in Russian), *Trudî Otd. Gorn. Dela i Metallurgii, Akad. Nauk KirgSSR* no. 1, 51–67; *Ref. Zb. Mekh.* no. 6, 1959, Rev. 6739.

The relation is theoretically established between Poisson's coefficient and the angle formed by the lines of slide. The authors derive different formulas for the determination of Poisson's coefficient by first investigating the cases of the deformation of a bar in its analogy to a beam of light and the friction of two hard bodies. The calculated values are compared with the experimental ones taken from the literature. There are numerous errors in the paper: (1) in Fig. 1 Δb should be $\frac{1}{2}\Delta b$; (2) the case of plane deformation is investigated (p. 53), consequently when referring to Poisson's coefficient μ, the authors are in reality operating with the magnitude of $(\mu/1 - \mu)$; (3) they obtain a value for μ differing from 0.5 for an incompressible material; (4) formula (13) should have the form of $S = P/F = P/F_1 \cos \varphi = \sigma_x$. With $\varphi = \frac{1}{2}\pi$ it is not $S = 0$ but $\sigma_n = 0$ in formula (15); (5) is an incorrect indication for, in conformity with the Coulomb-Amonton (20) principle of friction, the forces of friction depend on the area of the bodies being subjected to friction; (6) the deduction is made from table 5 of the convergence of the calculated values for Poisson's coefficient with the experimental; but for half of the materials listed in that table (bronze, steel, granite, limestones) there are actually marked divergencies. Exception must also be taken to the unsubstantiated parities introduced into formulas (3) and (22) and to the deduction on the identical nature of the concepts on Poisson's coefficients and of friction (pp. 66–67), and also of the calculations of Poisson's coefficient using the author's formulas which are incorporated in a table, where this coefficient exceeds 0.5 and reaches 6.40.

M. V. Malyshev

Courtesy Referativnyi Zburnal, USSR

6733. Guminski, R. D., and Willis, J., Development of cold-rolling lubricants for aluminium alloys, *J. Inst. Metals* **88**, 481–492, 1959/1960.

The requirements that should be met by a cold-rolling lubricant are stated, and an account is given of an investigation to develop improved lubricants. To enable the lubricants to be evaluated, tests for "reduction capacity," staining, surface finish, and stability have been devised. A good correlation was found between these tests and the performance of lubricants in a rolling mill. The tests were used to correlate the reduction capacity, staining, and stability of various lubricants with the physical and chemical characteristics (particularly structure and composition) of the base oils and additives from which the lubricants are compounded. On this basis a specification for a satisfactory cold-rolling lubricant has been formulated.

From authors' summary

6734. Karatyshkin, S. G., Theory of a lubricating oil layer in a dynamically loaded bearing (in Russian), Friction and wear in

machines (Trenie i Iznos v Mashinakh), Moscow, SSSR **12**, 163–180, 1958; *Ref. Zb. Mekb.* no. 6, 1959, Rev. 6409.

This is an experimental investigation on the distribution of pressure in a dynamically loaded main bearing of a single-cylinder two-stroke motor. The following items were varied: the delivery pressure of the lubricant to the bearing, the number of revolutions of the shaft and the character of the loading on the bearing. An oscillograph record was made of the distribution of pressure in the layer of lubricant in the bearing by means of three piezo-quartz transmitters, two of which were placed on the shaft and one on the bushing. The curves of the relation of pressure in the lubricant layer to the angle of rotation of the shaft are situated above the atmospheric line. In consequence, the transmitters did not record the rarefaction in the layer. Having reached this point the author deduces the continuity of the lubricant layer in the dynamically loaded bearing. Such a deduction cannot be considered as wholly justified, as the transmitters only recorded the pressure at determined points in the lubricant layer. Besides, when the delivery of lubricant to the bearing was discontinued the pressure curves remained positive which also contradicts the author's deduction regarding the continuous nature of the lubricant layer.

A. I. Golubev

Courtesy Referativnyi Zhurnal, USSR

6735. Savitskii, K. V., An investigation on the distribution of residual deformations under the surface of friction (in Russian), Investigations on the physics of the solid body (Issled. po Fiz. Tverdogo Tela), Moscow, Akad. Nauk SSSR, 1957, 107–114; *Ref. Zb. Mekb.* no. 6, 1959, Rev. 6986.

The deformation under the surface of the friction zone was measured along graduation lines etched on the lateral sides of test samples, made of aluminum and carbon steel, pressed on to the surface of a steel roller. Friction was carried out using a vaseline oil lubricant at rates of sliding varying from 2.2 to 712 m/min. The residual transposition of the insets on their surface during a wear test of one hour was 30–200 μ. More than 50% of the general deformation was concentrated in a thin layer with a thickness of ≈ 0.1 of the full depth of the extent of deformation (50 to 400 μ). The deformation increases in the normal and more especially in the tangential directions to the friction surface with increase in the normal loading. Increase in the speed of sliding reacts to a larger degree on the transposition of the metal in the tangential direction and on the friction surface. Adding to the time of duration of the friction action increases to the largest extent the deformation along the sliding surface; with time the deformation increments decrease and show a tendency to stabilize at some limit.

V. M. Gol'farb

Courtesy Referativnyi Zhurnal, USSR

6736. Burgvits, A. G., The stability of the motion of a journal in a bearing with consideration for the stiffness of the shaft and the resistance of the lubricating oil layer (in Russian), Calculations for design and construction of machines (Raschet i Konstruir. Machin.), Chelyabinsk, 1957, 4–16; *Ref. Zb. Mekb.* no. 3, 1959, Rev. 2257.

An analytical method is demonstrated for the determination of the boundaries of the zone of stable motion of a shaft with a heavy disk, while taking into account the resistance of the lubricating oil layer in the bearings. The forces of elastic resistance of the oil film are presented as proportional to the coordinates of the center of the journal, while the damping forces are treated as being proportional to the velocity of the center of the journal. The method of *D* breaking down is used for the investigation of the stability of the motion by the characteristic equation of the sixth order. A graph is drawn for the determination of the boundaries of stable regimes in relation to the parameters expressing the influence of the magnitude of the radial clearance, the elastic and damping properties of the oil film and also the stiffness of the shaft. The deduction is advanced, in conformity with the graphs, that the decrease of viscosity, the increase of the clearance and pressure on the bearing help to ensure the stable regimes of the work of the journal. The conditions of stability formulated by Hegg and Warner, obtained for the problem by the method of electrical analogy, are given for purposes of comparison.

O. N. Romaniv

Courtesy Referativnyi Zhurnal, USSR

6737. Daniels, C. M., High-load oscillating bearings—load and friction design data for abnormal conditions, *Mach. Design* **32**, 15, 136–141, July 1960.

Letters to the Editor

6738. Re: AMR **13** (1960), Rev. 2673: **Neuber, H., General frequency characteristics of linear elastic systems** (in German), *Ing.-Arch.* **28**, 229–241, Mar. 1959.

The second line of the review reads ... by $\nu \leq \frac{n}{2}$ linear springs .... It should read ... by $\nu \leq \binom{n}{2}$ linear springs ....

The editors regret this error.

6739. Re: AMR **13**(1960), Rev. 2965: **Schuh, H., Determination of the temperature distribution in thin plates with attached supersonic boundary layer** (in German), Symposium on boundary layer research, 209–223.

I believe that the review underestimates the difficulties in obtaining analytical solutions for transient temperature distributions in conducting plates with external heat transfer by boundary-layer flow, even in "simple cases." This is evident from the fact that no satisfactory solution exists for the fundamental case of a laminar boundary-layer flow at zero pressure gradient when the wall temperature is nonuniform, as actually occurs in transient heating. The purpose of my paper was to show first the accuracy of the finite difference method for problems of this type in a simple case for which an approximate analytical solution exists. Then, the method was used to solve the following cases: (a) Partly laminar and partly turbulent boundary layer and (b) heat transfer at the surface of the plate by convection and thermal radiation. An analytical treatment would appear very difficult, and therefore finite difference methods may be the only practical available technique for problems of this kind, particularly if high-speed calculating machines are used.

H. Schuh, Sweden

Books Received for Review

ALBERTSON, M. L., BARTON, J. R., AND SIMONS, D. B., Fluid mechanics for engineers, Englewood Cliffs, N. J., Prentice-Hall, Inc., 1960 xiii + 567 pp. $13. (Classroom edition $9.75)

ANDERSEN, P., AND NORDBY, G. M., Introduction to structural mechanics, New York, The Ronald Press Co., 1960, vii + 340 pp. $5.50.

BATEL, W., Einfuhrung in die Korngrossenmesstechnik, Berlin, Springer-Verlag, 1960, viii + 156 pp. DM 27.

BIRD, R. B., STEWART, W. E., AND LIGHTFOOT, E. N., Transport phenomena, New York, John Wiley & Sons, Inc., 1960, xxi + 780 pp. $13.75.

BRESLER, B., AND LIN, T. Y., Design of steel structures, New York, John Wiley & Sons, Inc., 1960, xiii + 710 pp. $9.75.

CASHWELL, E. D., AND EVERETT, C. J., A practical manual on the Monte Carlo method for random walk problems, New York, Pergamon Press, 1959, ix + 153 pp. $6.

EGLI, P. H., edited by, Thermoelectricity (includes the Proceedings of the Conference on Thermoelectricity sponsored by the Naval Research Laboratory, September 1958), New York, John Wiley & Sons, Inc., 1960, x + 407 pp. $10.

FLUGGE, W., Stresses in shells, Berlin, Springer-Verlag, 1960, xi + 499 pp. DM 58.80.

FORSYTHE, G. E., AND WASOW, W. R., Finite-difference methods for partial differential equations, New York, John Wiley & Sons, Inc., 1960, x + 444 pp. $11.50.

FUCHS, N. A., Evaporation and droplet growth in gaseous media (translated from the Russian by J. M. Pratt), New York, Pergamon Press, 1959, 72 pp. $5.50.

GOL'DENBLAT, I. I., AND NIKOLAENKO, N. A., Creep and load carrying capacity of shells [Polzuchest' i nesushchaya sposobnost' obolochek], Moskva, Gosudarstvennoe Izdatel'stvo Literatury po Stroitel'stvu, Arkhitekture i Stroitel'nym Materialam, 1960, 58 pp. 2 r 10 k.

GOL'DENBLAT, I. I., AND NIKOLAENKO, N. A., Theory of creep of structural materials and its application [Teoriya polzuchesti stroitel'nykh materialov i ee prilozheniya], Moskva, Gosudarstvennoe Izdatel'stvo Literatury po Stroitel'stvu, Arkhitekture i Stroitel'nym Materialam, 1960, 256 pp. 8 r 25 k.

GUROV, A. F., Bending vibrations of members and the assembly of aircraft gas turbine engines [Izgibnye kolebaniya detalei i uzlov abiatsionnykh gazoturbinnykh dvigatelei], Moskva, Gosudarstvennoe Izdatel'stvo Oboronnoi Promyshlennosti, 1959, 359 pp. 17 r 90 k.

HALEY, A. C. D., AND SCOTT, W. E., editors, Analogue and digital computers, New York, Philosophical Library Inc., 1960, viii + 308 pp. $15.

IVAKHNENKO, A. G., Engineering cybernetics [Tekhnicheskaya kibernetika], Kiev, Gosudarstvennoe Izdatel'stvo, 1959, 422 pp. 10 r 95 k.

JENSEN, A., AND CHENOWETH, H. H., Applied engineering mechanics, 2nd ed., New York, McGraw-Hill Book Co., Inc., 1960, xiii + 409 pp. $6.50.

KARPLUS, W. J., AND SOROKA, W. W., Analog methods, computation and simulation, 2nd ed., New York, McGraw-Hill Book Co., Inc., 1959, xiii + 483 pp. $12.50.

LEE, Y. W., Statistical theory of communication, New York, John Wiley & Sons, Inc., 1960, xvii + 509 pp. $16.75.

LORENZ, H., Grundbau-Dynamik, Berlin, Springer-Verlag, 1960, viii + 308 pp. DM 46.50.

MAXWELL, R. L., Kinematics and dynamics of machinery, Englewood Cliffs, N. J., Prentice-Hall, Inc., 1960, xii + 477 pp. $9.75.

MILNE-THOMSON, L. M., Theoretical hydrodynamics, 4th ed., New York, The Macmillan Co., 1960, xxi + 660 pp. $11.

MULLER, W., Theorie der elastischen Vervormung; Mathematik und Ihre Anwendungen in Physik und Technik, Band 27, Leipzig, Geest & Portig K.-G., 1959, xi + 327 pp. DM 31.50.

OBRAZTSOV, I. F., Methods of analysis of the strength of caisson-type wing construction [Metody rascheta na prochnost' kessonnykh konstruktsii tipa dryla], Moskva, Gosudarstvennoe Izdatel'stvo Oboronnoi Promyshlennosti, 1960, 312 pp. 10 r 65 k.

OLVER, F. W. J., edited by, Bessel functions, Part 3: Zeros and associated values, New York, Cambridge University Press, 1960, lx + 79 pp. $9.50.

OSTROWSKI, A., Vorlesungen uber Differential- und Integralrechnung, Vol. 1, 2nd ed., Basel, Birkhauser Verlag, 1960, 330 pp. Fr 35.

Proceedings of an International Symposium on High Temperature Technology, Asilomar Conference Grounds, Menlo Park, California, Oct. 6–9, 1959; New York, McGraw-Hill, Co., Inc., 1960, 348 pp. $15.

SHAMES, I. H., Engineering mechanics: Statics and dynamics (combined volumes 1 and 2), Englewood Cliffs, N. J., Prentice-Hall, Inc., 1960, xiv + 656 pp. $9.75.

SOROKIN, E. S., The theory of internal friction in the vibration of elastic systems [K teorii vnutrennego treniya pri kolebaniykh uprugikh sistem], Moskva, Gosudarstvennoe Izdatel'stvo Literatury po Stroitel'stvu, Arkhitekture i Stroitel'nym Materialam, 1960, 131 pp. 5 r 50 k. (Paperbound)

TIETJENS, O., Stromungslehre, Vol. 1: Hydro- und Aerostatik Bewegung der idealen Flussigkeit, Berlin, Springer-Verlag, 1960, xvi + 536 pp. DM 66.

WEBB, H. A., AND ASHWELL, D. E., Mathematical tool kit for engineers, 2nd ed., London, Longmans, Green and Co. Ltd., 1959, vii + 116 pp. 15 d.

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